Seeking out and taking advantage of opportunities for mathematical literacy (numeracy) across the curriculum

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Prof Helen Forgasz – Monash University
Prof Shelley Dole – Sunshine Coast University
Dr Anne Bennison – The University of Queensland
South Africa    1,221,037 km²
Australia        7,692,024 km²
Brisbane

**Monday 31 July**

- **Min**: 10
- **Max**: 26
- **Sunny.**
- **Possible rainfall**: 0 mm
- **Chance of any rain**: 0%

**Brisbane area**
The chance of fog in the early morning. Sunny day. Light winds becoming north to northeasterly 15 to 25 km/h in the early afternoon then becoming light in the late evening.

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Grahamstown

**Monday, Jul 31st**

- **Mix of Cloud and Sun**
- **Temperature**: 27°C / 9°C
Can South Africa and Australia feed themselves?

Arable land
- Australia 6.1%
- South Africa 10.3%

What other information is needed?
Can South Africa and Australia feed themselves?

- What are the populations of the two countries?
- What is a typical diet?
- What water resources are available?
- ........

What mathematics is needed and how should be applied?
Which is the most liveable city – Capetown or Melbourne?
It’s not just about knowing mathematics, it’s also about how to select the right information and use appropriate mathematics in a particular context.
Being numerate in the 21st Century
Some questions

• What is meant by “numeracy” (mathematical literacy) in the 21st century?
• What are the expectations in Australian education?
• What might this look like?
What is mathematical literacy/numeracy?

- No widely accepted definition for mathematical literacy internationally, e.g., numeracy, quantitative literacy, matheracy, mathemacy
- Policy and curriculum documents portray mathematical literacy as a vital skill for informed, participatory and productive citizenship that must be continuously developed over a lifetime.
- A challenge for conducting research in this area and for developing effective programs for improvement.
Definitions of mathematical literacy/numeracy

- Crowther Report - defined as the mirror image of literacy (Ministry of Education, 1959).
- Cockcroft, 1982; emphasised the importance of numeracy as a focus for schooling.
Definitions of mathematical literacy/numeracy

AAMT (1997):

To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life.
Definitions of mathematical literacy/numeracy

Mathematical literacy (PISA - OECD, 2013)

Mathematical literacy is an individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD 2013a, p. 17)
Definitions of mathematical literacy/numeracy

PIAAC survey of Adult (16-65 years) skills (OECD 2013)

Numeracy... the ability to use, apply, interpret, and communicate mathematical information and ideas. It is an essential skill in an age when individuals encounter an increasing amount and wide range of quantitative and mathematical information in their daily lives. Numeracy is a skill parallel to reading literacy, and it is important to assess how these competencies interact, since they are distributed differently across subgroups of the population (p. 75).
Elements of quantitative literacy

- confidence with mathematics
- number sense
- symbol sense
- cultural appreciation
- logical thinking
- making decisions
- prerequisite knowledge
- interpreting data
- practical skills
- mathematics in context
Current state of numeracy

- Considerable numbers of adults in OECD countries possess limited literacy and numeracy skills; 18.5% on average has poor reading skills (Level 1 or below) and even poorer numeracy skills (22.7% on average at Level 1 or below)
Australian perspective

Results from national numeracy testing (NAPLAN) and international comparisons of mathematical literacy (PISA) indicate that too many Australian students fail to meet numeracy benchmarks.

- 2013 NAPLAN results indicate nearly 10% of Year 9 students were below the national minimum standard for numeracy.
- 2012 PISA results show that 20% of Australian 15 year olds did not meet the international proficiency Level 2 for mathematical literacy – the level of competence necessary to use mathematics effectively in real-life situations.
- Further, 43% of students were placed below the Australian nationally agreed baseline of Level 3 (Thomson, De Bortolic, & Buckley, 2013).
Numeracy across the curriculum

Steen (2001, p. 18):

To enable students to become numerate, teachers must encourage them to see and use mathematics in everything they do … Fortunately, because numeracy is ubiquitous, opportunities abound to teach it throughout the curriculum.
Numeracy across the curriculum

That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment.

(Council of Australian Governments, 2008, p. 7)
Numeracy as a General Capability in the Australian Curriculum

Using mathematical skills across the curriculum both enriches the study of other learning areas and contributes to the development of a broader and deeper understanding of numeracy. Therefore, a commitment to numeracy development is an essential component of learning areas across the curriculum and a responsibility for all teachers. This requires that teachers:

- identify the specific numeracy demands of their learning area
- provide learning experiences and opportunities that support the application of students’ general mathematical knowledge and skills
- use the language of numeracy in their teaching as appropriate
Numeracy in the Australian Curriculum

Students become numerate as they develop the knowledge and skills to use mathematics confidently across all learning areas at school and in their lives more broadly. Numeracy involves students in recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully.
Enhancing HPE teaching by taking numeracy opportunities

Numeracy Statement
The Australian Curriculum: Health and Physical Education (F–10) provides students with opportunities to recognise the mathematics that exists in Health and Physical Education learning experiences. As they engage with Health and Physical Education, students see the importance of numeracy, select relevant numeracy knowledge and skills, and apply these skills in a range of contexts. Students use calculation, estimation and measurement to collect and make sense of information related to, for example, nutrition, fitness, navigation in the outdoors or various skill performances.
Enhancing HPE teaching by taking numeracy opportunities

Numeracy Statement
They use spatial reasoning in movement activities and in developing concepts and strategies for individual and team sports or recreational pursuits. Students interpret and analyse health and physical activity information using statistical reasoning, identifying patterns and relationships in data to consider trends, draw conclusions, make predictions and inform health behaviour and practices.
The Australian Professional Standards for Teachers, which underpin the design and accreditation of all initial teacher education programs in Australia, require that all teachers know, understand, and can implement effective strategies to support students’ numeracy achievement (see http://www.aitsl.edu.au/australian-professional-standards-for-teachers/standards/list)

**Standard 2.5 Literacy and numeracy strategies**

Graduate career stage: Know and understand literacy and numeracy teaching strategies and their application in teaching areas.

Proficient career stage: Apply knowledge and understanding of effective teaching strategies to support students’ literacy and numeracy achievement.
The challenge of implementing NAC!

Despite support from the Australian Curriculum and the Australian Professional Standards for Teachers, these documents provide teachers and teacher educators with little guidance on how to identify the numeracy demands of different school subjects, how to embed numeracy learning opportunities across the whole school curriculum, and how to work with teachers to achieve these goals.
Research and development program 2009 - 2015

Develop a model for auditing the numeracy demands of the Australian curriculum and for supporting numeracy planning and teaching in all school subjects.

The model has been validated in primary and secondary schools across three states and all education sectors. Dozens of teachers and hundreds of students.
Typical project structure

- Design research approach – intervention, cycles of reflection and enhancement.
- Introductory workshop: Introduction to the ideal of mathematical literacy (through the numeracy model) and immersion activities.
- Cycles of implementation and feedback – problematising rather than telling.
- Whole project workshops follow rounds of implementation in which the numeracy model was clarified and ideas on implementation shared. Additional immersion activities.
- Data gathering included: classroom observation; semi-structured interviews of teachers and students; questionnaires concerning the trajectory of teachers’ numeracy development including confidence and elements of numeracy preferences; surveys of students’ attitudes to embedding numeracy across the curriculum.
- Reflexive approach to analysis with a focus on change and growth.
What is numeracy?

- Numeracy involves ...
- A numerate person can ...
- A numerate person knows ...
- A numerate person is ...
What is numeracy?

Some lay views of numeracy:

- numbers ("doing sums", arithmetic)
- the basic maths people need in order to survive in the real world
- the name for primary school maths
- the name for a lower status maths subject in secondary school
21st century numeracy

- Dispositions
- Mathematical Knowledge
- Contexts
- Tools
21st century numeracy

Dispositions

Tools

Contexts

Problem Solving
Estimation
Concepts
Skills

Mathematical Knowledge
21st century numeracy

Use numbers to solve problems

Contexts

Budget

Tools

Read maps

Understand the odds

Problem Solving

Estimate

Read timetables

Estimation

Mathematical Knowledge

Concepts

Think logically

Skills

Interpret a graph
21st century numeracy

Dispositions
- Confidence
- Flexibility
- Initiative
- Risk

Contexts

Problem Solving
- Estimation
- Concepts
- Skills

Mathematical Knowledge

Tools
21st century numeracy

Dispositions

Confidence
Flexibility
Initiative
Risk

Confident in contemplating a range of thinking pathways to pursue a challenge

Flexible in their mathematical thinking

Problem Solving

Confident to take learning risks

Tools

Mathematical Knowledge

Concepts
Skills

Problem Solving

Confidence
21st century numeracy

Dispositions

Confidence
Flexibility
Initiative
Risk

Contexts

Problem Solving
Estimation
Concepts
Skills

Mathematical Knowledge

Tools

Representational
Physical
Digital
21st century numeracy

Dispositions:
- Flexibility
- Initiative
- Risk

Mathematical Knowledge
- Concepts
- Skills
- Confidence
- Flexibility
- Initiative
- Risk

Problem Solving
- Estimation
- Skills

Contexts
- Physical
- Digital

Tools
- Representational
- Use technology effectively

Using mathematical tools to solve problems
Using various equipment to solve problems
Apply mathematical tools appropriately
21st century numeracy

Dispositions

Confidence
Flexibility
Initiative
Risk

Problem Solving
Estimation
Concepts
Skills

Mathematical Knowledge

Tools

Representational
Physical
Digital

Contexts
21st century numeracy

Dispositions

Confidence
Flexibility
Initiative
Risk

Personal and Social

Problem Solving
Estimation
Concepts
Skills

Mathematical Knowledge

Contexts

Tools

Representational
Physical
Digital
21st century numeracy

Dispositions
- Confidence
- Flexibility
- Initiative
- Risk

Personal and Social

Problem Solving
- Estimation
- Concepts
- Skills

Mathematical Knowledge

Contexts

Tools
- Representational
- Physical
- Digital

Work
21st century numeracy

Dispositions
- Confidence
- Flexibility
- Initiative
- Risk

Personal and Social

Problem Solving
- Estimation
- Concepts
- Skills

Representational
- Physical
- Digital

Contexts

Tools

Work

Mathematical Knowledge

Citizenship
21st century numeracy

Dispositions

Use maths to be successful in everyday life

Personal and Social

Problem Solving

Estimation

Mathematical Knowledge

Contexts

Digital

Physical

Real

Risk

Initiative

Confidence

Flexibility

Risk

Representational

Tools

Able to function successfully in the real world

Work

Apply mathematical processes in real life
21st century numeracy

Dispositions
- Confidence
- Flexibility
- Initiative
- Risk

Personal and Social

Contexts
- Problem Solving
- Estimation
- Concepts
- Skills

Representational
- Physical
- Digital

Tools

Work

Mathematical Knowledge

Critical Orientation
21st century numeracy

Dispositions
- Confidence
- Flexibility
- Initiative
- Risk
- Personal and Social

Mathematical Knowledge
- Concepts
- Skills

Representational
- Physical
- Digital

Critical Orientation
- Knowing whether the answer makes sense
- Knowing when a graph is misleading
- Making informed choices
- Being critical of information and data

Tools
- Problem Solving
- Estimation
- Work

Citizenship
- Knowing whether the answer makes sense
- Knowing when a graph is misleading
- Making informed choices
- Being critical of information and data
21st Century Numeracy

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Numeracy in Health and Physical Education: Thinking about gender equity

- Year 8 Health and Physical Education
- “Area school” in a remote area in South Australia
- Teachers expected to teach a range of subjects
- The teacher was expected to teach HPE as well as mathematics - but had limited mathematics training
Numeracy in Health and Physical Education: Thinking about gender equity

A project in HPE where media coverage of sports was investigated.

Sports reports from an Adelaide newspaper each day for a week.

Attention devoted to both female and male sports.
Numeracy in Health and Physical Education: Thinking physical well being

Project in Physical Education (PE) where Year 8 students investigated their physical activity during a typical week.
Numeracy in Health and Physical Education: Thinking physical well being

- Measured via pedometer which the students wore for a week.
- The number of paces walked each day was entered into a shared Excel spreadsheet.
- Students negotiated a group approach to converting their total number of paces to km.
- Students analysed their own data using Excel and compared the results with other students.
- Students began to pose their own questions of the data.
Brooke

This graph shows the girls total throughout the week.

NUMBER 1: Brooke
NUMBER 2: Sophie
NUMBER 3: Teanne
NUMBER 4: Tiff
NUMBER 5: Laura
NUMBER 6: Mrs Peters

On the male steps, Zane did the most and Justin did the least.

It would have been better if I used the same graphs for the girls and boys.
Numeracy in HPE

“Wow – that’s nearly all the way to Whyalla!”.
Dimensions of numeracy

- Context
- Mathematical knowledge
- Dispositions
- Use of tools
- Critical orientation
A numeracy enhanced English lesson: Communicating emotion

- Year 8
- Aimed at developing students’ understanding of the role of pace when reading poetry.
- Improving student’s oral presentation skills by providing insight into the relationship between the emotions being communicated in a poem and the associated pace at which different sections of a poem should be read.
A numeracy enhanced English lesson: Communicating emotion

- Pace - is the speed at which a poem is read and that this speed is affected by both the content and the context of the ideas or events explored in a poem (beats per minute, number of words against time, and syllables against time).

- A student read a short poem, So Fast by Rick Roth, while also asking one group of students to count the number of syllables in the poem and another group to time how long it took for the poem to be read.
So Fast

So fast, so fast
an eye’s quick blink
had always heard
but didn’t think
it possible for all to go
so fast
more fast
than I could know

I wish I’d stopped
to linger more
to take it in
and feel the core
of that which mattered

most to me
I didn’t see
I didn’t see that folks would
age
and babes would grow
friends would travel
to and fro

The times I loved
fleeting they were
too many now
become a blur

and as I contemplate the past
I wish it hadn’t
gone so fast
and that today would better
last
A numeracy enhanced English lesson: Communicating emotion

120 syllables read in 24.4 seconds – a rate they concluded was too fast because it did not convey the emotion they felt in the poem.
A numeracy enhanced English lesson: Communicating emotion

- What emotions were being communicated within the poem? Students suggested a number of emotions including “regretful”, “sad”, and “angry”.
- Could different emotions be read at a matching pace?. This led to a discussion about how fast the poem should be read.
- Students reached a consensus that it should be “slow for sad and faster for angry”.
- Three students asked to the front of the room, with one student reading a section of the poem slowly, another student reading at a pace that related feelings of anger, another for excited, and, on suggestion from the students, for “pumped” (very excited).
A numeracy enhanced English lesson: Communicating emotion

- Students rearranged their order; students were asked to write the emotion they represented on a sticky note and place this in the appropriate position on a line drawn on the whiteboard.
- Annotated the line by placing scale marks against each sticky note and writing the emotion in larger print so the whole class could inspect the position of each named feeling.
- What emotions were missing?
- Students provided the additional feelings of laziness, maudlin, relaxed, and sickened for the Emotion Scale.
Emotion Scale

SLOW  Depression  Mourning  Sad  Boredom  Happy  Angry  Excited  Pumped  Fassi

Slow  SAD  Boredom  Fine  Happy  Angry  Excited  Pumped  Fassi

+ Laziness
+ Maudlin
+ Relaxed
+ Sickened
In what ways does this activity attend to:

- Context
- Mathematical knowledge
- Dispositions
- Use of tools
- Critical orientation
An additional example drawn from English: Promoting civic responsibility

**LAW & ORDER**

*Pet bans, jail time for irresponsible dog owners under new government crackdown*

Figure 4: Headline of article related to potential new laws about dog attacks (Source: Herald Sun 2013)
Figure 5: Graphical representation of the body parts most injured during dog attacks (Source: Herald Sun 2013)
Numeracy in History: Applying mathematics to promote empathy

Year 8 History class
- Australia is being invaded
- You have one chance to escape by using your father’s four metre “tinny”
- The closest safe refuge is New Zealand
- What do you need by way of food, fuel and other necessities?
- What are the likely weather conditions?
- How confident are you that your journey will be successful?
Some facts

- Distance – 4155 km
- Weather “bomb” predicted
  - The term "weather bomb" relates specifically to a low pressure system that drops 24hPa in 24 hours. "It's the weather equivalent of having a tyre blow out" says head weather analyst Philip Duncan. "The air pressure plummets and conditions quickly become unstable".
  - "These weather bombs tends to have the severe gales, or hurricane force winds, right near the centre, just like tropical cyclones.
  - "The models show the air pressure dropping from around 1000hPa on Thursday evening to 976hPa on Friday morning”. The low is expected to generate big seas on the western coastline, torrential rain for the West Coast..."
Purpose of numeracy across the curriculum

- Each illustration was tagged with a learning area and another purpose.
- Numeracy across the curriculum is not about turning every teacher into a mathematics teacher – it is about using mathematics to strengthen whichever subject they are teaching.
- ...but at the same time exposes students to using mathematics in different contexts – naturalising the use of mathematics.
Conclusions from this research

- It is possible to take advantage of numeracy *opportunities* in every subject at all levels (even when numeracy *demands* are limited).
- Numeracy tasks are most effective when the “critical” dimension of numeracy is integrated into a task.
- Digital tools can be employed to advantage when designing and implementing numeracy activities.
- Numeracy across the curriculum tasks are best supported through investigative pedagogies – especially in relation to developing a critical orientation.
- Teachers can design effective numeracy enhanced tasks for implementation across the curriculum provided they have a clear understanding of what numeracy means and receive support at early stages when developing activities.

- Little is known about the principles of task design effective numeracy task.
Teachers as designers of effective numeracy tasks

- How do teachers design effective numeracy opportunities for students across the curriculum?
- What are the conditions that best support numeracy across the curriculum approaches?
Teachers as designers of effective numeracy tasks

- 2015-2017
- Working initially with 5 expert teachers of numeracy identified through previous projects
- Data collection: Classroom observation; video analysis; video-stimulated semi-structured interviews; semi-structured student interviews
- Co-development of framework for task design
Principles of Numeracy Task Design and Implementation

**Identifying**

*Looking*

A disposition to look for numeracy opportunities and to take advantage of demands.

*Noticing*

Identifying the source of potential numeracy tasks – a) selection or adaptation of existing intra-school activities/resources, b) creation of new activities based on extra-school experiences.

*Seeing*

Identifying how a proto-activity (preliminary task idea) might align with curriculum ( -ula) – knowledge of curriculum
Principles of Numeracy Task Design and Implementation

Shaping

Structuring

- Numeracy model
- Context
- Mathematical knowledge
- Dispositions
- Tools
- Critical orientation
Principles of Numeracy Task Design and Implementation

Shaping

*Fit to circumstance*: Creating, selecting/adapting tasks to accommodate or take advantage of a teacher’s/school’s unique circumstances.

- specific student needs
- school’s interpretation of curriculum; focus a school places on aspects of curriculum
- available resources
- aspects of the built environment
- aspects of the nature environment
- colleagues – people management/convincing others
Principles of Numeracy Task Design and Implementation

Actualising

Pedagogical architecture

- Initial setup (explaining and building understanding of context and task; asking a critical question(s) accessibility/transparency)
- Selection of pedagogy (ies) (investigative, teacher directed etc)
- Pedagogical repertoire and flexibility/adaptability (ability to flip)
- Task adaptability (ability to change a task “on the fly”)
- Measured responsiveness (providing just enough information/feedback for students to remain engaged in a task; accessibility/transparency)
- Bringing student learning together at the conclusion of a lesson (back to the critical question, insistence on providing evidence).
Numeracy in a mathematics lesson

Teacher/Students

- Secondary mathematics teacher of more than 20 years experience
- All girls non-government school in a state capital city.
- Year 7 class
Numeracy in a mathematics lesson

Task
What is a typical student Year 7 student?

Ratios students were required to calculate to respond to the task:
- foot and shin length
- foot and leg length
- index finger and hand length
- hand and forearm length
- hand and length
- arm and leg length
- head and torso circumference
- torso circumference and leg length
- foot and forearm length
- height and leg length
- height and arm length
Numeracy in a mathematics lesson

Foot/leg length
5/8, 22/93, 22/85

Tanya then asked students if they could convert 22/93 to simpler ratio that it might be compared to other ratios

Students struggled with this, so Tanya decided to change her approach and asked students to convert their ratios to decimal fractions and write them on the white board

0.625, 0.24, 0.26 !!!!!
Numeracy in a mathematics lesson

Accurate measurements get you off to a good start toward jeans that fit well—your experience with other pants may help too.
Numeracy in a mathematics lesson

- Tanya stopped the class and began a whole class discussion about the correct way to measure the length of a leg – an activity she had assumed all students would have experienced in earlier year levels.

- After checking all students now understood that they should be standing while measuring their leg length, and the other body parts listed, Tanya asked them to go back to the task.
Numeracy in a mathematics lesson

- At the end of the lesson, Tanya asked students about what they had achieved over the previous 45 minutes, bringing them back to the question she had proposed at the beginning for the session.

- During this discussion, she talked with her students about what needed to be done in the next lesson in order to provide an adequate response to the critical question.
Principles of Numeracy Task Design and Implementation

Actualising

**Pedagogical architecture**

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Where to next?

• Establish the “adoptably” of the framework by teachers with less developed numeracy practice capabilities
• Assessment of students’ numeracy capability development
Thank you!
Acknowledgement:

This project is funded by the Australian Research Council (Discovery Grant DE150100269).
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