

## Abstracts of Invited Lectures

# On refinable functions, subdivision and wavelets

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An introduction is given to the mathematical analysis of the related concepts of refinable functions, subdivision and wavelets. Iterative methods based on subdivision are widely applied in geometric modelling, whereas wavelet decomposition techniques represent a viable alternative to Fourier methods in signal analysis. Of fundamental importance in both subdivision and wavelet analysis is the issue of the existence of a solution, called a refinable function, of a certain refinement equation. We demonstrate the link between subdivision convergence and refinable function existence, and we show how a given refinable function can be used as a building block for the multi-resolutional method of wavelet construction. Also, we present an overview of refinable function results with respect to existence and uniqueness, regularity (or smoothness), as well as a variety of other properties.

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# An algebraic view of weaker forms of realcompactness

Themba Dube

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In their study of realcompactness in pointfree topology, Banaschewski and Gilmour [1] define a completely regular frame to be realcompact in case every  $\sigma$ -proper maximal ideal in the cozero part of the frame is completely proper.

In this talk we investigate the effects of requiring every  $\sigma$ -proper ideal in the Booleanization of an arbitrary frame to be completely proper. It turns out that the frame of open sets of a topological space has this property if and only if the space is almost realcompact in the sense of Frolik; namely: every open ultrafilter on the space that has empty adherence has a countable subfamily with empty adherence.

## References

- [1] B. Banaschewski and C. Gilmour, *Realcompactness and the cozero part of a frame*, Appl. Cat. Struct. **9** (2001), 395-417.
- [2] Z. Frolik, *A generalization of realcompact spaces*, Czechoslovak Math. J. **13** (1963), 127-138.

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## Recent developments in permutation decoding

Jennifer D. Key

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Recent advances in technology have produced a requirement for new implementations of good error-correcting codes. Such applications of codes also require efficient encoding and decoding methods.

The method of permutation decoding was first developed by Jessie MacWilliams in the early 60's and can be used when a code has a sufficiently large automorphism group to ensure the existence of a set of automorphisms, called a PD-set, that has some specific properties.

This talk will describe this method and give an update of recent developments in the search for good PD-sets. Background concepts in coding theory will be given.

# The Introduction of Mathematical Literacy to South African Education

Aarnout Brombacher

The introduction of the South African Qualifications Act and National Qualifications Framework has led to the requirement that all candidates for qualifications at levels 2, 3 and 4 must offer “16 credits from the subfield Mathematics.” In earlier versions of the policy this read “16 credits in Mathematical Literacy.” The requirement that all candidates offer these credits has seen the emergence of a subject known as Mathematical Literacy and, in particular, the requirement that all National Senior Certificate candidates must offer either Mathematics or Mathematical Literacy as one of their seven subjects. There exists a widely accepted notion that Mathematics, in the broadest sense of the word, is a prerequisite for ordinary citizens to participate meaningfully in the technologically driven twenty-first century world—a fundamental human right. While it is generally acknowledged that the mathematical needs of ordinary citizens and of professional mathematicians are different, there is as yet a poorly developed and generally agreed to understanding of what it means to be mathematically literate and hence of what should constitute a Mathematical Literacy curriculum and of how it should be taught. This paper explores this tension and the impact of that tension on the implementation of the policy.

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## Influence of Nutrition, Treatment and Beliefs on the Spread of the HIV/AIDS

E.M. Lungu

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We model the development of HIV/AIDS in a population consisting of two classes of individuals namely, those who are nutrient sufficient and those who are nutrient deficient. We consider a situation in which an individual must make a choice between

the type of treatment, that is, either ARV therapy or traditional remedies. We ask the question “how far can nutrition, type of treatment, and education on prevention influence the spread of HIV/AIDS in a population subjected to inequity access?” We have derived the model reproduction number,  $R_0$ , which has provided (i) insight into the transmission dynamics of the disease and (ii) estimates for the critical proportion of individuals,  $\epsilon^*$ , who must be exposed to education and the corresponding parameter,  $\psi^*$ , which describes the critical level of effectiveness of the education instruments that must be achieved in order to control the disease.

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## **Abstracts of contributed lectures**

These abstracts are arranged in alphabetical order.

# Refinement of covers by partitions: a new equivalent of the Axiom of Choice

Bernhard Banaschewski

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In a complete Boolean algebra  $M$ , a *cover* is a family  $(a_\alpha)_{\alpha \in I}$  of elements of  $M$  whose join (= supremum) is the top of  $M$ , and if  $a_\alpha \wedge a_\beta = 0$  whenever  $\alpha \neq \beta$  the cover is called a *partition*. Further, a family  $(b_\alpha)_{\alpha \in I}$  in  $M$  is said to *refine*  $(a_\alpha)_{\alpha \in I}$  if  $b_\alpha \leq a_\alpha$  for all  $\alpha \in I$ . It will be shown that, in Zermelo-Fraenkel Set Theory, the Axiom of Choice holds if and only if, in any complete Boolean algebra, every cover is refined by a partition (with P. Schuster).

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## Diffusion-kinetic equations with applications

Jacek Banasiak

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University of KwaZulu-Natal

Many systems in nature can be described by a deterministic equations modelling Markov type processes in which we describe the time evolution of the density  $u(t, x)$  of some quantity  $Q$ , where  $x \in \Omega$  and  $\Omega$  is a state space. Equations describing the evolution of  $u$  are typically constructed by balancing, for any state  $x$ , the loss of  $Q$  due to the transfer of a part of it to the other states, and the gain due to the transfer of parts of  $Q$  from other states to the state  $x$ . General form of such equations is:

$$\partial_t u = T_0 u + Nu + Bu, \tag{1}$$

where  $A$  is the loss, and  $N$  is the gain, operator and  $T_0$  describes transport in the state space (e.g. free streaming or diffusion). Such equations can be derived from first principles, like e.g. the space dependent fragmentation equation:

$$\begin{aligned} \partial_t u(x, \mathbf{r}, t) &= d(x) \Delta_{\mathbf{r}} u(x, \mathbf{r}, t) - a(x) u(x, \mathbf{r}, t) \\ &\quad + \int_x^\infty a(y) b(x|y) u(y, \mathbf{r}, t) dy. \end{aligned}$$

Here  $u$  is the density of particles of mass  $x$ ,  $a$  is the fragmentation rate,  $b$  describes the distribution of particle masses  $x$  spawned by the fragmentation of a particle of mass  $y$  and  $\mathbf{r}$  describes the position of a particle in the physical space  $\mathbb{R}^3$ . Similar equations appear also in asymptotic analysis of the interplay between various scattering mechanisms in the linear Boltzmann equation.

We shall discuss solvability of such equations and properties of the solutions related to their conservativeness.

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## Lattice Boltzmann and Relaxation Methods in Incompressible Flow

Mapundi Banda

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In the low Mach number limit of the lattice Boltzmann type models the incompressible Navier-Stokes equations are obtained. This is achieved by asymptotic analysis. Moreover in the course of this analysis, the Lattice Boltzmann Model reduces to a relaxation system which can be discretized using higher-order non-oscillatory upwind relaxation schemes. These approaches offer an alternative to the traditional solvers for incompressible Navier-Stokes problems based on the projection method. The implementation of both these approaches is simple and numerical computations are carried out on various test problems. Their performance on these test problems will be discussed.

# Emergent Universe in Starobinsky Model

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Sailo Mukherjee  
Physics Department  
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Bikash Paul  
Physics Department  
North Bengal University

Sunil Maharaj  
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We present an emergent universe scenario making use of a new solution of the Starobinsky model. The solution belongs to a one parameter family of solutions, where the parameter is determined by the number and the species (spin-values) of primordial fields. The general features of the model have also been studied.

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## On domination of inessential elements in ordered Banach algebras

Darren Behrendt

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University of Johannesburg

If  $A$  is an ordered Banach algebra ordered by an algebra cone  $C$ , then we reference the following problem as the 'domination problem': If  $0 < a < b$  and  $b$  has a certain property, then does  $a$  inherit this property? We extend the analysis of this problem in the setting of radical elements and introduce it for inessential, rank one and finite elements. We also introduce the class of  $r$ -inessential operators on Banach lattices



and prove that if  $S$  and  $T$  are operators on a Banach lattice  $E$  such that  $0 < S < T$  and  $T$  is  $r$ -inessential then  $S$  is also  $r$ -inessential.

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## Weak mixing over semigroups

Conrad Beyers, Rocco Duvenhage, Anton Stroh  
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Universiteit van Pretoria

With the dynamics given by the action of a semigroup, we define a dynamical system, a measure preserving dynamical system, and also weak mixing and ergodicity (relative to a net). Some characterizations of weak mixing is proved and it is shown that the definition of M-weak mixing relative to a space-filling net is independent of the space-filling net being used.

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## Linearized solutions of the Einstein equations for the Bondi-Sachs metric

Nigel Bishop

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We use the common technique of linearization and separation of variables to construct solutions of the Einstein vacuum equations, for the case when the zeroth order solution is Schwarzschild, and when the metric is written in Bondi-Sachs form.

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# Using diagnostic assessment for mathematics at Unisa

Carol Bohlmann

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Economies of scale usually suggest that admissions testing has no place within a distance learning context. However, at the University of South Africa diagnostic assessment, particularly with respect to reading comprehension and quantitative literacy, was found to play an important role in student performance in mathematics. The primary objective of the research described in this paper is to establish whether the introduction of diagnostic assessment for mathematics at UNISA is feasible, and whether students can be placed into meaningful categories according to potential risk. In the design of assessment tools, assessment procedures used elsewhere were taken into account; specific assessment tools were then selected. The main outcomes of the first phase of diagnostic testing (2004) suggested that such assessment is possible within a distance-learning environment, that initial categorisation procedures were reasonably accurate, and that specific aspects of the questions assessing quantitative literacy appeared to cluster well together. In addition, this phase of the testing led to a repeat of the process in 2005; it also led to the introduction of a diagnostic assignment which will be piloted on a different group of students in 2006. Further research into the predictive validity of different aspects of the tests should yield information that could be useful in test design and in informing aspects of student support. Ongoing debate as to what is sensible for mathematics in a South African context is critical. A discussion of this 'work-in-progress' will hopefully stimulate such debate.

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# On lattices of matric-extensible special radicals

Geoff Booth and Halina France-Jackson

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Nelson Mandela Metropolitan University

In this talk, we consider the lattice of matric-extensible radicals of associative rings, and characterise some of its atoms. This work extends work previously completed by the authors on the lattice of matric-extensible radicals.

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## Game Generalized Colourings of Graphs

Mieczyslaw Borowiecki

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University of Zielona Gora

We consider the two-players game defined as follows. The two players are Alice and Bob and they play alternatively with Alice having the first move. Given a graph  $G$  and a set  $C$  of colours, the players take turns colouring of  $G$  with colours from  $C$ . If after  $|V(G)|$  moves the graph is coloured then Alice wins, otherwise Bob wins, i.e., Bob wins whenever an impass is reached before the whole graph is coloured.

The game chromatic number of  $G$ , denoted by  $\chi_g(G)$ , is defined as the least cardinality of  $C$  for which Alice has a winning strategy. The number  $\chi_g(G)$  is well defined since we have  $\chi(G) \leq \chi_g(G) \leq |V(G)|$ .

In talk we are interested in some other versions of this game. We combine concepts: this game and generalized colouring of graphs.

A survey of results concerning the game colouring, new results and open problems on the game partitions of graphs will be presented.

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# On-line graph coloring with respect to hereditary properties

Piotr Borowiecki

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Among many generalizations of classical graph coloring problem we are especially interested in on-line  $\mathcal{P}$ -colorings where  $\mathcal{P}$  is some additive induced hereditary property of graphs. On-line  $\mathcal{P}$ -coloring turned out to be advantageous in analysis of various on-line resource management problems.

On-line coloring can be viewed as a game of two adversaries called *Presenter* and *Painter*. Painter (on-line algorithm) does not know the structure of a graph to be colored. Presenter reveals subsequent vertices of graph  $G$  in some order  $(v_1, \dots, v_n)$  which is unknown to Painter. Vertex  $v_i$  is presented together with edges  $E_i \subseteq E(G)$  to its already presented neighbors. When vertex  $v_i$  is presented, Painter has to irrevocably assign  $c(v_i)$  - one of the permissible colors and it has to be done before the next vertex is presented. The goal of Painter, to use as few colors as possible, is opposed to the strategy of Presenter which aims at finding the vertex ordering that forces Painter to use as much colors as possible.

One of the best known heuristics for on-line graph coloring is the First-Fit algorithm, which assigns to each vertex as small color as possible.

A family of greedy on-line  $\mathcal{P}$ -coloring algorithms is defined. We give some lower and upper bounds for on-line  $\mathcal{P}$ -chromatic number and prove that for some classes of graphs our algorithms are best possible. A finite basis theorem for the family of on-line First-Fit  $(\mathcal{P}, k)$ -colorable graphs is given.

# On a unification result of A.R. Sourour concerning commutators and products of involutions.

Johan Botha

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University of South Africa

In a paper by A.R. Sourour, a factorization theorem is presented that could be used to unify some matrix factorization results concerning commutators and products of involutions over an arbitrary field with sufficiently many elements (in particular, an infinite field). We hope to present similar results that are valid over an arbitrary field with at least four elements.

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# A symmetric presentation of the sporadic group $J_3$

John Bradley

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University of KwaZulu-Natal

We make use of the primitive permutation action of degree 120 of the automorphism group of  $L_2(16)$ , the special linear group of degree 2 over the field of order 16, to give an elementary definition of the automorphism group of the third Janko group  $J_3$  and to prove its existence. This definition enables us to construct the 6156-point graph which is preserved by  $J_3 : 2$ , to obtain the order of  $J_3$  and to prove its simplicity. A presentation for  $J_3 : 2$  follows from our definition, which also provides a concise notation for the elements of the group.

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# The Andre-Oort conjecture for Drinfeld modular varieties

Florian Breuer

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The André-Oort conjecture states that a subvariety of a complex Shimura variety contains a Zariski-dense set of special points if and only if it is a subvariety of Hodge type.

In this talk, we introduce a characteristic- $p$  analogue of this conjecture for subvarieties of Drinfeld modular varieties. We prove this conjecture in two special cases, most notably in the case of subcurves. As a consequence, the analogue of the André-Oort conjecture holds for Drinfeld modular surfaces.

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## Rank in Banach algebras

Rudi Brits

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Without the use of analytic multifunctions, we develop the theory of spectral rank in a general semisimple Banach algebra with nonzero socle.

# A note on localized directional weak lower semi-continuity

Erwin Brüning

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SAMS Subject Classification: 11 – PDE's & Dynamical Systems

In minimization problems for functionals  $f : M \rightarrow \mathbf{R}$ ,  $M \subseteq E$  a subset of some infinite dimensional Banach space  $E$ , we typically have to rely on weak (sequential) lower semi-continuity of  $f$  on the whole space  $E$  even if  $M$  is a proper subset of  $E$ . The main reason for this lack of ‘localized’ weak lower semi-continuity seems to be that it is not known how to get and/or to characterize weak sequential lower semi-continuity on a subset  $M$  without knowing it on the whole space. As a first step to overcome this difficulty we propose the concept of ‘localized directional weak sequential lower semi-continuity’ and offer a way to implement it, namely in terms of conditions on the Gateaux derivative  $f'$  of  $f$  (weak  $K$ -monotonicity). This allows to formulate a criterium and new sufficient conditions for the existence of a minimizer.

We conclude with a discussion of applications to the variational approach to the solution of (systems of) nonlinear partial differential equations where we focus on the case of integral functionals of vector fields for which the integrand is not assumed to be quasi-convex.

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## Enumeration of Optimal Lottery Sets of Cardinality Three

Alewyn Burger

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University of Stellenbosch

In this talk it is demonstrated how the notion of a set contraction operation (developed in the first talk) may be applied to determine the number of non-isomorphic optimal (unlabelled) lottery set structures of cardinality three analytically.

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# On non-conservative solution of a Boltzmann-like semiconductor model

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SAMS Subject Classification: 9, 11, 13.

Evolution of a low-density gas of charged particles interacting with a fixed background (e.g. electrons in a crystalline lattice of a semiconductor) can be modeled by a one-particle distribution function that satisfies the linear Maxwell-Boltzmann equation. Well-posed and other properties of this model can be studied using tools of functional analysis and, in particular, the theory of positive semigroups. It turns out, however, that for some classes of coefficients the solution lacks some important physical properties, e.g. it fails to conserve the total number of particles in the system.

In this talk, we start with the birth-and-death system of equations for which we present necessary and sufficient conditions for conservativity of solutions. Furthermore we analyze the linear Boltzmann equation of semiconductor theory with unbounded collision term modelling both elastic and inelastic scattering of electrons on the crystalline lattice and present an example of a cross-section growing at an exponential rate for which the semigroup is not conservative. The construction of a non-conservative solution here is based on the birth-and-death model.



# The Soberification of Sigma Spaces

Jumani Clarke

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In classical topology, there is a notion of sober space and the corresponding reflection given by soberification. We define an analogous notion for sigma-spaces (where the "opens" are only required to be closed for the taking of countable unions) and see that, as in the classical case, soberification preserves products for sigma spaces. A special case of this was already observed by Hugh Gordon, in showing that a product of the Hewitt realcompactifications of regular sigma spaces (= zero set spaces) is isomorphic to the Hewitt realcompactification of the product.

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## Normal, sigma-normal and singular functionals

Jurie Conradie

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University of Cape Town

A bounded linear functionals on von Neumann algebras can be written as a sum of a normal and a singular part. We look at the possibility of finding explicit expressions for these parts. In the process we come up against the difference between normal and sigma-normal functionals and non-measurable cardinals.

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# A description of norm-convergent martingales on vector-valued $L^p$ -spaces

Stuart F. Cullender  
Coenraad C. A. Labuschagne

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University of the Witwatersrand

Norm-convergent martingales on tensor products of Banach spaces are considered in a measure-free setting. As a consequence, we obtain the following characterization for convergent martingales on vector-valued  $L^p$ -spaces: Let  $(\Omega, \Sigma, \mu)$  be a probability space,  $X$  a Banach space and  $(\Sigma_n)$  an increasing sequence of sub  $\sigma$ -algebras of  $\Sigma$ . In order for  $(f_n, \Sigma_n)_{n=1}^\infty$  to be a convergent martingale in  $L^p(\mu, X)$  ( $1 \leq p < \infty$ ) it is necessary and sufficient that, for each  $i \in \mathbb{N}$ , there exist a convergent martingale  $(x_i^{(n)}, \Sigma_n)_{n=1}^\infty$  in  $L^p(\mu)$  and  $y_i \in X$  such that, for each  $n \in \mathbb{N}$ , we have

$$f_n(s) = \sum_{i=1}^{\infty} x_i^{(n)}(s)y_i \quad \text{for all } s \in \Omega,$$

where  $\left\| \sum_{i=1}^{\infty} \left| \lim_{n \rightarrow \infty} x_i^{(n)} \right| \right\|_{L^p(\mu)} < \infty$  and  $\lim_{i \rightarrow \infty} \|y_i\| \rightarrow 0$ .

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## Dirichlet-Neumann Bracketing for boundary value problems on Graphs

Sonja Currie and Bruce Watson

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University of Witwatersrand

We consider the spectral structure of second order boundary value problems on graphs. A variational formulation for boundary value problems on graphs is given. As a consequence we can formulate an analogue of Dirichlet-Neumann bracketing

for boundary value problems on graphs. This in turn gives rise to eigenvalue and eigenfunction asymptotic approximations.

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## The Conway group and the Leech lattice revisited - a recent application of symmetric generation.

Robert Curtis

Department of Mathematics  
University of Birmingham

During the last decade or so the techniques of symmetric generation of groups have been used by the author and several of his collaborators to define and construct a number of the fascinating sporadic simple groups, often in a remarkably straightforward and revealing manner. Besides the group itself, the combinatorial object on which it most naturally acts and through which it is most usefully studied, often appears as a by-product.

The Leech lattice was discovered in the 1960s by John Leech in connection with efficient packing of spheres in 24-dimensional real space. John Conway found that the group of orthogonal transformations of 24-dimensional space which preserves the lattice is the double cover of a sporadic simple group.

In this talk the process is turned around: the group is defined in terms of the Mathieu group  $M_{24}$  and it is then used to construct the lattice. The procedure is highly combinatorial in flavour and the techniques used are elementary but highly effective.

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# Complexity and Turing Reducibility

George Davie

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University of South Africa

For finite “c.e.” strings, complexity and Turing reducibility are almost the same. We discuss this.

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## Minimum Degree of Graphs with Small Domination Number

Peter Dankelmann

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Let  $G = (V, E)$  be a graph. A dominating set of  $G$  is a subset of the vertex set with the property that every vertex of  $G$  is either in the set or adjacent to a vertex in the set. The domination number of  $G$ ,  $\gamma(G)$ , is the minimum cardinality of a dominating set of  $G$ . Several bounds on the the domination number of  $G$  in terms of its order and minimum degree  $\delta$  are known, for example

$$\gamma(G) \leq \frac{1}{2}|V| \quad \text{if } \delta \geq 2,$$

and

$$\gamma(G) \leq \frac{2}{5}|V| \quad \text{if } \delta \geq 3,$$

which holds for all but seven exceptional graphs.

In our talk we investigate the relation between order, minimum degree and domination number from a different perspective and consider the question: Given the order and domination number of a graph, how large can its minimum degree be?

This is joint work with Henda C. Swart and David Day.

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# On refinement mask symbols and their factorisation

Deter de Wet

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Refinement equations, which are fundamental to wavelet analysis and subdivision schemes, have been extensively studied over the last two decades. Many results have been obtained by means of the Fourier transform, often leading to fairly involved constructions. Use of the mask symbol and its factorisation often present a way of stating (or proving) results on refinement equations in a more direct manner. We briefly show some applications of this idea to the questions of convergence of subdivision and the regularity of the refinable function.

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## Discontinuous Galerkin Method for variational inequalities

Jules Djoko Kamdem and Daya Reddy

Department of Pure and Applied Mathematics  
University of Cape Town

We present the error analysis for the elliptic variational inequality of the first kind and second kind based on a Discontinuous Galerkin space approximation. For both problems discussed spatial rates in mesh dependent as well as  $L^2$ -norm errors are suboptimal.

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# On refinable function existence and the cascade algorithm

Guy Blaise Dongmo

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University of Stellenbosch

We consider the problem of proving the existence of a finitely-supported continuous solution to the refinement equation

$$\phi = T_a\phi := \sum_{j \in \mathbb{Z}} a_j \phi(2 \cdot -j), \quad (2)$$

where a finite number of the coefficients  $a_j$  are non-zero, and where

$\sum_j a_{2j} = \sum_j a_{2j+1} = 1$ . It is well known that in case of positive masks, i.e.  $a_j > 0$ , for all non-zero  $a_j$ , the cascade algorithm  $\phi_{r+1} = T_a\phi_r$ ,  $r = 0, 1, \dots$ , with  $\phi_0$  appropriately chosen, converges to a solution  $\phi$  of (2), called a refinable function.

For general masks, i.e. including also those where some  $a_j < 0$ , Fourier transform techniques have generally been used to investigate the existence of a solution to equation (2).

In this talk, we consider the use of the cascade algorithm to provide a constructive existence proof for a solution  $\phi$  of the refinement equation (2) in the setting of general masks.

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## On joins of additive hereditary graph properties

Ewa Drgas-Burchardt

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University of Zielona Gora

Let  $\mathcal{L}^a$  denote a set of additive hereditary graph properties. It is a known fact that a partially ordered set  $(\mathcal{L}^a, \subseteq)$  is a complete distributive lattice. We decide when a

join of additive hereditary graph properties in  $(\mathcal{L}^a, \subseteq)$  has a finite or infinite family of forbidden subgraphs. Moreover we show that the analysed class of graph properties is disjoint with the set of reducible properties. That means each consequence of the document is not included in the only known result of this type obtained by A.Berger and stated that for any additive hereditary reducible property of graphs the family of forbidden subgraphs is infinite.

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## Weak mixing of all orders

Rocco Duvenhage, Conrad Beyers, and Anton Ströh

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University of Pretoria

We consider a weakly mixing measure preserving dynamical system, where the dynamics is given by the action of an abelian second countable topological group with invariant measure. It is then shown that this system is weakly mixing of all orders.

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# $\gamma$ -labelings of graphs

David Erwin

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Gary Chartrand and Ping Zhang

Department of Mathematics

Western Michigan

Donald Vanderjagt

Grand Valley State

For a connected graph  $G$  of order  $n$  and size  $m$ , a  $\gamma$ -labeling of  $G$  is a one-to-one function  $f : V(G) \rightarrow \{0, 1, 2, \dots, m\}$  that induces a labeling  $f' : E(G) \rightarrow \{1, 2, \dots, m\}$  of the edges of  $G$  defined by  $f'(uv) = |f(u) - f(v)|$ . Hence, a graceful labeling is a  $\gamma$ -labeling for which  $f'$  is one-to-one. For a  $\gamma$ -labeling  $f$ , define  $\text{val}(f) = \sum_{e \in E(G)} f'(e)$ , and, for a graph  $G$ ,  $\text{val}_{\min}(G) = \min\{\text{val}(f)\}$  and  $\text{val}_{\max}(G) = \max\{\text{val}(f)\}$ . We discuss  $\text{val}_{\min}$  and  $\text{val}_{\max}$  for several classes of graphs  $G$ .

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## A Theoretical Model of the formation of Cerebral Fluid in the brain

Sunday Faleye

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We present a theoretical model of the production of cerebrospinal Fluid. Cerebrospinal Fluid is formed by the filtration of blood through the fenestration of the choroidal capillaries within the ventricular and fill the subarachnoid space. A system of Intracranial capillaries produces the cerebrospinal Fluid by pressing blood through the pia matter and an epithelial membrane into the cerebrospinal Fluid compartment. One may say that the cerebrospinal Fluid is blood without haemoglobin. Production



of cerebrospinal Fluid is therefore, reduced to the filtration of Newtonian fluid from the Intracranial capillaries into the subarachnoid space. The permeability is modelled by a special boundary condition to describe the flow through the boundaries.

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## A Multistep Collocation Method Based On Exponential Basis Functions for Stiff IVPs.

Johnson Fatokun

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Peter Onumanyi

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In this paper, we consider a multistep collocation for the derivation of methods for the numerical solution of stiff initial value problems for differential systems. We attempt to fit some multistep collocation methods with a special exponential basis function to gain stable and accurate schemes for stiff systems. Onumanyi et al. [7,8] have dealt with non-stiff systems of ordinary differential equations Odes using the idea of multistep collocation (MC) proposed by Lie and Norset [5]. This paper has two aims. The first is to present a simplified analysis of the derivation of the MC methods. The second is to identify some methods, which can solve stiff initial value problem (IVP) for ordinary differential equations adequately.

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## Continuous Finite Difference Methods for Solving Systems of First Order ode's with very large Lipschitz Constants.

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This paper presents a computational procedure for obtaining the solution of the system of first order ordinary differential equations with very large Lipschitz constants. Such systems involving very large stiffness/oscillatory parameters are inclusive. The idea of continuous finite difference equations (interpolant) (Onumanyi et al. (1994, 1999) based on interpolation and collocation is extended to arbitrary basis vector function. An application using a simple case to illustrate the effect of the basis function is given by the trapezoidal method. The conventional trapezoidal method is obtained by the power basis function that yields order two accuracy and is not L-stable for highly stiff i.v.p.s. But the use of the exponential basis function leads to order three accuracy and L-stable method. Two numerical examples are used to demonstrate the theoretical results obtained.

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## $(p, q)$ -Integral functions and $(p, q)$ -summing sequences.

Jan Fourie

School of Computer, Statistical and Mathematical Sciences  
North-West University

Multiplication functions and multiplication sequences of operators appear naturally in the study of  $p$ -summing operators. We discuss a general framework in which the properties of so called  $(p, q)$ -integral functions and  $(p, q)$ -summing sequences of operators are applied to study (geometrical) properties on Banach spaces. Several natural examples of  $(p, q)$ -integral functions and  $(p, q)$ -summing sequences will be discussed and their involvement in recent results about Rademacher boundedness and the Grothendieck Theorem, will be showed.

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# On coatoms of the lattice of matric-extensible radicals

Halina France-Jackson

Department of Mathematics and Applied Mathematics  
Nelson Mandela Metropolitan University

A radical  $\alpha$  in the universal class of all associative rings is called matric-extensible if for all natural numbers  $n$  and all rings  $A$ ,  $A$  is in  $\alpha$  if and only if the  $n \times n$  matrix ring with entries from  $A$  is in  $\alpha$ . We show that there are no coatoms, that is, maximal elements in the lattice of all matric-extensible radicals of associative rings.

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# New Perspectives on the Path Partition Conjecture

Marietjie Frick\*, Carol Whitehead and Ingo Schiermeyer

Department of Mathematical Sciences  
University of South Africa

The *order* of a graph  $G$  is the number of vertices in  $G$  and the *detour order* of  $G$ , denoted by  $\tau(G)$ , is the order of a longest path in  $G$ . The difference between the order and the detour order of  $G$  is called the *detour deficiency* of  $G$  and is denoted by  $p(G)$ . A graph with detour deficiency  $p$  is called *p-deficient*. A partition  $(A, B)$  of  $V(G)$  such that  $\tau(\langle A \rangle) \leq a$  and  $\tau(\langle B \rangle) \leq b$  is called an  $(a, b)$ -partition of  $G$ . We investigate the *path partition function*  $f : \mathbb{Z}^+ \cup \{0\} \rightarrow \mathbb{Z}$ , defined by:  $f(p)$  is the greatest integer for which every connected  $p$ -deficient graph  $G$  has an  $(a, b)$ -partition for every pair  $a, b$  such that  $a + b = \tau(G) - f(p)$ . The Path Partition Conjecture (which was formulated in 1981 and has not yet been settled) is that  $f(p) \geq 0$  for all  $p \geq 0$ . We show that  $-p \leq f(p) \leq 1$  for all  $p \geq 0$ . In particular,  $f(0) = 0$ ,  $f(1) = f(2) = 1$  and  $f(3)$  is either 0 or 1.

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## Primeness in zero square near-rings

Lungisile Godloza

Department of Mathematics  
Walter Sisulu University for Technology and Science

In this talk we show that a dg near-ring cannot be 0-prime. We also show that a commutative near-ring cannot even have a proper 0-prime ideal. These results extend to near-rings, a theorem of Levitzki which states that a zero square ring cannot be prime. Our talk gives a complete answer to a question posed by Howard Bell in the International Conference on near-rings and Nearfields held in Hamburg, Germany in 2003. The question arose in the context of derivations in near-rings. It seeks an answer to the question of existence of a 3-prime near-ring  $R$  with the property that the square of each element of  $R$  is zero. In this talk we go far beyond answering that question and seek to obtain some structural theorems for zero square near-rings.

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# Hidden Symmetries for PDEs

Kesh Govinder

School of Mathematical Sciences  
University of KwaZulu-Natal

Barbara Abraham-Shrauner

Department of Electrical and Systems Engineering  
Washington University

The provenance of Type II hidden symmetries of differential equations reduced from nonlinear partial differential equations is analyzed. The hidden symmetries are extra symmetries in addition to the inherited symmetries of the differential equations when the number of independent and dependent variables is reduced by a Lie point symmetry. These Type II hidden symmetries do not arise from contact symmetries or nonlocal symmetries as in the case of ordinary differential equations. The Lie point symmetries of a model equation and the two-dimensional Burgers equation and their descendants are used to identify the hidden symmetries. The significant new result is the provenance of the Type II Lie point hidden symmetries found for differential equations reduced from partial differential equations.

# Counting finite order automorphisms of the $p$ -adic disc

Barry Green

Department of Mathematics  
University of Stellenbosch

In this talk we report on recent results on the fixed point geometry of order  $p$  automorphisms of the  $p$ -adic open disc, the associated semistable models as well as work in progress on those of higher  $p$ -power order. In particular we discuss how these results determine the number of such automorphisms up to change of parameter in special cases. We also draw attention to recent work on the lifting conjecture for galois covers of smooth curves from characteristic  $p$  to characteristic 0.

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## Necessary and sufficient conditions for the closedness of the sum or marginal spaces

Jacobus J Grobler

Unit for Business Mathematics and Informatics  
North-West University

In the talk conditions mentioned in the title will be discussed for marginal function spaces. The problem's origin comes from the problem of estimating the probability measure if the marginal measures are known.

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# Realization theorems for formal power series satisfying a von Neumann inequality

Gilbert Groenewald

Department of Mathematics  
North-West University

The operator-valued Schur class consists of functions analytic on the unit disc with values equal to contraction operators between two Hilbert spaces. Equivalently, the operator-valued function  $S(z)$  belongs to the Schur class if and only if  $[I - S(z)S(w)^*]/(1 - z\bar{w})$  is a positive kernel, or equivalently, if and only if  $S(z)$  can be realized as  $S(z) = D + zC(I - zA)^{-1}B$  where  $U = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$  is a unitary operator. We discuss a generalized Schur-class where  $S(z)$  becomes a formal power series in noncommuting indeterminates  $z = (z_1, \dots, z_d)$  with operator coefficients. Such formal power series appear in the theory of automata and formal languages, robust control and the search for Linear-Matrix-Inequality representations for quadratic functions of noncommuting formal variables in control theory applications. This is joint work with Joseph Ball (Virginia Tech, USA) and Tanit Malakorn (Naresuan University, Thailand).

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## Designing an online mathematics 1 course

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Durban Institute of Technology

Department of Mathematics  
Durban Institute of Technology

Students studying at a University of Technology encounter many difficulties with regard to mathematics. Misconceptions created at secondary school and the perception that mathematics is an ancillary subject creates a major problem in the teaching and

learning of mathematics. Exacerbating the problems are high failure rate, spread of course content, limited time for lecture support, timetabling, and venues allocations led to the development alternative teaching and learning programmes. Furthermore, research has demonstrated that conventional lecturing has failed. A method mooted to ease the teaching and learning burden for technology students is the WBL. Design and implementation of a WBL followed the Neuron and Cognitivists models of teaching and learning. Specifically the design of the WBL concentrated on modifying the mathematical thinking of the students from a study of existing errors performed by R.Naidoo et al (SAARMSE, 1996).

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## Total domination in graphs with minimum degree at least three

Michael Henning

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University of KwaZulu-Natal

It is shown in several manuscripts (Small transversals in hypergraphs. *Combinatorica* 12 (1992), 19–26 and Some remarks on domination. *J. Graph Theory* 46 (2004), 207–210) that if  $G$  is a graph with minimum degree at least three, then the total domination of  $G$  is at most one-half its order. In this talk, we use transversals in hypergraphs to characterize the connected graphs with minimum degree at least three and with total domination number exactly one-half their order. In order to do this we characterize all 3-uniform hypergraphs of order  $n$  and size  $m$ , which do not have transversals of size less than  $(n + m)/4$ .

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# Approach structures and measures of connectedness

David Holgate

Department of Mathematics  
University of Stellenbosch

If  $X$  and  $Y$  are connected and totally disconnected spaces respectively, then any continuous map  $f : X \rightarrow Y$  is constant. This observation leads to the natural definition of a “left/right-constant” relation between classes of spaces and a general study of “connectedness” versus “disconnectedness”.

In this talk we introduce the notion of a *measure of connectedness* (or disconnectedness) motivated by the above considerations, where a measure is simply taken to be a function  $\mu : \mathbf{X} \rightarrow [0, \infty]$  on a category  $\mathbf{X}$ .

Analysing such measures we see that they are essentially “approach notions” in the sense of the approach spaces of Lowen ([1]).

## Reference

[1] R. Lowen. *Approach Spaces: The Missing Link in the Topology-Uniformity-Metric Triad*. Oxford University Press, 1997.

\* Joint work with Mark Sioen, Vrije Universiteit Brussel.

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## “Mathematical Work” in the teaching of mathematics: Teachers’ Voice

Prince Senyukelo Jaca

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Walter Sisulu University for Technology and Science

This paper reports work in progress. This paper attempts to understand mathematics teaching through a construct called “Mathematical work”. “Mathematical work”

refers to the those explanations, definitions, analogies etc, the analyses of learners work etc. It moves from the premises that there is something inherently “mathematical” in the teaching of mathematics. Response of three teachers under Dinaledi initiatives are analysed and argument is put forward in support of study projects that are based on grounded theory in general, and “mathematical work” in particular.

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## Integral representations and asymptotic zero distribution of some hypergeometric polynomials

Sarah Jane Johnston and Kathy Driver

School of Mathematics  
University of Witwatersrand

The Euler integral representation of the  ${}_2F_1$  Gauss hypergeometric function is well known and plays a prominent role in the derivation of transformation identities and in the evaluation of  ${}_2F_1(a, b; c; 1)$ , among other applications. The general  ${}_{p+k}F_{q+k}$  hypergeometric function has an integral representation where the integrand involves  ${}_pF_q$ . We give a simple Euler integral representation for a special class of  ${}_{q+1}F_q$  functions for  $q \geq 2$ . We then obtain an asymptotic expansion for the Euler integral representation of some  ${}_3F_2$  hypergeometric polynomials which leads us to our main result that, asymptotically, the zeros of these  ${}_3F_2$  hypergeometric polynomials cluster on the loops of specified lemniscates.

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## The $L(d, 1)$ -hole index of paths and cycles

E. Jonck\*, C.J.Ras

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University of Johannesburg

J.H.Hattingh  
Department of Mathematics and Statistics  
Georgia State University

An  $L(j, k)$ -labeling of a graph  $G$ , where  $j \geq k$ , is defined as a function  $f : V(G) \rightarrow \mathbf{Z}^+ \cup \{0\}$  such that if  $u$  and  $v$  are adjacent vertices in  $G$ , then  $|f(u) - f(v)| \geq j$ , while if  $u$  and  $v$  are vertices such that  $d(u, v) = 2$ , then  $|f(u) - f(v)| \geq k$ . The largest label used by  $f$  is the span of  $f$ , denoted  $\text{span}(f)$ . The smallest span among all  $L(j, k)$ -labelings of  $G$ , denoted  $\lambda_{j,k}(G)$ , is called the span of  $G$ . An  $L(j, k)$ -labeling of  $G$  that has a span of  $\lambda_{j,k}(G)$  is called a span labeling of  $G$ . We say that a span labeling  $f$  has  $\ell$  holes if the set  $\{i \mid f^{-1}(\{i\}) = \emptyset, \text{ where } 1 \leq i \leq \text{span}(f) - 1\}$  has cardinality  $\ell$ . The hole index of  $G$ , denoted  $\rho_{j,k}(G)$ , is defined as the minimum number of holes over all span  $L(j, k)$ -labelings of  $G$ . We determine the hole index of paths and cycles for the case when  $k = 1$ .

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## Spouse-Avoiding Mixed Doubles Tennis Tournaments

Rickus Jooste and JH van Vuuren

Department of Applied Mathematics  
University of Stellenbosch

A spouse-avoiding mixed doubles tennis tournament (*samdt*) is a tennis tournament for married where husband and wife are not allowed to be partnered with or against each other.

In this session it is shown that such a tournament of order  $n$  exists if and only if a self-orthogonal latin square  $L$  of order  $n$  exists.

Furthermore, a symmetric latin square  $S$  of order  $n$  which is orthogonal to  $L$ , may be used to schedule the matches of such a tournament into the minimum number of rounds.

However, the construction methods for the combinatorial pair  $(L, S)$ , presented in this session, are only applicable if the order of  $L$  and  $S$  is a prime power or coprime to 6. Methods are presented by which such a *samdt* may be constructed but which is sub-optimal.

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# Separation theorems for the zeros of certain hypergeometric polynomials

Kerstin Jordaan and Kathy Driver

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University of Witwatersrand

This talk is based on the paper (cf. [1]) by Driver and Jordaan. We discuss interlacing properties of the zeros of two contiguous  $2F_1$  hypergeometric polynomials. We use the connection between hypergeometric  $2F_1$  and Jacobi polynomials, as well as a monotonicity property of zeros of orthogonal polynomials due to A Markoff (cf. [2]), to prove that the zeros of contiguous hypergeometric polynomials separate each other. We also discuss interlacing results for the zeros of  $2F_1$  and those of the polynomial obtained by shifting one of the parameters of  $2F_1$  by  $t$  or  $-t$  where  $0 < t < 1$ .

[1] K. Driver and K. Jordaan, "Separation theorems for the zeros of certain hypergeometric polynomials", *Journal of Computational and Applied Mathematics*, to appear.

[2] G. Szego, *Orthogonal Polynomials*, (American Mathematical Society, New York, 1959).

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## On Total Restrained Domination in Trees

J.H.Hattingh, E. Jonck, E.J.Joubert\*

Department of Mathematics and Statistics  
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Department of Mathematics and Statistics  
Georgia State University

Let  $G = (V, E)$  be a graph. A total restrained dominating set is a set  $S \subseteq V$  where every vertex in  $V - S$  is adjacent to another vertex in  $S$  as well as to another vertex in  $V - S$ , and every vertex in  $S$  is adjacent to another vertex in  $S$ . The total restrained

domination number of  $G$ , denoted by  $\gamma_{tr}(G)$ , is the smallest cardinality of a total restrained dominating set of  $G$ . We determine the lower bound of the total restrained domination number for trees and characterize those trees attaining this bound. Then we consider trees with  $n \equiv 0 \pmod{4}$  and again determine the lower bound of the total restrained domination number and characterize those trees attaining this bound.

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## Information sets for generalized Reed-Muller codes and bases of minimum-weight vectors

J D Key

Department of Mathematics  
Clemson University

Knowledge of which sets of coordinate positions can be used as information sets for linear codes is essential for some decoding methods. In this talk a particularly simple way to get information sets for the generalized Reed-Muller codes will be described, along with a consequence involving finding bases of minimum-weight vectors for some of the codes.

This is joint work with T. P. McDonough and V. C. Mavron of the University of Wales, Aberystwyth.

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## Lax Proper Maps of Locales

Mareli Korostenski and Coenraad Labuschagne

School of Mathematics  
University of Witwatersrand

We give a proof of localic Priestley duality. Our approach is based on lax proper maps of locales, which provide a vehicle for presenting the Priestley version of full Stone duality constructively and preserve spacial intuitions.

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# The transformation of the even parity perturbations of Shwarzschild black hole in terms of Chandrasekhars radial function to Bondi-Sachs form, and the calculation of the hypersurface, evolution, and the constraints equations.

Amos Kubeka

Department of Mathematical Sciences  
University of South Africa

We begin with the definition of the odd and the even parity perturbations and then show how they are related, by doing that we will be showing that they describe the same physical phenomenon and that the only difference between them is their parities. Then we extend the results of Chandrasekhar on the linear perturbations of Schwarzschild Black Hole to numerical relativity. We do that by transforming the perturbed Schwarzschild metric to Bond-Sachs formalism and then compare the results obtained with that obtained by Bishop (2005). The hypersurface equations, evolution equations, and the constraints equations are presented. The form of these equations were derived by Nigel (2005).

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## Infima of quasi-uniform anti-atoms

Hans-Peter A. Kunzi and Eliza P. de Jager

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University of Cape Town

It is well known that each filter on a set is the intersection of the family of ultrafilters containing it.

On the other hand it has been shown that there are uniformities on an infinite set  $X$  that cannot be written as the infimum of a family of anti-atoms in the complete lattice of uniformities on  $X$ .

In this article we study the corresponding question in the complete lattice  $(q(X), \subseteq)$  of all quasi-uniformities on a given set  $X$ . In recent years the theory of quasi-uniformities has found numerous interesting applications in topological algebra, functional analysis and the theory of hyper and function spaces.

While the general question remains open, we obtain the following positive partial results:

Each quasi-uniformity on a set  $X$  that is proximally fine, totally bounded or has a linearly ordered base is the infimum of a family of anti-atoms in  $(q(X), \subseteq)$ .

Moreover, each quasi-uniformity  $\mathcal{U}$  on  $X$  such that the topology of the associated supremum uniformity  $\mathcal{U} \vee \mathcal{U}^{-1}$  is resolvable also has the latter property. (A topological space is called resolvable if it has two disjoint dense subsets.)

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## On the min and max norms on tensor products of $C^*$ -algebras

Coenraad Labuschagne

School of Mathematics  
University of Witwatersrand

The usual descriptions of both the min and max norms on tensor products of  $C^*$ -algebras rely on the Gelfand-Naimark representation theorem for  $C^*$ -algebras.

We consider properties of these norms . In particular, we give a description of the max norm which is free of the Gelfand-Naimark theorem.

(This research was funded by the John Knopfmacher Centre for Applicable Analysis and Number Theory)

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# Automorphic forms, Langlands' programme and trace formulas

Laurent Lafforgue

Institut des Hautes Études Scientifiques, Bures-sur-Yvette

Langlands' programme is certainly at the heart of many parts of present day mathematics: number theory, group theory, harmonic analysis, arithmetic algebraic geometry. The purpose of this talk is to give an idea of what Langlands' programme is all about and what are the available techniques. The special importance of Langlands' spectral decomposition theorem will be stressed, as well as the fascinating simplicity of the statement of the functoriality principle. It will be made clear that new ideas are needed...

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## From Numerical Relativity to Critical Phenomena

Kevin Lai

Department of Mathematical Sciences  
University of South Africa

Numerical relativity has been a subject of intense interest among relativists for more than 40 years. It becomes an important technique which enable researchers to investigate systems which are extremely difficult to handle analytically. In this talk I will give two main reasons why we need numerical relativity: the first being able to simulate systems closely resemble astrophysical relevant scenarios, and provide results which can be compared with those obtained from gravitational wave detectors. Another reason is that it allows us to explore the solution space of the theory and may reveal unexpected phenomena. I will discuss critical phenomena as an outstanding example, and what implication it may have on the cosmic censorship conjecture.

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# Spatial aspects of species richness: location, scale and the possibility of location-invariant species-area relations

Henri Laurie

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University of Cape Town

Classical ecological theory says that species richness  $N$  is a function of area  $A$ , in its most influential form expressed as  $N = cA^z$ . However, this leaves open the selection of  $\Omega \subset \mathbb{R}^2$  such that  $\text{area}\Omega = A$ . Protea species richness data reveal that the parameters  $c$  and  $z$  as estimated by least-squares fits from several  $\Omega_i$  (for  $i = 1, 2, \dots, k$  for some small  $k$ ) strongly depends on how the  $\Omega_i$  are chosen. I show that this phenomenon is unavoidable and discuss possible modifications of the classical approach.

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## Contact Structures with Basic Potentials

G. Lubczonok

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Rhodes University

A *contact manifold*  $M$  of dimension  $2n + 1$  admits a *contact structure* which is a global differential 1-form  $\alpha$  such that

$$\Omega = \alpha \wedge (d\alpha)^n \neq 0$$

everywhere. We define the *basic potential* as a 1-form  $\beta$  such that

$$X \lrcorner \beta = 0, \quad \mathfrak{L}_X \beta = 0, \quad d\beta = d\alpha,$$

where  $X$  is the *characteristic field*. Contact manifolds that admit a basic potential are shown to have additional foliated structure of co-dimension one. The properties of the new foliation and its relation to the characteristic field  $X$  are explored.

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# Influence of Nutrition, Treatment and Beliefs on the Spread of the HIV/AIDS

E.M. Lungu

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University of Botswana

We model the development of HIV/AIDS in a population consisting of two classes of individuals namely, those who are nutrient sufficient and those who are nutrient deficient. We consider a situation in which an individual must make a choice between the type of treatment, that is, either ARV therapy or traditional remedies. We ask the question “how far can nutrition, type of treatment, and education on prevention influence the spread of HIV/AIDS in a population subjected to inequity access?” We have derived the model reproduction number,  $R_0$ , which has provided (i) insight into the transmission dynamics of the disease and (ii) estimates for the critical proportion of individuals,  $\epsilon^*$ , who must be exposed to education and the corresponding parameter,  $\psi^*$ , which describes the critical level of effectiveness of the education instruments that must be achieved in order to control the disease.

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## Embeddings in relativity

S D Maharaj

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University of KwaZulu-Natal

We motivate the reasons for studying the embedding of a 4-dimensional manifold into higher dimensional manifolds in relativity. Some recent results of embedding in this context are reviewed. In particular we consider the governing equations that arise in the embedding of a 4-dimensional Riemannian manifold into a 5-dimensional Einstein space. Explicit examples of relativistic spacetimes embeddable in 5-dimensional Einstein space are constructed which illustrates the general method.

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# On steady flow of a reactive variable viscosity fluid in a channel with isothermal wall

Oluwole D. Makinde

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University of Limpopo

In petrochemical industries as well as petroleum refineries, the study of thermal ignition in a combustible reacting variable viscosity fluid is of great important in order to ensure safety of life and properties ([1]). This paper studies the steady flow of a reactive variable viscosity fluid in a channel with isothermal wall. It is assumed that the reaction is strongly exothermic under Arrhenius kinetics, neglecting the consumption of the material. The dimensionless nonlinear governing boundary-value equation together with the corresponding boundary conditions is

$$\frac{dW}{dy} = -ye^{\left(\frac{\theta}{1+\varepsilon\theta}\right)}, \quad \frac{d^2\theta}{dy^2} \lambda(1 + \beta y^2) ye^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} = 0, \quad \frac{d\theta}{dy}(0) = 0, \quad \theta(1) = W(1) = 0,$$

where  $\lambda$ ,  $\varepsilon$ ,  $\beta$  represent the Frank-Kamenetskii parameter, activation energy parameter and the viscous heating parameter respectively. Analytical solutions are constructed using regular perturbation technique (RPT) coupled with computer-extended series solution (CESS) and a special type of Hermite-Padé approximants [2, 3, 4]. The important properties of the overall flow structure including velocity field, temperature field, bifurcations and thermal ignition criticality are discussed. Keywords: Poiseuille flow, Thermal ignition, Arrhenius kinetics, Hermite-Padé approximants.

References:

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# THOMAS MUIR IN THE EASTERN CAPE AND KAROO

Pieter Maritz

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University of Stellenbosch

In 1892, Thomas Muir, mathematician and educator, became the third Superintendent-General of Education (SGE) of the Cape Colony under British rule. He will be remembered as one of the greatest organisers and reformers in the history of Cape education. As mathematician, his magnum opus was a five-volume work: *The Theory of Determinants in the Historical Order of Development* (London, 1890–1930). Most of his more than 320 papers were on determinants and related subjects. Muir introduced the term *wronskian* into the mathematical vocabulary, and he discovered Hadamard's determinant inequality before Hadamard. During the years 1892–1915, Muir made several trips by train or horse-cart into the interior of the Colony. The main object of this talk is to join him on some of those trips.

## References

- [1] P. Maritz: *Sir Thomas Muir, 1844–1934*, to appear in *Linear Algebra and its Applications*, 2005.
  - [2] P. Maritz: *Thomas Muir, Educator. Part IV: His trips to the Karoo and the Eastern Cape*. Submitted.
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# A Mathematical study of the formation of Cerebrospinal Fluid in the Brain

Riëtte Maritz and Sunday Faleye

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University of South Africa

A system of Intracranial capillaries produces the Cerebrospinal Fluid by pressing blood through the pia matter and epithelia membrane into the Cerebrospinal Fluid compartment. Production of Cerebrospinal Fluid is therefore, reduced to the filtration of Newtonian fluid from the Intracranial capillaries into the subarachnoid space. A special boundary condition has been provided to describe the flow through the boundaries. This unique boundary condition is different to the model of Starling's Law which is the usual permeability condition in capillaries. We use the Navier Stokes equations to model both an evolution as well as a stationary problem. Uniqueness and existence are proved for a local weak solution of the stationary problem.

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## The role of non-monogenic $R$ -groups in Matrix near-rings

Anthony Mpho Matlala

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University of Witwatersrand

Let  $R$  be a zero symmetric right near-ring with identity. In this paper it is shown how certain non-monogenic  $R$ -groups affect the interplay between ideals of a near-ring  $R$  and their corresponding Matrix near-ring ideals. It is concluded that if  $R$  has no non-monogenic  $R$ -groups then there is a relationship between the  $s$ -radicals, the socle ideals, and the  $s$ -socles.

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# Random Network having power-law distribution

Mohammed Mohammed, Edith Perrier, and Gareth Witten

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Recently, there has been a great interest in network theory and its applications to many different fields including database mining, disease spread, sociology etc. In particular, the statistical properties of the networks (for example, average path length, cluster size, and degree distribution) provides valuable insight into how these networks function. There is evidence that many real-world networks cannot be represented as a random graph. In particular, the degree distribution of most real-world complex networks can be approximated by a power law distribution, meaning that the number of vertices with degree  $k$  is proportional to  $k^{-\alpha}$ , for some exponent  $\alpha > 1$ , and that is an unexpected feature has a crucial impact on many phenomena of interest. Since then, many models have been introduced to capture this feature. Barabasi and Albert (1999) introduced the term *Scale-free* networks for this type of graphs and pointed out that there are important features of such graphs that are missed if they are approximated by Erdős Renyi graphs.

It is important to be able to generate random graphs with other degree distributions than Poisson. In this paper we will build networks and report on the cluster size distribution and degree distribution. We will also present the implications of these properties for the robustness of these networks.

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## Similarity solutions of coupled PDEs with application in diffusion processes

Motlatsi Molati

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A one-dimensional model for heat and moisture transfer through fabrics is considered. The Lie group theory is employed to obtain symmetries of the model which are then used to construct similarity solutions whenever possible. Furthermore, the reduced model is analyzed numerically.

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## Limit-point/limit-circle classification for Sturm-Liouville problems whose coefficients depend rationally on the eigenvalue parameter

Manfred Möller

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University of Witwatersrand

We will consider the  $2 \times 2$  system

$$\mathbb{A}_0 = \begin{pmatrix} -DpD + q & -Dc + a \\ cD + a & r \end{pmatrix} \quad (3)$$

of formal differential operators, where the coefficient functions  $p, q, c, r, a : (0, \infty) \rightarrow \mathbb{R}$  are measurable functions with  $p \neq 0$  a. e. and  $D$  denotes differentiation with respect to the single variable.

For the classical Sturm-Liouville operator  $-DpD + q$ , the famous limit-point/limit-circle classification says that if for one  $\lambda \in \mathbb{C}$  all solutions of  $(-DpD + q - \lambda)$  belong to  $L_2(0, 1)$ , then this is true for all  $\lambda \in \mathbb{C}$ . In this talk we will show that, with certain restrictions, this is also true for suitably defined solutions of  $\mathbb{A}_0 - \lambda$ . Indeed, the scalar  $\lambda$ -rational problem

$$-(\omega(\cdot, \lambda)y_1')' + (\tilde{q}(\cdot, \lambda) - \lambda)y_1 = 0,$$

where

$$\omega(\cdot, \lambda) = p + \frac{c^2}{\lambda - r}, \quad \tilde{q}(\cdot, \lambda) = q - \left( \frac{ca}{\lambda - r} \right)' + \frac{a^2}{\lambda - r},$$

associated with (3) plays a crucial role, and this already shows that some restrictions on  $\lambda$  are needed.

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# The boundary layer on a spherical liquid drop

GM Moremedi

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University of South Africa.

The results from a flow visualization study of a slow viscous flow past a spherical liquid drop with non-uniform interfacial tension are considered. The fluid dynamic solution contains an undetermined parameter,  $\alpha$ , which depends on the terminal velocity of the liquid drop. The dependence on  $\alpha$  of the interfacial tension, the fluid velocity on the interface and the fluid flow inside and outside the liquid drop as  $\alpha$  takes values in the range

$$\frac{(2 + 3k)^2}{8(1 + k)} Re \leq \alpha \leq 1 + \frac{9}{8}(1 + k) Re$$

is analysed.

A thin transition layer attached to the liquid drop occurs in the exterior flow. The results are found to be in qualitative agreement with known properties of surfactants.

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## On completeness in distance spaces

Seithuti Moshokoa

Department of Mathematical Sciences  
University of South Africa

It is well known that completeness in metric spaces is equivalent to absolute closure. In this talk we characterize completeness in bounded separated distance spaces.



# Polynomial containment for refinement spaces

Désirée V. Moubandjo

Department of Mathematics  
University of Stellenbosch

Given a refinable function  $\phi$  with mask  $a = \{a_j : j \in \mathbb{Z}\}$ , we denote the associated refinement spaces by  $V^{(r)} = \text{span}\{\phi(2^r \cdot -j) : j \in \mathbb{Z}\}, r \in \mathbb{Z}$ . For the corresponding wavelet spaces  $\{W^{(r)} : r \in \mathbb{Z}\}$ , as defined by  $V^{(r+1)} = V^{(r)} + W^{(r)}, r \in \mathbb{Z}$ , it is important in the context of wavelet decomposition that, for each  $r \in \mathbb{Z}$ , the space  $V^{(r)}$  contains polynomials up to some degree. We present here sufficient and necessary conditions on the mask for this polynomial containment result to hold. Also, we give an explicit construction procedure for a quasi-interpolant operator from  $\mathcal{C}(\mathbb{R})$  into  $V^{(r)}$ .

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# Spectral Continuity of Positive Elements

S Mouton

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University of Stellenbosch

SAMS Subject Classification: 9

It is well-known that if  $A$  is a non-commutative Banach algebra, then the spectrum and spectral radius functions are only upper semicontinuous on  $A$ , while if  $A$  is a commutative Banach algebra, then these functions are uniformly continuous on  $A$ . More generally, if  $x \in A$ , then  $|\rho(y) - \rho(x)| \leq \rho(x - y)$  for all  $y \in \{x\}^c$ , where  $\rho$  denotes the spectral radius and  $\{x\}^c$  the commutant of  $x$  (so that the spectral radius is continuous at  $x$ , considered an element of  $\{x\}^c$ ). In this talk we present certain generalisations of this result for positive elements in an ordered Banach algebra.

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# Blocks of characters of finite groups

Zwelethemba Mpono

Department of Mathematical Sciences  
University of South Africa

Let  $G$  be a group,  $Irr(G)$  be the set of all the irreducible ordinary characters of  $G$ ,  $IBr(G)$  be the set of all the irreducible Brauer characters of  $G$  and  $S = Irr(G) \cup IBr(G)$ . Then  $S$  can be divided into nonempty subsets whose union is all of  $S$ . These subsets are called the BLOCKS of  $G$  and the set of all these blocks partitions  $S$ . For  $B \in Bl(G)$ , we write

$$Irr(B) = Irr(G) \cap B$$

$$IBr(B) = IBr(G) \cap B$$

## References

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  - [2] G. Navarro, *Characters and Blocks of Finite Groups*, London Mathematical Society Lecture Note Series **250**, Cambridge University Press, Cambridge, 1998.
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# Emergent Universe with Exotic Matter

S Mukherjee

Department of Physics  
University of North Bengal

It has recently been pointed out that Einstein's Equations permit an emergent universe scenario in which the universe is nonsingular, exists eternally and always remains large enough so that the spacetime can be treated as classical entities at all epochs. Explicit solutions have been obtained in three cases: (1) a closed universe with radiation and a cosmological constant (2) a closed universe with a scalar field with a special self- interaction potential and (3) a flat universe driven by semiclassical vacuum energy. In this paper we have discussed the question if the emergent universe solutions occur only in isolated cases or a generic class of solutions are possible. We have obtained an equation of state, describing exotic matter, which allow generic emergent solutions. The problem of time-symmetry is also solved for these solutions. It appears that the Emergent universe scenario is a viable alternative cosmological model, at least for the distant past.

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## A Theoretical Model of Bloodflow in the Capillaries

Justin MW Munganga and Soares Clovis Oukouomi Noutchie

Department of Mathematical Sciences  
University of South Africa

We review the basic physiology of the cardiovascular system and highlight some cardiovascular equations. The main emphasis is placed on blood circulation through the capillaries. We present a theoretical model of blood flow in the capillaries. A special boundary condition is provided to describe the flow through the walls of the capillaries. This condition simplifies to Starling's Law in a special case. Stability, uniqueness and existence are proved for a local and global weak solution in the talk by S.C Noutchie.

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# Boundary layer flow and its stability

Eunice W. Mureithi

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University of Pretoria

SAMS Subject Classification: 17

Boundary layer flow past a moving wall with velocity  $U_w \sim x^n$  in a streaming flow with free-stream velocity  $U_\infty \sim x^n$  is considered. The wall is subjected to fluid suction/injection. Self-similar boundary layer equations are sought. The equations admit closed form solutions for  $n = -1/3$ . The analytical solutions are complemented by numerical solution of the self-similar boundary layer equations using a shooting technique. Critical values of the velocity ratio parameter  $\lambda = U_\infty/(U_w + U_\infty)$  exist beyond which no physical solutions are realized. Linear stability analysis of the boundary layer flow is carried out. The governing Orr-Sommerfeld equation is solved using the Chebyshev spectral collocation method. Results show the destabilizing effect of fluid injection, pressure gradient and the parameter  $\lambda$  on both the viscous and the inviscid modes.

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## The Complete Symmetry Group of Linear Evolution Equations

Senzosenkosi Myeni

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Howard College, University of KwaZulu-Natal

We introduce the concept of the complete symmetry group, already known for some time in ordinary differential equations, to partial differential equations, in particular to the classical heat equation. For a  $1+1$  linear evolution equation the representation of the complete symmetry group is found to be a six-dimensional algebra isomorphic to  $sl(2, R) \oplus_s A_{3,1}$ , where the second subalgebra is commonly known as the Heisenberg-Weyl algebra.

# On Errors in Elementary Differential Calculus

Kristie Naidoo and Richard Naidoo

Department of Education  
Durban Institute of Technology

Department of Mathematics  
Durban Institute of Technology

The purpose of the study was to determine the effect of technology on the understanding of key concepts in elementary calculus. We investigated the mathematical cognitive errors made in elementary calculus by first year students. A sample of 33 first year students(experimental group), invited to participate in a project in elementary calculus using computer technology (CT). A second group, the control group, of 33 first year engineering students from the same program were given a conventional test. The experimental group on completion of the project in elementary calculus using computer technology (CT) was given the same test. The major finding was that technology helps students to make connections, analyse ideas and develop conceptual frameworks for thinking and problem solving.

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## MacCormack finite difference solution for a non-linear electrostatic fluid equation

Richard Naidoo

Department of Mathematics  
Durban Institute of Technology

A MacCormack finite difference scheme was employed to solve numerically a one dimensional electrostatic 3 equation coupled system. For initial density profiles we explore the nonlinear system as it evolves over many time periods. We then compare the electrostatic system behaviour to a general gas fluid flow 3 equation coupled system.

# Numerical solution of a fourth-order ordinary differential equation modelling thin power-law fluids

Serge N. Neossi Nguetchue and Ebrahim Momoniat

School of Computational and Applied Mathematics  
University of the Witwatersrand

In this talk we derive a fourth-order nonlinear ordinary differential equation modelling the behavior of a thin power law fluid on a vertical plane. We solve this fourth-order ode numerically by imposing contact angle boundary conditions at the trailing and leading contact line. We match the boundary conditions by imposing a fixed film height and gradient on the axis  $x = 0$ . The results obtained here offers new insight into the relationship between the contact angle and power law coefficient  $k$ . We also show a dependence of the capillary ridge height on the coefficient  $k$ .

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## On the effect of Bushes in Banach spaces with either the RNP and the KMP or both.

Thabang Nthebe

Department of Mathematics  
University of Stellenbosch

For decades, the Radon-Nikodym property (RNP) and the Krein-Milman property (KMP) have been studied in an attempt to establish their equivalence in Banach spaces. Their equivalence is still an open question today, and it has been simplified to only proving that the KMP implies the RNP in Banach spaces, since the converse has been proved already.

Existence, and perhaps a lack of, a bush in Banach spaces plays an important role in determining the relationship between these properties. This discussion is on the extent to which the bushes can be used to simplifying the open question, hence a step closer to proving the open question.

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# On Tangent Cones of Frölicher Spaces

Patrice Ntumba

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University of Pretoria

Brett Dugmore

Cadiz Financial Strategists (Pty) Ltd  
Cape Town

Tangent spaces, tangent cones, invertible pairs, and various other notions common to differential geometry are defined for Frölicher spaces in a natural way and seen to coincide with their counterparts for smooth finite dimensional manifolds. The geometry of the wedge product space  $\mathbb{R} \vee \mathbb{R}$ , where  $\mathbb{R}$  is the canonical one-dimensional Euclidean Frölicher space, is studied in great detail. Invertible pairs, as shown in the paper, are very indispensable tools when understanding the geometry of a Frölicher space locally. Unfortunately the inverse function theorem for smooth manifolds does not exist in the context of Frölicher spaces.

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## On the Hölder regularity of refinable functions

Akwum A. Onwunta

Department of Mathematics  
University of Stellenbosch

Given the existence of a compactly supported continuous solution  $\phi$  of a refinement equation of the type  $\phi = \sum_k a_k \phi(2 \cdot -k)$ , it is known that the regularity (or degree of smoothness) of  $\phi$  is preserved by the limit function of its corresponding subdivision scheme, as well as by its associated wavelet. Here, we derive sufficient conditions on the mask coefficient sequence  $\{a_k : k \in \mathbb{Z}\}$  for the Hölder regularity of  $\phi$  by estimating the decay rate of the Fourier transform of  $\phi$ , before using results on the embedding of certain Sobolev spaces into Hölder spaces. Numerical and graphical illustrations are provided.

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# Model of HIV-1 infection with periodic treatment that includes delays: an optimal strategy for treatment

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(SACEMA)

Gareth Witten

Department of Mathematics and Applied Mathematics  
University of Cape Town

Many mathematical models that represent HIV-1 pathogen-immune system interaction assumed that the process of viral infection occurs instantaneously, that is, a target cell begins producing virus as soon as it is infected. In reality, there is a time delay between the initial entry into a cell and subsequent viral production. Moreover, under therapy, the drugs given affect different stages of the virus life-cycle which is divided into pre-and post-drug action allowing for one or two more delays. We consider a model that includes triple drug therapy (HAART) and two more intracellular delays: one associated with the loss of target cells by infection, and the second delay represents the time for the newly produced virions to become infectious. We first assume a constant drug efficacy, we show that the introduction of two delays destabilizes the infected steady state, improves the critical efficacy of the treatment, and leads to an infected steady state with more healthy cells and less infected cells and viruses. We also consider the case with periodic treatment and investigate the stability of the viral free steady state and derive an optimization problem for the optimal strategy of treatment that leads to a minimal amount of virus. Finally, we account for drug toxicity and virus resistance by allowing some constraints of the optimization problem. Numerical evidence shows that under periodic treatment the viral load tends to zero by oscillating with the same period as the treatment. This corresponds exactly to experimental data.

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# Adaptive Finite Element Methods for Singular Perturbation Problems

Kailash C. Patidar and Jean M.-S. Lubuma

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University of Pretoria

It is well known that the standard finite element method based on the space  $V_h$  of continuous piecewise linear functions is not reliable in solving singular perturbation problems. It is also known that the solution of a two-point boundary-value singular perturbation problem admits a decomposition into a regular part and a finite linear combination of explicit singular functions. Taking into account this decomposition, first, we design a finite element method (which we call Singular Function Method) where the space of trial and test functions is the direct sum of  $V_h$  and the space spanned by these singular functions. The second method, based on the finite element discretization on a suitably refined mesh, is referred to as Mesh Refinement Method. Both of these methods are proved to be  $\varepsilon$ -uniformly convergent. Numerical examples which confirm the theory are presented.

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## Fuzzy Vector lattices

Andrew Pinchuck

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University of Witwatersrand

We consider the concept of a fuzzy vector lattice, a generalization of an ordinary vector lattice and we examine its properties. We also look at the problems that are facing the theory.

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# $\lambda(d, 1)$ –Minimal trees

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Department of Mathematics and Statistics  
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An  $L(j, k)$ -labeling of a graph  $G$ , where  $j \geq k$ , is defined as a function  $f : V(G) \rightarrow \mathbf{Z}^+ \cup \{0\}$  such that if  $u$  and  $v$  are adjacent vertices in  $G$ , then  $|f(u) - f(v)| \geq j$ , while if  $u$  and  $v$  are vertices such that  $d(u, v) = 2$ , then  $|f(u) - f(v)| \geq k$ . The largest label used by  $f$  is the span of  $f$ , denoted  $\text{span}(f)$ . The smallest span among all  $L(j, k)$ -labelings of  $G$ , denoted  $\lambda_{j,k}(G)$ , is called the span of  $G$ . An  $L(j, k)$ -labeling of  $G$  that has a span of  $\lambda_{j,k}(G)$  is called a span labeling of  $G$ . Let  $\ell = \Delta + d - 1$ , and  $U = \min\{\Delta + 2d - 2, 2\Delta + d - 2\}$ . A tree  $T$  is in class  $c$  if  $\lambda_{d,1}(T) = \ell + c - 1$ . For each  $c \in \{1, \dots, \min\{d, \Delta\}\}$ , we characterize the class of trees  $T$  for which  $\lambda_{d,1}(T) = \ell + c - 1$ , and also show that this class of trees is non-empty.

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## On the Exponential Spectrum in Banach Algebras

Heinrich Raubenheimer

Department of Mathematics  
University of Johannesburg

We compare the exponential spectrum with the usual spectrum of a Banach algebra element.

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# Monotone maps as upclosed multirelations

Ingrid Rewitzky

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University of Stellenbosch

In the study of semantic models for programs two independent views predominate: relational models and predicate transformer semantics. Relational models view programs as binary relations between states, while in predicate transformer semantics programs are viewed as certain monotone maps from predicates to predicates. Recently the traditional relational view has been generalised to multirelations between states and properties allowing programs and specifications (with angelic and demonic nondeterminism) to be modelled in a single framework. In this talk monotone maps are shown to be, in the precise sense of duality, up-closed multirelations. Moreover, the two-level nature of multirelations is exploited to provide a factorisation of up-closed multirelations which clarifies how multirelations model both angelic and demonic nondeterminism.

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## A design and a code invariant under the simple group $Co_2$

Bernardo Rodrigues

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University of KwaZulu-Natal

We examine a design  $\mathcal{D}$  and a binary code  $C$  constructed from a primitive permutation representation of degree 2300 of the sporadic simple group  $Co_2$ . We prove that  $\text{Aut}(C) = \text{Aut}(\mathcal{D}) = Co_2$  and determine the weight distribution of the code and that of its dual. We show that for a word  $w_i$  of weight  $i$ , where  $i \in \{1024, 1136, 1152, 1200, 1408\}$  the stabilizer  $(Co_2)_{w_i}$  is a maximal subgroup of  $Co_2$ .

# Every strictly positive asset is a generalized geometric Brownian motion

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and

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Let  $(\Omega, \mathcal{F}, P)$  be a probability space on which is defined a Brownian motion  $W(t)$ ,  $0 \leq t \leq T$ . Let  $\mathcal{F}(t)$ ,  $0 \leq t \leq T$ , be a filtration generated by this Brownian motion. Assume there is a risk-neutral measure  $\tilde{P}$ , and let  $\tilde{W}(t)$ ,  $0 \leq t \leq T$ , be the Brownian motion under  $\tilde{P}$  obtained by an application of Girsanov's Theorem. Let  $V(T)$  be an almost surely positive  $\mathcal{F}(T)$ -measurable random variable. According to the risk-neutral pricing formula, the price at time  $t$  of a security paying  $V(T)$  at time  $T$  is

$$V(t) = \tilde{E} \left[ e^{-\int_t^T R(u)du} V(T) | \mathcal{F}(t) \right], \quad 0 \leq t \leq T$$

where  $\tilde{E}$  is the expectation with respect to the measure  $\tilde{P}$ . We show that  $V(t)$  is driven by the equation

$$dV(t) = R(t)V(t)dt + \sigma(t)V(t)d\tilde{W}(t), \quad 0 \leq t \leq T.$$

**keywords:** Risk-neutral measure, Risk-neutral pricing formula, generalized geometric Brownian motion, discount process.

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# Schröder Numbers of Arbitrary Integer Slope

Joachim Schröder and Koos Oosthuizen

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Given a point-lattice  $(m+1) \times (n+1) \subseteq \mathbb{N} \times \mathbb{N}$  and  $\ell \in \mathbb{N}$ , we determine the number of royal paths from  $(0, 0)$  to  $(m, n)$  with unit steps  $(1, 0)$ ,  $(0, 1)$  and  $(1, 1)$ , which never go below the line  $y = \ell x$ . We will specify their generating functions, find equivalent recursion formulae and exhibit a connection to coordination sequences in the cubic lattice  $\mathbb{Z}^d$ . As combinatorial tools we employ the reflection and the rotation principle.

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## Convection in boundary layer flows in the presence of a chemical reaction

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In this study we investigate the stability of two dimensional disturbances imposed on a boundary layer flow over a semi-infinite flat plate in the presence of a reacting chemical species. Species concentration levels are assumed small, as is typical for many processes in water and in atmospheric air. We exploit the multi-deck structure of the flow in the limit of large Reynolds numbers to analyze asymptotically the

perturbed flow. The neutral eigenrelations are obtained implicitly and limiting cases for large buoyancy and reaction kinematics are investigated. The results show some interesting effects of the Damkohler number on the wave number and wave speed of the disturbed flow.

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## Game list colouring of graphs

Elzbieta Sidorowicz

Faculty of Mathematics, Computer Science and Econometrics

Let  $N$  denote the set of positive integers and  $2N$  denote the power set of  $N$ . A list assignment  $L$  for  $G$  is a function  $L : V \rightarrow 2N$ . A function  $f : V \rightarrow N$  is an  $L$ -colouring of  $G$  if  $f(v) \neq f(u)$  whenever the vertices  $v$  and  $u$  are adjacent and  $f(v)$  is in  $L(v)$  for every vertex  $v$ .

We consider the two-players game defined as follows. Let  $(G, L)$  be a graph  $G$  with a list assignment  $L$ . The two players are Alice and Bob and they play alternatively with Alice having the first move. Alice's goal is to provide an  $L$ -colouring of  $G$  and Bob's goal is to prevent her from doing so. A move consists in choosing an uncoloured vertex  $v$  and assigning it a colour from the set  $L(v)$ . This game will be called game list colouring. We say that a graph  $G$  is game list colouring if Alice has winning strategy. The game choice number of  $(G, L)$ , denoted by  $\text{chg}(G)$ , is defined as the least  $k$  such that Alice has a winning strategy for any  $k$ -list assignment of  $G$ . We characterize the class of graphs with  $\text{chg}(G) = 2$  and determine the game choice number for some class of graphs.

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## Unsteady Flow of Linear Visco-elastic Fluid through porous medium between two parallel plates

SHOBH NSTH Singh

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Walter Sisulu University for Science and Technology

In the present article, an exact solution of the governing equations of the flow has been obtained using Laplace transform techniques to study the behaviour of the flow when both the plates are at rest. A detailed discussion of two particular cases has been given. The changes in the velocity profiles of the fluid have been demonstrated graphically for the examination of the physical situation of the flow. This type of flow is of great importance in the petroleum industry concerned with the movement of oil, gas and water through the reservoir of an oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineer in the filtration process.

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## The Impact of the new FETC Mathematics and Mathematical Literacy on HEI's

Maritz Snyders

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Nelson Mandela Metropolitan University

The paper makes comparisons between Mathematics that is currently offered to learners at secondary schools and the new FETC mathematics and mathematical literacy that will be introduced in grade 10 in 2006. The impact of these changes on the extent to which the new subjects will prepare learners for studies in Mathematics at Higher education institutions will be discussed.

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## Somos sequences

Christine Swart

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University of Cape Town

Somos sequences are rational sequences defined by a certain quadratic recurrence relation. They have some interesting integrality properties, and periodicity properties when reduced modulo a prime power. We collect the old results and give some new ones.

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## Partial Integral Differential Equations for Option Prices in Exponential Levy Models

Isaac Takaidza

Department of Applied Mathematics  
University of Limpopo

Empirical data shows that the distributions of stock/asset prices have peaks, a generic property of models with jumps. Models with jumps allow for more realistic representation of price dynamics and a greater flexibility in modelling.

We derive partial integral-differential equations (PIDE) for pricing European and American options when the dynamics of an asset/stock price is described as the exponential of a Levy process.

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## Energy estimation and the effects of the container curvature on the permeable boundary Navier-Stokes flows

Tanki Thelejane and Joe Hlomuka

Department of Mathematics and Applied Mathematics  
Nelson Mandela Metropolitan University

We analyze the energy identity for the nonlinear, nonstationary permeable boundary problem for the Navier-Stokes flows, through the permeable walls of a container. We highlight the conditions on the flow energy and the container curvature that may



lead to the existence of the weak solutions to the problem. We also confirm the boundedness of a very crucial integral, generated by the permeation model, as part of the energy identity for the problem.

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## Maximal Nontraceable Oriented Graphs

Susan van Aardt\*, Marietjie Frick, Jean Dunbar and Ortrud Oellermann

Department of Mathematical Sciences  
University of South Africa

$D$  is a maximal nontraceable (MNT) oriented graph if  $D$  is not traceable but for any two nonadjacent vertices  $u$  and  $v$  in  $D$ ,  $D + uv$  is traceable.

We characterize acyclic and unicyclic MNT oriented graphs as well as the strong component digraphs of MNT oriented graphs. This enables us to characterize MNT oriented graphs of order  $n$  that have size  $\binom{n}{2} - 1$  and we show that no MNT oriented graph of order  $n$  has size  $\binom{n}{2} - 2$ . We also show the maximum size of a *strong* MNT oriented graph of order  $n$  is  $\binom{n}{2} - 3$ .

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## Model Theory for Riesz Spaces

Clint van Alten and Coenraad Labuschagne

School of Mathematics  
University of Witwatersrand

A very natural mathematical object is the system of real numbers considered as a vector space and equipped with its natural lattice ordering. The theory of such ‘lattice ordered vector spaces’, a.k.a. Riesz spaces, is a well developed area of functional analysis; it includes the study of ordered function spaces and sequence spaces. We

approach Riesz spaces as purely algebraic objects; they are, after all, defined by a set of algebraic equations. This implies that the class of Riesz spaces forms a ‘variety’ in the sense of universal algebra, meaning that it is closed under homomorphisms, subalgebras and products. It is known that all Riesz spaces may be obtained by taking homomorphisms, subalgebras and products of linearly ordered Riesz spaces. We give an improvement of this result: each Riesz space may be obtained in this way from the Riesz space of real numbers.

In this talk we give an introduction to the theory of Riesz spaces and discuss the above-mentioned result and some of its consequences.

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## A Banach-Steinhaus Theorem for sigma-order Continuous Operators

Jan Harm van der Walt

Department of Mathematics and Applied Mathematics  
University of Pretoria

We show that order convergence of sequences on an Archimedean vector lattice is induced by a vector space convergence structure. Questions of completeness and completion are considered. We apply these results to the space of  $\sigma$ -order continuous linear mappings  $T : E \rightarrow F$  where  $E$  and  $F$  are Archimedean vector lattices. We show that if the  $\sigma$ -order continuous linear functionals on  $F$  separate the points of  $F$ , then the pointwise limit of a sequence of positive operators is  $\sigma$ -order continuous. If the space  $F$  is moreover Dedekind complete, the same is true for general  $\sigma$ -order continuous operators.

# A Framework for the Enumeration of Set Covers and Related Structures

Jan van Vuuren and Alewyn Burger

Department of Applied Mathematics  
University of Stellenbosch

In this session a theoretical framework is developed by which the enumeration of labelled and unlabelled overlapping set structures satisfying various constraints may be achieved. This framework is then applied to two specific enumeration problems: (a) counting the number of minimum (labelled) set covers and (b) counting the number of optimal (unlabelled) lottery sets of cardinality three. The session comprises three talks, of which the first is a prerequisite in terms of following the other two.

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## Viewing Set Covers as Contractions

Jan van Vuuren and Alewyn Burger

Department of Applied Mathematics  
University of Stellenbosch

In this talk the notion of a set contraction operation, capable of capturing the overlap properties of discrete structures such as finite set covers and optimal lottery playing sets, is developed. Certain properties of and a suitable notation for this set operation are established.

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# Enumeration of Labelled, Balanced Minimum Set Covers

Jan van Vuuren and Alewyn Burger

Department of Applied Mathematics  
University of Stellenbosch

In this talk it is demonstrated how the notion of a set contraction operation (developed in the first talk) may be applied to determine the number of non-isomorphic minimum (labelled) set cover structures via a generating function.

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## A matrix field representation of some factor rings of $k[X]$ , $k$ any field

Leon Van Wyk\*  
Department of Mathematics  
University of Stellenbosch

Kirby C. Smith  
Department of Mathematics  
Texas A&M University

We provide a concrete description of some factor rings of the polynomial ring  $k[X]$ ,  $k$  a field, as fields of matrices, namely of the factor rings  $k[X]/(X^n - X - 1)$  for which  $n$  is such that the polynomial  $X^n - X - 1$  is irreducible in  $k[X]$ . These factor rings include the Galois fields  $\text{GF}(p^n)$  for which  $X^n - X - 1$  is irreducible in  $\mathbb{Z}_p[X]$ . Both MAGMA and MAPLE show that there are many such fields.

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# A unified theory of gravitation and electromagnetism

Pieter Wagener

Department of Physics  
Nelson Mandela Metropolitan University

A model of gravitation presented at the 2004 congress is applied to a closed electromagnetic system. The fine spectrum of the hydrogen atom is derived, as well as expressions for the classical electron radius and Diracs Large Numbers. An alternative interpretation of electron spin is also presented.

A brief explanation of how the model is applied to the nuclear force is also given.

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# Universal Relativity and Its Mathematical Requirements

Sanjay Wagh

School of Mathematical Sciences  
University of KwaZulu-Natal

In this presentation, I review physical principles behind a recently proposed Universal Theory of Relativity and speculate on the mathematical requirements implied by these physical principles. Some unresolved issues will also be discussed.

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# Stability Analysis of Within-Cell Positive Strand Viral Dynamics

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We focus on the viral replication strategies within an infected/target cell for any single-stranded, positive sense RNA virus. The binding of a viral particle to a receptor on a target cell initiates a cascade of events (translation, replication and encapsidation) that can ultimately lead to the target cell becoming productively infected. Within a target host cell, the viral genome (which could be DNA or RNA) is replicated and directs the synthesis of virion components while at the same time using host cellular pathways. We consider a model that represents the within cell dynamics i.e. translation, replication and encapsidation processes of a single-stranded, positive sense RNA virus and report on the stability analysis of the model and results concerning the extinction and persistence of the virus.

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## Arbitrarily vertex decomposable graphs

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A graph  $G$  of order  $n$  is called arbitrarily vertex decomposable if for each sequence  $(n_1, \dots, n_k)$  of positive integers with  $n_1 + \dots + n_k = n$ , there exists a partition  $(V_1, \dots, V_k)$  of the vertex set of  $G$  such that  $V_i$  induces a connected subgraph of order  $n_i$ , for all  $i = 1, \dots, k$ .

We characterize some families of arbitrarily vertex decomposable trees and unicyclic graphs. We consider also the ‘on-line’ version of the problem.

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## Fischer-Clifford Matrices of $B_S(p, n)$

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A combinatorial method for constructing the Fischer-Clifford matrices of generalized symmetric groups  $B(m, n)$ , where  $m$  and  $n$  are positive integers, has been given in [1] and [5]. In [5] Moori and Zimba have given a computer programme for combinatorially constructing in GAP [2] the Fischer-Clifford matrices of generalized symmetric groups. In [3] List has presented a method for constructing the Fischer-Clifford matrices of some group extensions of a faithful irreducible- $S_n$  constituent of  $N = 2^n$ . This is done by deducing the Fischer-Clifford matrices of these groups from those of  $B(2, n)$ . In this paper we use a similar technique to construct the Fischer-Clifford matrices of the group  $B_S(p, n)$ , which is a subgroup of the generalized symmetric group  $B(p, n)$ , where  $p$  is prime. In sequel we present some results which describe the construction of the Fischer-Clifford matrices of  $B_S(p, n)$  from those of  $B(p, n)$ . This gives a combinatorial method for constructing the Fischer-Clifford matrices of the groups  $B_S(p, n)$ .

**Key-words:** combinatorial method, Fischer-Clifford matrices, generalized symmetric group

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