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# PROMOTING TEACHING AND LEARNING OF STRATEGIC CALCULATION AND THINKING THROUGH DIAGNOSTIC ASSESSMENTS

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## ABSTRACT

This paper shares the results of the first of a series of diagnostic assessments focused on promoting the teaching and learning of calculation strategies for Grade 3 learners in South Africa. The series of assessments address calculation strategies such as bridging through ten, jump strategies and doubling and halving. They are accompanied by reasoning chains for teacher use in eight ten-minute mental mathematics sessions designed to develop learner fluency in related skills and the focal strategy. The assessments are then used as a post-test to gauge the improvement in student learning. Initial trials were conducted for the 'bridging through ten' strategy in six classes across two provinces in South Africa. Results show positive learner outcomes. The format of these assessments and the accompanying reasoning chains are informing the national landscape where summative end year assessments have been abandoned in favor of assessments that can inform teaching throughout the year.

**Keywords:** *Diagnostic assessment; reasoning chains; strategic thinking*

## INTRODUCTION AND CONTEXT

Mathematics education in South Africa is argued to be 'in crisis' (Fleisch, 2008) with learners performing below expectations on national, regional and international studies. Furthermore, performance is highly polarised indicating among the greatest performance gaps internationally in mathematics between rich and poor (Reddy, 2006). Spaull and Kotze (2015) argue that by Grade 4 a majority of learners are already two grades behind expectations.

A factor widely identified as contributing to poor performance and weak progression is a lack of number sense and a dominance of concrete methods of calculation. Schollar's (2008) study for example found that 79.5% of the Grade 5 test scripts from 154 schools across all 9 provinces relied on simple unit counting to solve problems. Our own research across the multiple schools concurs with this and we have widespread evidence of learners using drawn tallies for simple calculations such as 10+10+10 (e.g., Weitz & Venkat, 2013).

The implementation of the Annual National Assessments (ANAs) by the Department of Basic Education in 2011 for Grades 1-6 and 9 did little to address poor performance and weak number sense. As Diamond (2007, p. 306) argues while high-stakes assessments 'may get teachers' attention, they provide few resources for addressing issues of inequality in schools.'

The ANAs were criticised by teachers and teacher unions and ended with refusal by some schools to write them. They were abandoned in 2016. Among the criticisms was that they did little to encourage the teaching of number sense and the focus on correct answers fed into acceptance of counting based strategies thus perpetuating rather than addressing problems of progression (Graven, Venkat, Westaway, Tshesane, 2013; Graven & Venkat, 2014).

South Africa's national curriculum policy however includes the development of number sense which is connected with developing mental models and strategies for computation. For example, the Curriculum and Assessment Policy document (CAPS) includes that Mathematics should "develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision making" (DBE, 2011, 8-9).

The document includes a range of basic facts (fluencies) that learners should know instantly (such as adding ten to a number; knowing number bonds to ten and so forth) as well as a range of calculation strategies (such as Bridging through 10 and Doubling and Halving). Through our professional development work we focused on supporting teachers to understand (and use) the relationship between using such fluencies and strategies to move students beyond one to one concrete methods of calculation.

The Foundation Phase diagnostic assessment investigation emerged from this context and was led by the two South African Numeracy Chair (authors) who are mandated to search for ways forward to the challenges of mathematics teaching and learning in primary schools in South Africa. While their Chairs are located in two separate universities and provinces they have worked closely together since their Chairs began in 2011 (see Graven & Venkat (2017) for a range of research based on the work of members of their research teams).

This assessment project began with an initial meeting in 2016 which was attended by members of both Chair teams, with representation from the Department of Basic Education (at national, provincial and district level), the Association of Mathematics Education of South Africa, the Southern African Association of Research in Mathematics Science and Technology Education; the Non-Government Organisation community, and two international experts in early mathematics teaching and learning: Professor Mike Askew and Professor Bob Wright.

Given widespread acknowledgement that assessment influences practice (e.g., Elmore, Ablemann & Fuhrman, 1996), absence of attention to number sense that underlies fluent, flexible and strategic mental and written working in previous Annual National Assessments was seen as problematic. Furthermore, it was noted that to shift teacher practice on a more national scale it would be important to influence national assessment practices to foreground number sense and non-concrete strategies. Knowledge of such strategies is stipulated in the national curriculum (DBE, 2011).

Our representative from the national Department of Basic Education noted that there was policy level interest in diagnostic assessment formats that could be administered with an orientation grounded in feedback loops into teaching and learning. Thus, following our week of deliberations, consensus was reached that we should investigate a possible format for the design of a series of diagnostic assessments and reasoning chains to support the teachers and learners to move beyond concrete methods of calculation to using strategic awareness of number relations and structure in ways that promote effective and efficient calculation.

We report on the final format arrived at following our ongoing deliberations and small scale piloting. We then report on the findings of our formal pilot across six classrooms in two provinces.

## THEORETICAL ORIENTATION AND LITERATURE REVIEW

A socio-constructivist perspective broadly guided our deliberations and the design of our diagnostic assessments and reasoning chains. Kilpatrick, Swafford & Findell's (2001) model of five strands of mathematical proficiency (namely: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence and productive disposition) and the inter dependence of these strands broadly informed our thinking. We particularly drew on Askew's (2012) work suggesting that it is practical to foreground, in working with teachers, fluency, reasoning and problem-solving (strategic competence) as these are the strands that are both most 'visible' in learner working and also useful to design for in teaching. The focus on these three strands links to the three categories of assessment used in the design of our pre- and post- diagnostic assessments, discussed below, particularly in relation to our emphasis on the interrelationship between fluencies (such as adding ten to any number) and strategies (such as using jump strategy).

In terms of designing assessment items our work was guided by earlier seminal assessment work developed in England and Australia by our two international participants namely Mike Askew and Bob Wright (respectively) and their colleagues. In particular we drew on the types of assessment items included in the Leverhulme study published as *Effective Teachers of Numeracy* conducted in England in the nineties (Askew, Brown, Rhodes, Wiliam & Johnson, 1997). In particular, the finding from their study that indicated having a connected understanding of mathematics is important for the effective teaching of numeracy resonated with our guiding assumptions.

We also drew on the work of Bob Wright and his colleagues who focus on using carefully constructed assessment items to enable mathematical recovery of learners falling behind grade level expectations. For example we drew on the books of Wright, Martland, Stafford, & Stanger (2006) and Wright, Martland and Stafford (2006) which focus on early numeracy assessment for teaching and intervention.

Next we explain our research design, the strategies we selected to focus on; the categories of assessment items developed and the way in which we put these together into two-week assessment-teaching cycles.

## RESEARCH DESIGN

Following our initial meeting the following strategies were selected for the development of assessment items and reasoning chains:

- Bridging through ten
- Jump strategy
- Doubling and halving
- Understanding the relationship between addition and subtraction
- Re-ordering
- Compensation

These were identified within the South African Foundation Phase curriculum document (DBE, 2011) as important. Using the number line was considered an essential tool (including as a mental image) for working across these strategies. A range of fluencies were considered essential for successful use of these strategies (such as being able to: add ten to any number; double and halve numbers). Thus, for each of the above strategies we decided to design three categories of assessment items, namely: rapid recall (fluency), strategic calculating (strategic competence/ problem solving) and strategic thinking (adaptive reasoning) items.

Our focus on these three categories of items, was driven by the literature discussed above which draws attention to the usefulness of focusing on these strands as well as the lack of attention to these in South Africa in the Foundation Phase.

Rapid recall items - noted in the CAPS curriculum for mental mathematics: e.g., multiplying by 2, adding and subtracting 1, 2, 3, 4, 5 and 10 to any number; place value decompositions of number, and key fact triples between 1 and 20.

Strategic calculating items - include items such as 99 + 99 which are laborious to do in a one to one or 'procedural' calculation orientation – but very easy to do if the problem is recognised as one that is amenable to rounding to 100, doubling 100 and then compensating or to using a number line – written or mental – to 'bridge through 100' (i.e. 99 + 1 + 98 = 198 or 2 x 100 – 2 = 198).

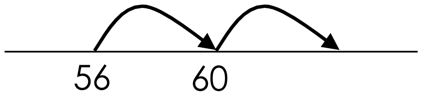
Strategic thinking items - focused on understanding number structure, properties and relationships, and the behaviour of operations. Items focus on using knowledge of number and relationships to limit the extent of calculation needed: e.g., Given 43 + 138 = 181 then what is 181 – 43 = __ ?

In follow-up meetings between smaller groupings of participants, possible formats for diagnostic assessment were discussed. Agreement was reached on the format of a 2-week cycle with a pre- and post- 'test-let' format focused on each of the above six strategies. (Data from the first 'bridging through ten' is shared in this paper). The two-week cycle commenced as follows:

- 2-week block begins and ends with a test-let in time limited format
- teacher marking follows guided by lesson starter 'reasoning chain' teaching activities aimed at developing fluencies and strategies in ten minute sessions on the eight days following the pre-test
- re-test - provides feedback on learning and, hence, success of teaching

In an initial pilot by the first author (in three classes in one Eastern Cape school) challenges were noted in the administration of the assessments and the teaching of the 'bridging through ten' reasoning chains. Thus, the format of the assessment items and time available for each category of items was changed resulting in three single pages of items administered separately as follows:

- 20 rapid recall items to be completed in two minutes (E.g., 10 = 7 + _; 50 + 6 = and 40 – 7 =)
- 5 strategic calculating items to be completed in one minute (e.g., 56 + 8 = _ and 93-7 = , the first two items were accompanied by a number line for example:




- 5 strategic thinking items to be completed in one minute (E.g., 98 + 56 = 98 + 2 + _)

The assessment of the 'bridging through ten strategy' in the above format was then conducted with classes across two provinces (discussed below). Assessments were then marked providing feedback to teachers on strengths and weaknesses of the learners and the eight adapted ten-minute session lesson starter outlines were provided to teachers (following 20-40 minute initial conversations with them about the use of the lesson starter outlines). Thereafter learners were re-assessed on the test in the same way.

An example of part of day one's lesson starter outline is given below:

**First minute mental warm up – playing games for bonds to ten and multiples of ten. E.g., I say 3 you say 7; I say 6 you say ?; What's the next ten after 47? 58? 32? – emphasis that this is not rounding to the nearest ten but finding the next ten on the number line.**
**Then consider: 46 + 7. We can show this on a number line:**

**We have to jump forwards 7. Let's jump to the next ten rather than jumping in 1s. What is the next ten after 46? Then show the 50 on the number line above. What do we add to get to the 50? Show this +4 with an arrow in the number line. We have added 4 but we need to add 7. How much more must we add? Show the +3 with an arrow on the number line from 50 to ?. So 50 + 3 is ? Show the 53 on the number line. So 46 + 7 = 46 + 4 + 3 = 53. Do another example and have learners solve a few more using this method independently.**

The authors sought and obtained access to two government primary schools in Gauteng and one government primary school in the Eastern Cape to trial the adapted test-let and reasoning chain activities. Below we share the results of our initial trials based on administering this test-let twice in a fortnight period (or in some cases just over as weather conditions in the Eastern Cape school caused delay in post testing), with teachers in government schools in these two provinces.

South African state schools are classified into five quintiles based on a range of catchment area factors (e.g. income, unemployment rate). Quintile 1 schools are the poorest while quintile 5 the 'least poor' (DoE, 1998). The two Gauteng schools were a township Quintile 1 school and a suburban Quintile 5 school. One township Quintile 3 school participated in the Eastern Cape trial.

In this school, two Grade 3 teachers and one Grade 2 teacher (who requested inclusion) participated. Since the assessments were trialed toward the end of the Grade 2 academic teaching year it was considered appropriate for these learners to participate.

While the tests were designed at Grade 3 level all fluencies and strategies assessed apply also to Grade 2 learners. Due to limited space, we combine results for the classes in each province below although we have kept the Grade 2 class results separate. The latter results indicate possible broader applicability of the diagnostic test-let assessment/ reasoning chain format beyond Grade 3.

## RESULTS AND DISCUSSION

Table 1 and Table 2 below provide the results for the mean average pre- and post-percentages obtained by participating learners across the two provinces.

**Table 1. Eastern Cape Grade 3 and Grade 2 outcomes**

| | Rapid Recall (20 items: 2 minutes) | | Strategic Calculating (5 items: 1 minute) | | Strategic Thinking (5 items: 1 minute) | |
|---|---|---|---|---|---|---|
| | MeanPre% | MeanPost% | MeanPre% | MeanPost% | MeanPre% | MeanPost% |
| (n=65) Gr 3 | 26.7 | 40.7 | 11.7 | 22.5 | 1.9 | 6.5 |
| (n=30) Grade 2 | 31.2 | 56.4 | 10.3 | 21.3 | 1.1 | 7.5 |

**Table 2. Gauteng Grade 3 outcomes**

| | Rapid Recall (20 items: 2 minutes) | | Strategic Calculating (5 items: 1 minute) | | Strategic Thinking (5 items: 1 minute) | |
|---|---|---|---|---|---|---|
| | MeanPre% | MeanPost% | MeanPre% | MeanPost% | MeanPre% | MeanPost% |
| All (n=134) | 56.3 | 75.3 | 37.6 | 35.8 | 6.1 | 24.5 |

The differences in performance on assessments across the two provinces cohere with national data which indicates Gauteng as one of the top performing provinces in the country and the Eastern Cape as one of the lowest. This also reflects the difference in economic wealth of these provinces with Gauteng being the wealthiest of South Africa's nine provinces and the Eastern Cape being among the poorest. There are several interesting points to make in relation to the above tables.

Predictably, and in both provinces, pre-test performance was highest in the rapid recall cluster of items, and lower in the other two item cluster types. What the pre-test results also point to though is problematic gaps within the rapid recall category given that these items represent fundamental skills that the curriculum expects almost all children to have mastery of, as the base upon which more strategic calculating and thinking are built. Our data points to little more than a third of these items being answered correctly in three of the four Grade 3 classes in this sample. Given this, and the broader evidence alluded to earlier, the weaker pre-test performance on the Strategic Calculating and Strategic Thinking items is predictable.

Post-test outcomes point to pleasing gains in both provinces in the Grade 3 classes. In the Eastern Cape, gains were seen across all three item categories; in Gauteng, there was a small drop in performance in the strategic calculating category in the township school, but substantial increases in all of the other categories across both schools. The feedback from the teachers involved also suggested positive experiences of working with the reasoning chains, and we saw reasonably good implementation in the mental mathematics starter sections that we observed.

## CONCLUDING REMARKS: WHERE TO FROM HERE?

These findings suggest that the diagnostic test-let/reasoning chain activity model can contribute to improvements in performance in ways that support the development of number sense. Our recommendation for the next stage would be a broader and more nationally representative DBE-led trial of the Bridging through 10 and Doubling and Halving test-lets and Reasoning Chain activities that have been developed.

Our suggestion for this trial would be to work via Foundation Phase provincial and district Mathematics specialists to support the running of trials in each district using the same model used in the preliminary trial. We could provide an outline of processes for Subject

Advisers to support this. Our sense is that broader and more representative trials would be needed to decide the robustness of the promising results that we have seen, prior to deciding the feasibility for roll-out of the diagnostic test-let format.

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