



SA
NUMERACY
CHAIR



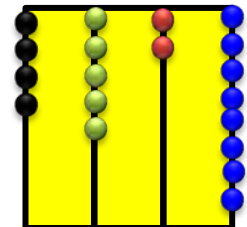
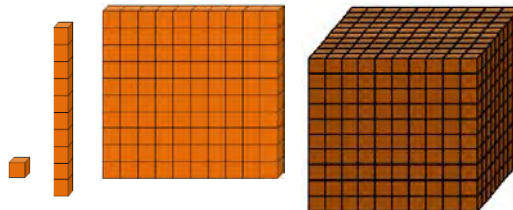
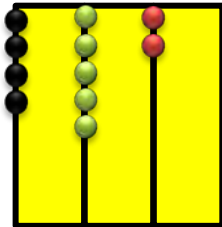
RHODES UNIVERSITY
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NICLE

Numeracy Inquiry Community Of Leader Educators
Tuesday 11 October 2011



Developing a concept of Place Value



Presenter & Materials Developer

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Developing a concept of Place Value

- In order for learners to experience the beauty and assumed simplicity of our base-ten number system, they should be presented with recurrent opportunities to construct meaning of place value. Some learners often experience difficulties with this concept.
- The development of a thorough understanding of place value could assist learners in effectively applying this knowledge to derive mental calculation multiplication and division facts, for example.
 - ✓ A learner with a deep sense of place value would be able to view 440 as 44 tens.
 - ✓ The learner would therefore know that 440 divided by 10 is 44 because there are 44 tens in 440; not because of the rule that you cross off zeros when dividing by 10 and multiples and powers of 10.
 - ✓ When asked how many boxes are needed to package 48 638 chocolates into boxes with 100 chocolates per box, the learner with a good place value concept would be able to reason mentally that 486 boxes are needed with 38 chocolates remaining (Only 9% of Grade 6 learners in the study of Sowder (1994) was able to solve this problem).
- Asking learners to give the place value of underlined digits, e.g. the value of 8 in 487, does not require of learners to illustrate understanding of place value. The answer could be supplied without conceptual understanding of place value.
- Place value is one of the basic concepts for developing flexible partitioning and decomposing of numbers – a skill needed for effective mental calculations and estimation.
- Learners who lack conceptual understanding of place value often overgeneralize knowledge of whole numbers to derive (incorrect) facts of fractions and decimals.
- The power of having deep conceptual understanding of place value is displayed in the thinking and reasoning of a Grade 1 learner in solving the problem $246 + 178 = \square$

How many boxes are needed to package 48 638 chocolates into boxes with 100 chocolates each?

48 638 is 486 hundreds remainder 38!



What is the place value of the underlined digits?

$$\begin{array}{r} 48 \\ \underline{679} \\ \underline{1285} \end{array}$$

91 are 7 tens and 21 ones.

$$\begin{array}{l} 91 \div 7 \\ = (70 \div 7) + (21 \div 7) \\ = 13 \end{array}$$

0,13 > 0,4 because 13 are more than 4!
 0,683 > 0,68 because thousands are more than hundreds!



Well, 2 plus 1 is 3, so I know it's 200 and 100, so now it's somewhere in the three hundreds. And then you have to add the tens on, and the tens are 4 and 7 . . . Well, um. If you started 70, 80, 90, 100. Right? And that's four hundreds. So now you're already in the three hundreds because of the [100 + 200], but now you're in the four hundreds because of that [40 + 70]. But you've still got one more ten. So if you're doing it 300 plus 40 plus 70, you'd have 410. But you are not doing that. So what you need to do then is add 6 more onto 10, which is 16. And then 8 more: 17, 18, 19, 20, 21, 22, 23, 24. So that's 124. I mean 424. Sowder, J. (1994). Number Sense-making. *Arithmetic Teacher*, p343-344.

Place Value in the Curriculum

Grade 3

RNCS

Grade 4

- 3.1.4. Know, read and write number symbols and names from 1 to at least 1 000.
- 3.1.5. Order, describe and compare whole numbers to at least 3-digit numbers.
- 3.1.6. Recognise the place value of digits in whole numbers to at least 3-digit numbers.
- 3.1.11. Use the following techniques:
 - building up and breaking down numbers
 - doubling and halving
 - number lines
 - rounding off in tens

- 4.1. 3. Recognise and represent numbers to describe & compare:
 - whole numbers to at least 4-digit numbers.
- 4.1.4. Recognise the place value of digits in whole numbers to at least 4-digit numbers.
- 4.1.8. Estimate and calculate using appropriate calculations:
 - rounding off to the nearest 10, 100 or 1 000
- 4.1.11. Use various techniques for written and mental calculations :
 - building up and breaking down numbers
 - rounding off and compensation
 - doubling and halving
 - use a number line
 - use a calculator

CAPS

Grade 3

- 1.1. Whole Numbers**
 - 1.3. Number symbols and number names**
Recognize, identify and read numbers
 - Number symbols 0 to 1 000
 - Write number symbols 0 to 1 000
 - Number names 0 to 1 000
 - 1.4. Describe, compare and order numbers to 999**
 - Describe & compare whole numbers up to 999 using smaller & greater than, more or less than & equal to
 - Describe & order numbers up to 999 from smallest to greatest and greatest to smallest
- Use ordinal numbers to show order, place or position**
- Use, read & write ordinal numbers

- 1.5. Recognize place value of 3-digit numbers to 999**
 - Decompose 3-digit numbers up to 999 into multiples of 10 & ones/units
 - Identify and state the value of each digit
- 1.12. Techniques (methods or strategies)**
Use the following techniques when performing calculations:
 - Building up and breaking down numbers
 - Doubling and halving
 - Number lines
 - Rounding off in tens

Grade 4

Grade 4: Content Clarification

- 1.1. Whole Numbers**
- Counting, ordering, comparing, representing of numbers and place value**
 - Order, compare and represent numbers to at least 4-digit numbers
 - Recognize place value of whole numbers to at least 4-digit numbers
 - Rounding off to the nearest 10, 100 and 1 000
- Calculation strategies & check written and mental calculations:**
 - Estimation
 - Building up and breaking down numbers
 - Rounding off and compensation
 - Doubling and halving
 - Using a number line

- Order, compare and represent**
 - Order, compare & represent numbers to at least 4-digit nos.
 - Arrange numbers in descending and ascending order
 - Compare numbers: Fill in <, > or =. 5 461 > 4 533
- Place value**
 - Recognize place value of whole numbers to at least 4-digit numbers
 - Break up numbers into 100s, 10s and units/ones
 - Number names/words
 - Place value or flard cards
 - Expanded notation
 - Dienes blocks
 - Practice in mental calculations
 - Increase number range from 999 to 4-digit numbers
 - Rounding off to the nearest 10, 100 and 1 000

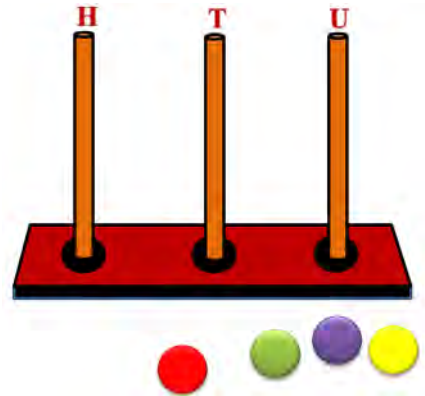
Grade 4 Time Allocation per Topic							
TERM 1	Hrs	TERM 2	Hrs	TERM 3	Hrs	TERM 4	Hrs
Mental Maths (10 min per day)	8	Mental Maths (10 min per day)	7	Mental Maths (10 min per day)	8	Mental Maths (10 min per day)	7
1.1. Whole numbers: Count, order, compare, represent & place value (3-digits)	2	1.1. Whole numbers: Count, order, compare, represent & place value (4-digits)	1	4.3. Capacity/Volume	6	1.1. Whole numbers: Count, order, compare, represent & place value (4-digits)	1
2.3. Number sentences	3	1.1. Whole numbers: Addition & subtraction (4-digits)	4	1.2. Common fractions	5	1.1. Whole numbers: Addition & subtraction (4-digits)	4
1.1. Whole numbers: Addition & subtraction (3-digits)	8	1.2. Common fractions	6	1.1. Whole numbers: Count, order, compare, represent & place value (4-digits)	1	4.2. Mass	6
2.1. Numeric Patterns	4	4.1. Length	6	1.1. Whole numbers: Addition & subtraction (4-digits)	4	3.2. Properties of 3-D objects	4
1.1. Whole numbers: Multiplication & division (1 by 1-digits)	4	1.1. Whole numbers: Multiplication (2 by 2-digit numbers)	6	3.5. Viewing objects	2	1.2. Common fractions	5
4.4. Time	6	3.2. Properties of 3-D objects	5	3.1. Properties of 2-D shapes	4	1.1. Whole numbers: Division (3 by 1-digits)	3
5. Data Handling	10	2.2. Geometric Patterns	4	5. Data Handling	7	4.6. Perimeter, Area & Volume	7
3.1. Properties of 2-D shapes	5	3.3. Symmetry	2	2.1. Numeric Patterns	4	3.6. Position & Movement	2
1.1. Whole numbers: Multiplication & division (2 by 1-digits)	5	1.1. Whole numbers: Addition & subtraction (4-digit numbers)	4	1.1. Whole numbers: Addition subtraction (4-digit numbers)	4	3.4. Transformations	3
		1.1. Whole numbers: Division (3 by 1-digits)	4	1.1. Whole numbers: Multiplication (2 by 2-digit numbers)	5	2.2. Geometric Patterns	2
				2.3 Number sentences	3	1.1. Whole numbers: Addition & subtraction (4-digits)	3
						5.4. Probability	2
REVISION	5	REVISION	4	REVISION	4	REVISION	5
		ASSESSMENT	1			ASSESSMENT	1

GRADE 3 CONTENT AREA & TOPIC ALLOCATION IN LESSONS						
Content Area	Topic	Number of Lessons				
		Term 1	Term 2	Term 3	Term 4	Total
Numbers, Operations & Relationships	All topics of Number, Operations & Relationships	26	22	19	27	94
Patterns, Functions & Algebra	Number Patterns	3	3	3	3	12
	Geometric Patterns	1	1	1	1	4
Space & Shape (Geometry)	2-D Shapes	2		2		4
	3-D Objects		3	3	1	7
	Position, orientation and views		2	3		5
	Symmetry	2			1	3
Measurement	Time	3	2	3	2	10
	Length		2	2		4
	Mass		2		1	3
	Capacity/Volume	2			1	3
	Perimeter			1		1
	Area				2	2
Data Handling	Whole Data Cycle	3		3		6
	Sections of Data Cycle		1		1	2
TOTAL LESSONS		40	40	40	40	160

Place Value Board

Use 4 counters.

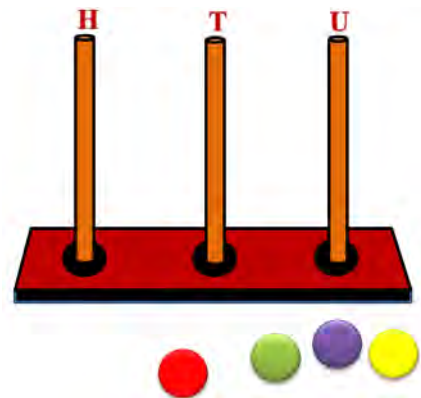
- Make the smallest number.
- Make the largest number.
- Make numbers that are less than 50.
- Make numbers that are more than 100.
- Make the smallest even number.
- Make the largest even number.
- Make the smallest odd number.
- Make the largest odd number.
- If there's a Thousands rod on the board, what is the biggest number you can make?



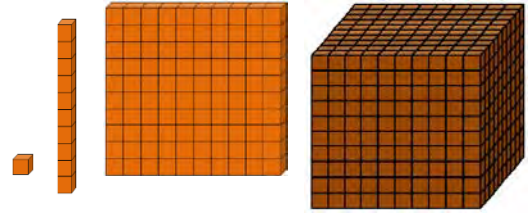
Plekwaarde Bord

Gebruik 4 tellers.

- Maak die kleinste getal.
- Maak die grootste getal.
- Maak getalle minder as 50.
- Maak getalle meer as 100.
- Maak die kleinste ewe getal.
- Maak die grootste ewe getal.
- Maak die kleinste onewe getal.
- Maak die grootste onewe getal.
- As daar 'n Duisende plek is, wat is die grootste getal wat jy kan maak?



Dienes Blocks

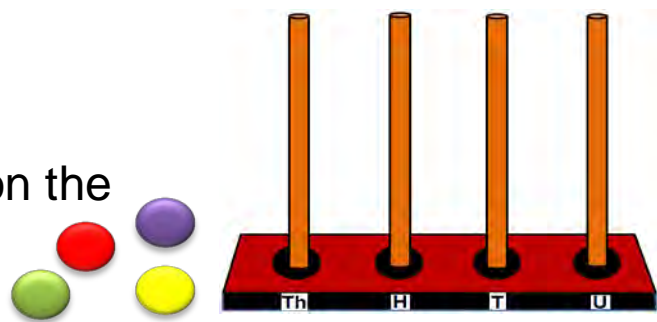


Use the Dienes block cards.
Show these numbers.

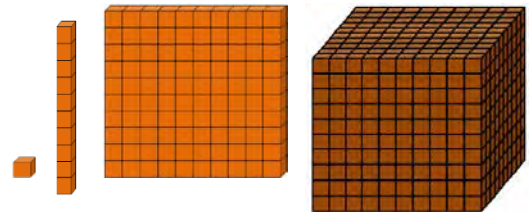
- a. 14
- b. 303
- c. 4 220
- d. 2 035

Use counters.

Show the above numbers on the
place value board



Dienes blokke

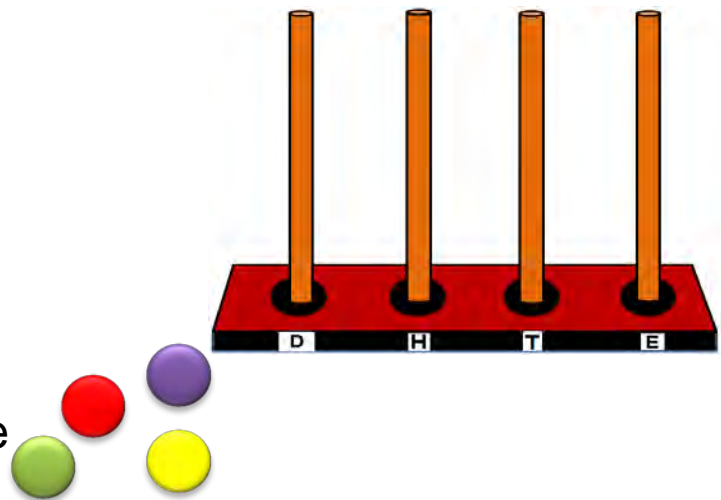


Gebruik die Dienes Blok kaarte.
Wys die getalle.

- a. 14
- b. 303
- c. 4 220
- d. 2 035

Gebruik tellers.

Wys die getalle hierbo op die
plekwaarde bord.



Ancient Number Representation (1)

Many, many years ago ancient mathematicians did not use zero as a place holder. They only used the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. To represent zero in numbers, they just left an open space if there were no Units, Tens, Hundreds or Thousands. This often caused confusion. Some spaces were smaller than others. They wrote the numbers 404 and 44 like this:

4 4 44

How would the ancient mathematicians have written these numbers?

- (a) 530 and 53 (c) 5003 and 503 (e) 2020 and 202
(b) 503 and 53 (d) 101 and 110 (f) 1500 and 15

What do you think of this way of representing numbers?

Years later the Indian mathematicians invented another idea. Instead of an empty space, they used a dot called *sunya*, meaning empty. They then represented the numbers 40, 404 and 4 004 like this:

4• 4•4 4••4

How would the Indian mathematicians have written these numbers using *sunya*?

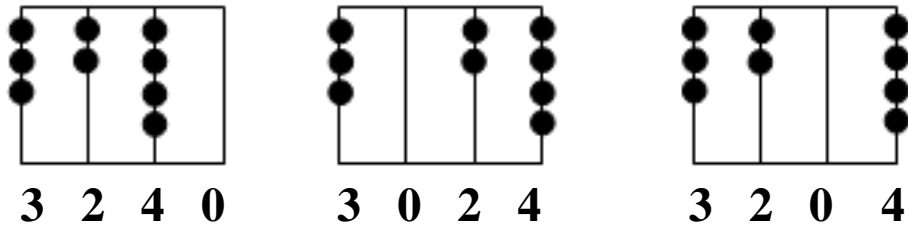
- a. 350 b. 305 c. 3 005 c. 3 050 d. 3 500

e. 600 f. 608 g. 7 020 h. 8 909 i. 770

Tell the class what you think about the Indian's invention.

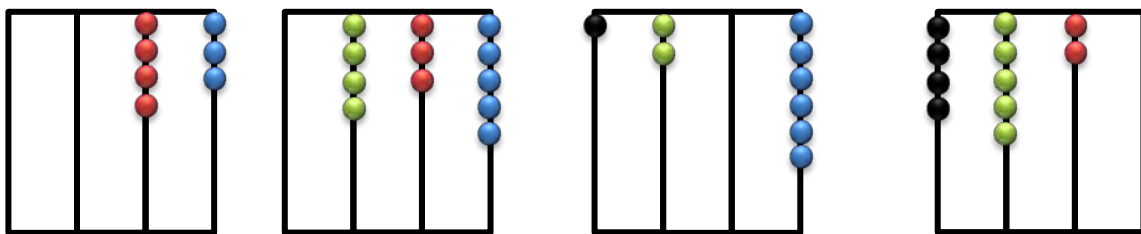
Ancient Number Representation (2)

Indian mathematicians invented the idea of zero as a placeholder. They created a counting board with beads to represent numbers. One of the rods on the board was used to show empty places in numbers, in other words to show zero digits.



What would have happened if there were no place on the board to represent zero digits?

Which numbers were represented on the ancient counting boards below? Write down the numbers and read them aloud.



Write the numbers above in expanded notation, i.e. break them up in Units, Tens, Hundreds and Thousands.

The *dot* that Indian mathematicians invented later became a *circle* – the *zero* we use today.

The word *digit* means ‘finger’. Ancient people used their fingers and other body parts to count and calculate. 305 is a 3-digit number and 3 500 is a 4-digit number.

Antieke voorstelling van getalle (1)

Baie, baie jare gelede het antieke wiskundiges nie 'n zero as 'n plekhouer gebruik nie. Hulle het net die syfers 1, 2, 3, 4, 5, 6, 7, 8, 9 gebruik. Vir die voorstelling van zero in getalle het hulle 'n oop spasie gebruik as daar nie Ene, Tiene, Honderde of Duisende was nie. Dit het gewoonlik verwarring veroorsaak. Sommige spasies was kleiner as ander. Hulle het die getalle 404 en 44 só geskryf:

4 4

44

Hoe sou die Antieke wiskundiges dié getalle geskryf het?

(a) 530 en 53

(c) 5003 en 503

(e) 2020 en 202

(b) 503 en 53

(d) 101 en 110

(f) 1500 en 15

Wat dink jy van hierdie manier om getalle voor te stel?

Jare later het Indiese wiskundiges 'n ander idee uitgevind. In plaas van die leë spasie het hulle 'n kol, genoem *sunya* wat 'leeg' beteken gebruik. Hulle het toe getalle soos 40, 404 en 4 004 só geskryf:

4•

4•4

4••4

Hoe sou die Indiese wiskundiges *sunya* gebruik het om hierdie getalle te voor te stel?

a. 350

b. 305

c. 3 005

c. 3 050

d. 3 500

e. 600

f. 608

g. 7 020

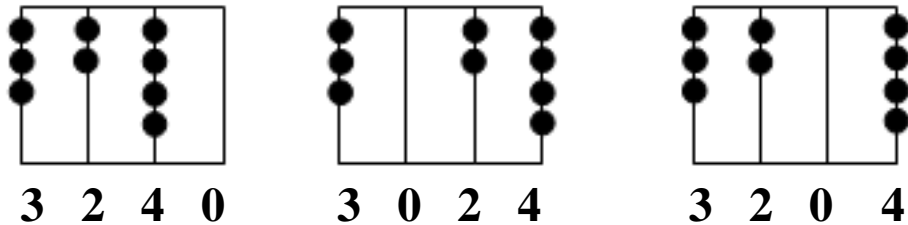
h. 8 909

i. 770

Sê vir die klas wat jy dink van die Indiese wiskundiges se uitvindsel.

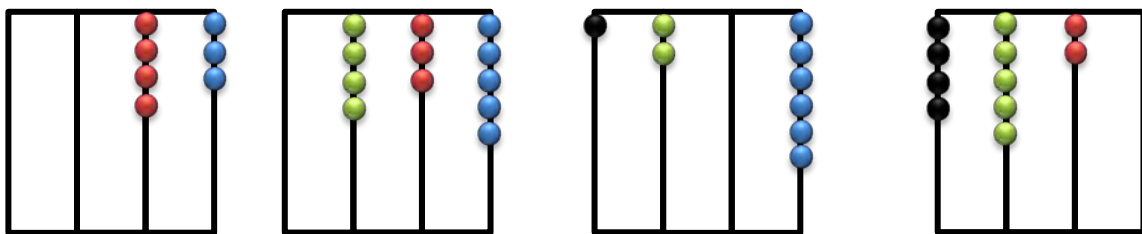
Antieke voorstelling van getalle (2)

Indiese wiskundiges het die idee van zero as 'n plekhouer uitgevind. Hulle het 'n telbord met krale wat getalle voor stel ontwerp. Een van die stafies op die bord was gebruik om leë plekke in getalle voor te stel, met ander woorde om zero syfers te wys.



Wat sou gebeur het as daar geen plek op die bord was om zero syfers voor te stel nie?

Watter getalle was voorgestel op die antieke telborde hieronder?
Skryf die getalle neer en lees hulle hardop.



Skryf die getalle hierbo in uitgebreide notasie, dit is breek die getalle op in Ene, Tiene, Honderde en Duisende.

Die *kol* wat Indiese wiskundiges uitgevind het het later 'n *sirkel* geword – die zero wat ons vandag gebruik.

Die woord *digit* (syfer) beteken 'vinger'. Outydse mense het hul vingers en ander liggaamsdele gebruik om te tel en te bereken. 305 is 'n 3-syfergetal en 3 500 is 'n 4-syfergetal.

Broken Calculator keys

Some of the keys on a calculator are broken.

The only keys that work are



(a) How would you make the number 510?

(b) Now make the number 643.

(c) Now make the number 401.

(d) Double 401.

(e) Make the number 884.

(f) Now half the number.

Stukkende Sakrekenaar Sleutels

Sommige sleutels op 'n sakrekenaar is stukkend.

Die enigste sleutels wat werk is



(a) Hoe sal jy die getal 510 maak?

(b) Maak die getal 643

(c) Maak nou die getal 401

(d) Verdubbel 401

(e) Maak die getal 884

(f) Halveer nou die getal

Creating Numbers



a. Which number fit the description below?

- Less than a thousand
- The sum of the digits is three
- More than two digits
- All digits are different
- An even number

Create as many numbers as you can.

b. The giant in *Jack and the Beanstalk* counted his golden eggs like this:

fee	fie	fum	fot	feefot	fiefot	foefot	fumfot	fotfot	feefotfot
-----	-----	-----	-----	--------	--------	--------	--------	--------	-----------

Which word did the giant use for twenty?

c. The reading on a car's speedometer is 12 342 km.

What is the least number of kilometers that the car must travel for the speedometer to have the same digit in every place in the reading?

Ontwerp Getalle



a. Watter getal pas die beskrywing hieronder?

- Minder as 'n duisend
- Die som van die syfers is drie
- Meer as 2 syfers
- Al die syfers is verskillend
- 'n Ewe getal

Ontwerp so veel getalle as jy kan.

d. Die reus in *Jannie en die Boontjierank* het sy goue eiers só getel:

fee	fie	fum	fot	feefot	fiefot	foefot	fumfot	fotfot	feefotfot
-----	-----	-----	-----	--------	--------	--------	--------	--------	-----------

Watter woord het die reus vir twintig gebruik?

c. Die lesing op 'n motor se spoedmeter is 12 342 km.

Wat is die minste aantal kilometers wat die motor moet ry sodat vir die spoedmeter dieselfde syfers in elke plek in die spoedlesing wys?

Hidden Numbers

In the boxes below are clues to find hidden numbers in the number square below. Use the clues to find the numbers. Circle the numbers in grid. Write the clue numbers next to the numbers you found.

A 1 unit 8 hundreds 6 thousands 3 tens	B 7 tens 9 units 3 hundreds	C 1 thousands 2 tens 9 units	D 5 thousands 3 hundreds 1 tens 8 units	E 9 units 6 thousands
F 8 tens 9 hundreds 3 units	G 9 units 2 tens 4 hundreds 7 thousands	H 1 thousands 4 hundreds 1 tens 3 units	I 5 units 8 hundreds 2 thousands	J 3 hundreds 4 units 5 thousands

The first one has been done for you.

	3	3	1	2	5	3	1	2	7	3
A	8	6	8	3	1	1	4	8	5	2
	1	0	2	9	1	2	5	3	1	8
	7	5	3	0	4	6	9	8	3	0
	4	3	9	3	4	2	2	8	0	5
	1	7	3	7	5	2	3	0	7	5
	1	3	7	9	0	5	1	8	2	3
	1	2	9	0	8	3	6	0	0	9
	1	7	4	2	9	1	8	2	3	8
	3	6	3	2	3	1	1	4	1	3

Versteekte Getalle

In die raampies hieronder is wenke om versteekte getalle in die getalvierkant daaronder te vind.

Gebruik die wenke om die getalle te vind. Omkring die getalle in die vierkant. Skryf die letters van die wenke langs die getalle neer.

A 1 ene 8 honderde 6 duisende 3 tiene	B 7 tiene 9 ene 3 honderde	C 1 duisende 2 tiene 9 ene	D 5 duisende 3 honderde 1 tiene 8 ene	E 9 ene 6 duisende
F 8 tiene 9 honderde 3 ene	G 9 ene 2 tiene 4 honderde 7 duisende	H 1 duisende 4 honderde 1 tiene 3 ene	I 5 ene 8 honderde 2 duisende	J 3 honderde 4 ene 5 duisende

Die eerste een is gedoen vir jou.

	3	3	1	2	5	3	1	2	7	3
A	8	6	8	3	1	1	4	8	5	2
	1	0	2	9	1	2	5	3	1	8
	7	5	3	0	4	6	9	8	3	0
	4	3	9	3	4	2	2	8	0	5
	1	7	3	7	5	2	3	0	7	5
	1	3	7	9	0	5	1	8	2	3
	1	2	9	0	8	3	6	0	0	9
	1	7	4	2	9	1	8	2	3	8
	3	6	3	2	3	1	1	4	1	3

Missing numbers

Which numbers should be written in the shaded squares?

0									9

Ontbrekende Getalle

Watter getalle moet in die gekleurde vierkante geskryf word?

1000

2000

3000

4000

5000

6000

7000

8000

9000

1000

2000

3000

4000

5000

6000

7000

8000

9000

100

10

1

200

20

2

300

30

3

400

40

4

500

50

5

600

60

6

700

70

7

800

80

8

900

90

9

100

10

1

200

20

2

300

30

3

400

40

4

500

50

5

600

60

6

700

70

7

800

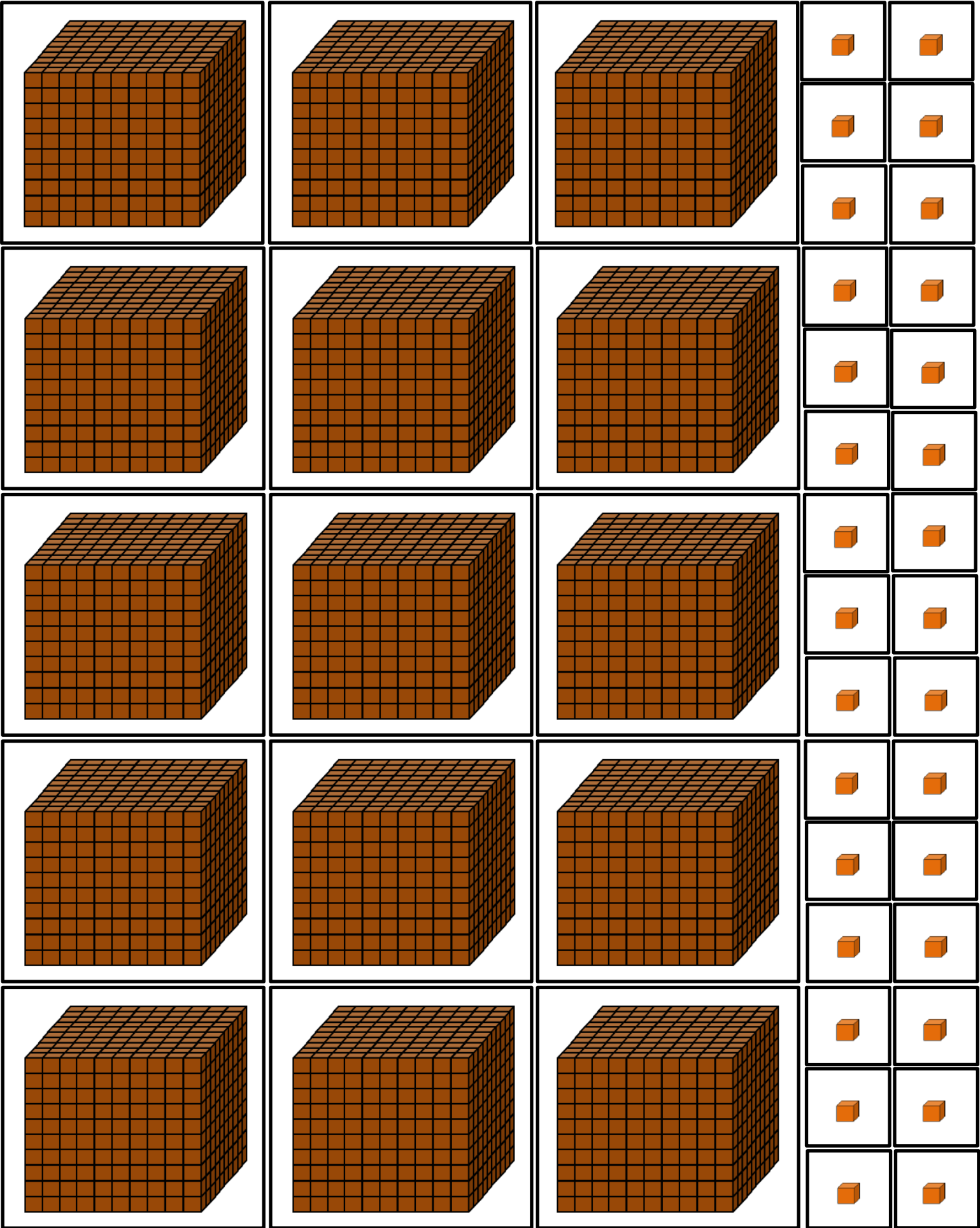
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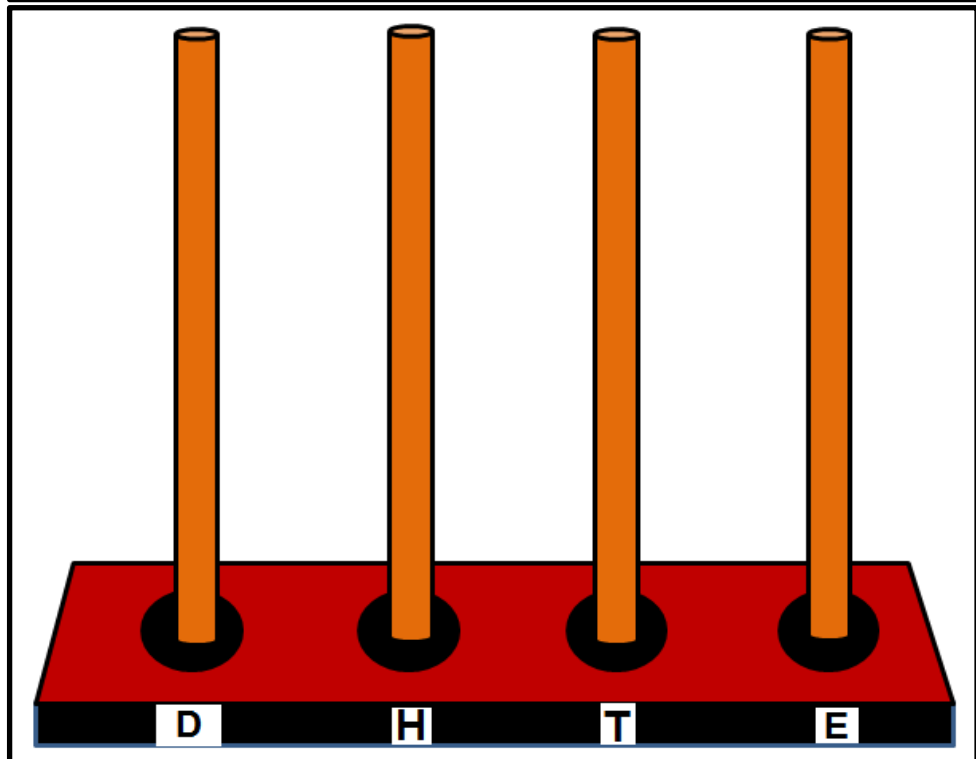
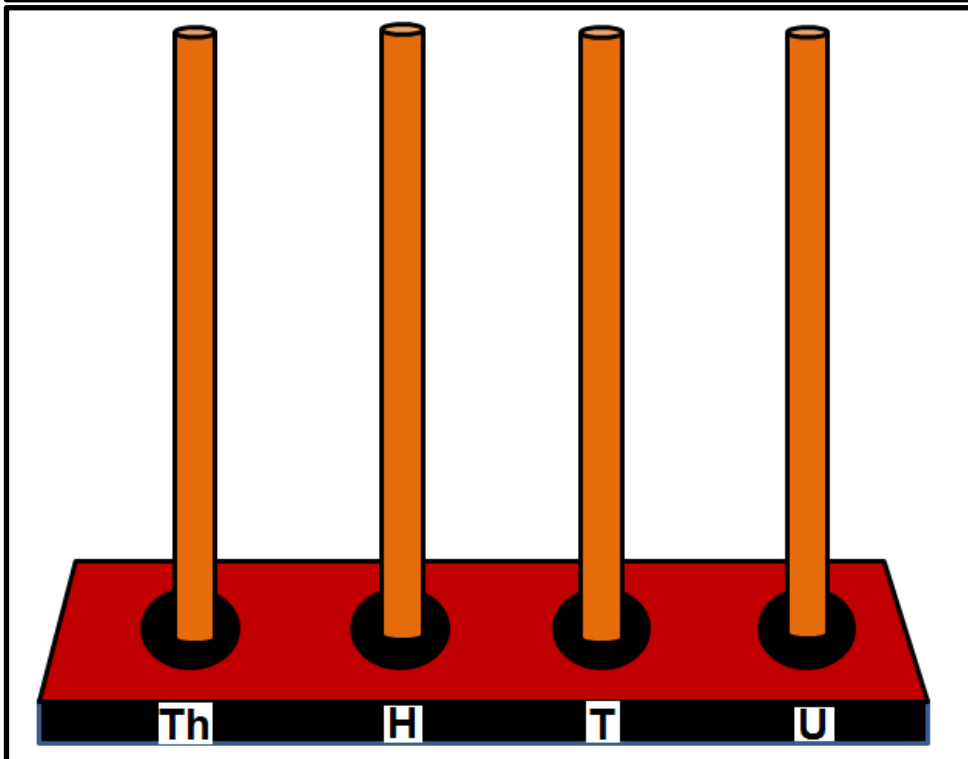
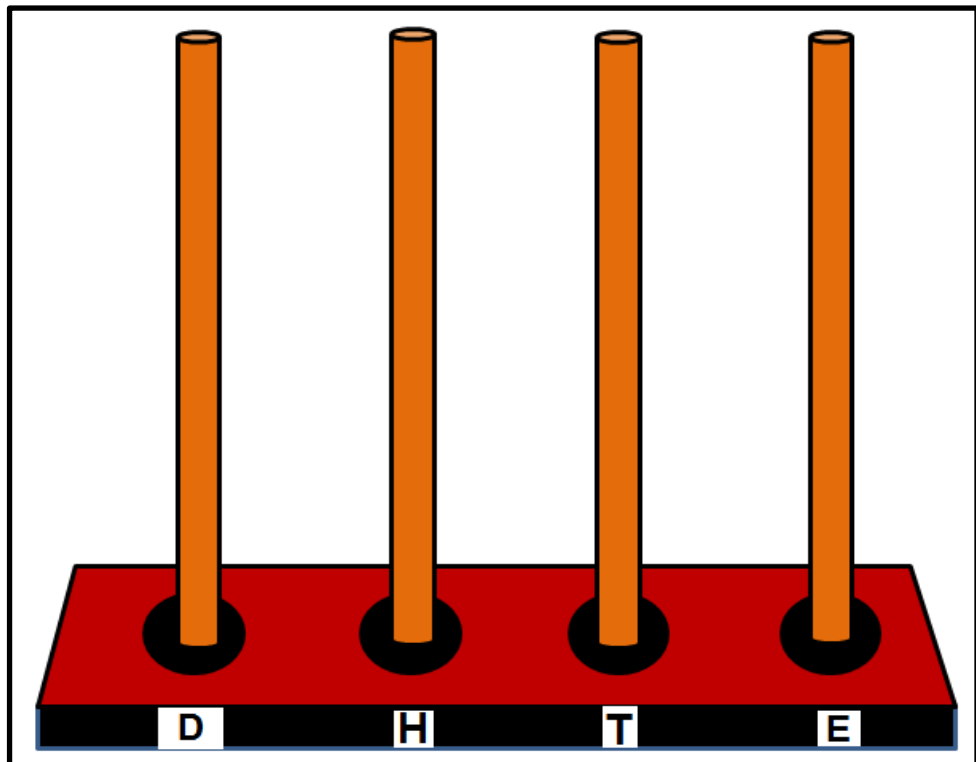
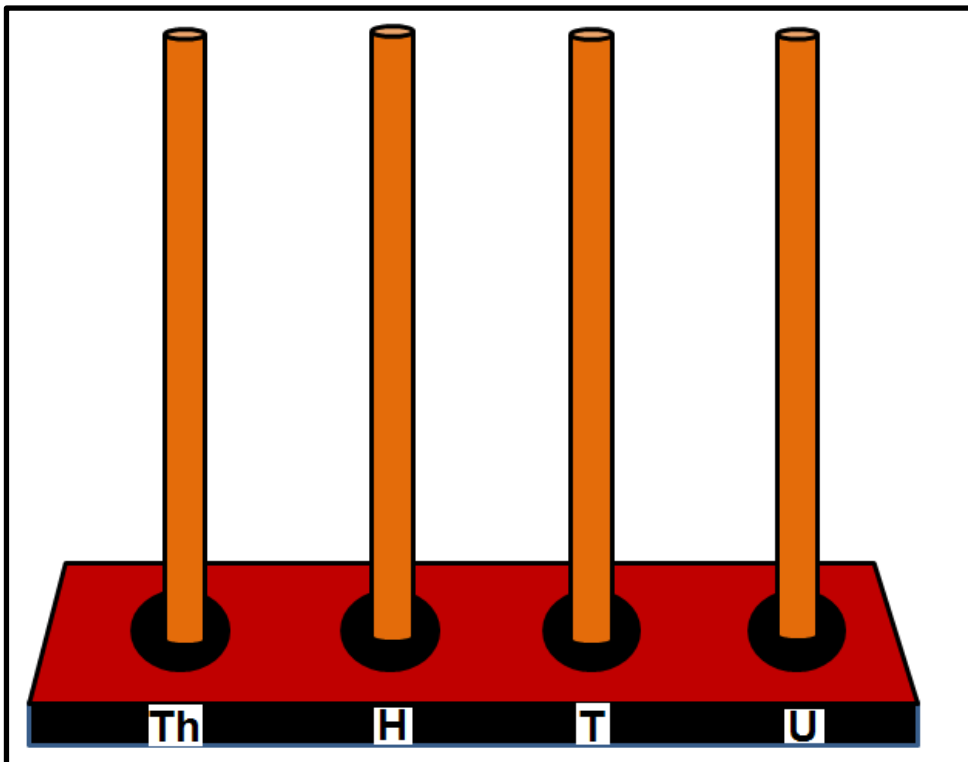
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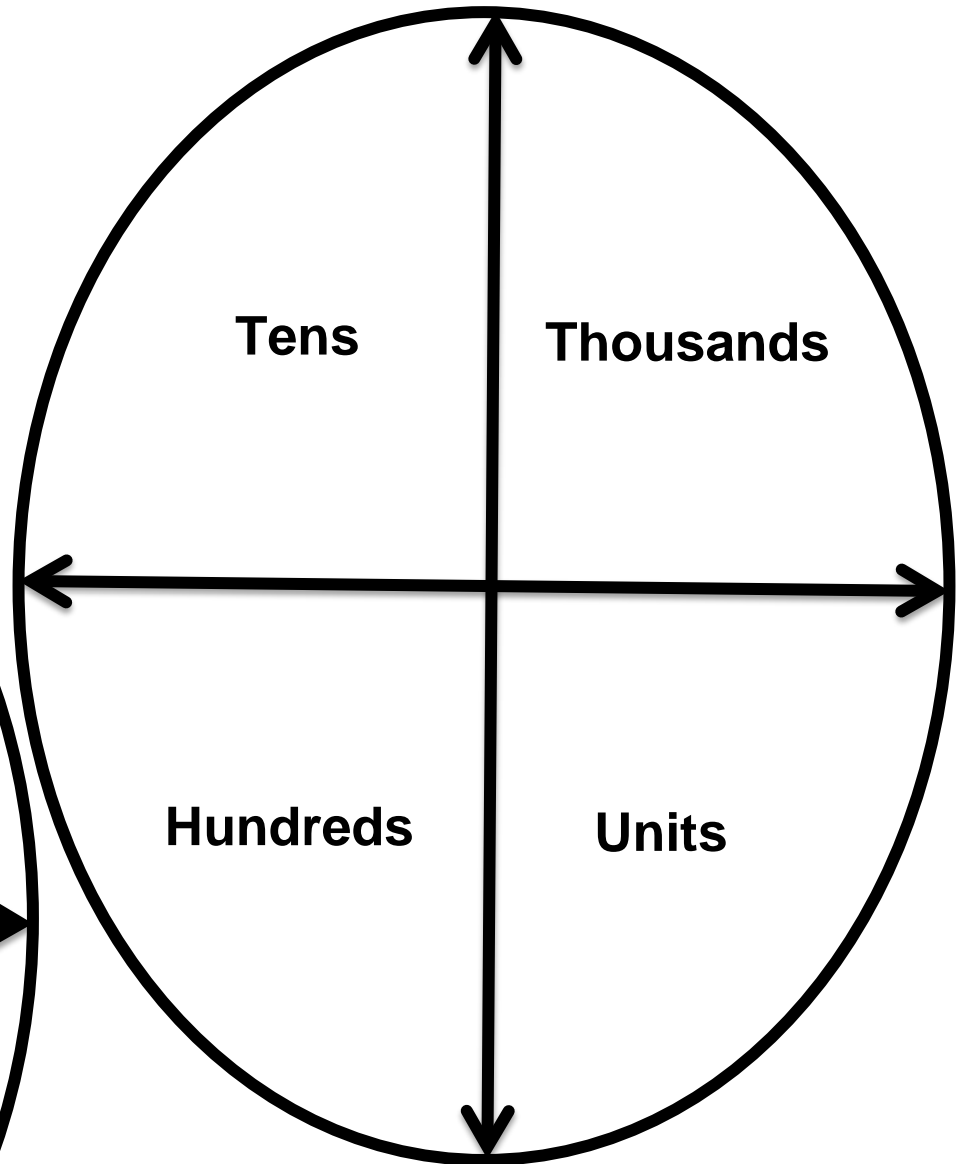
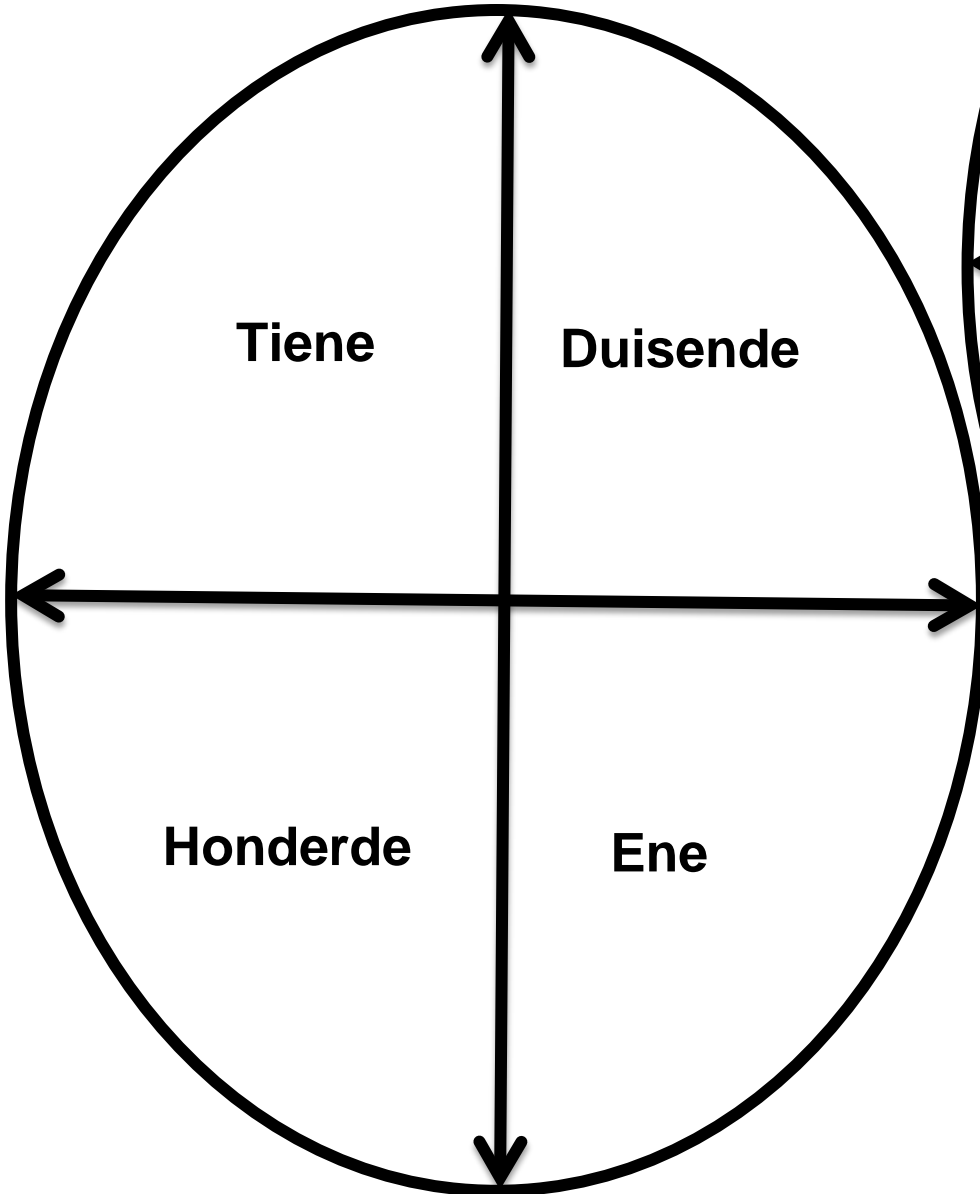
900

90

9







A	0	1	2	3	4	5	6	7	8	9
B	0	1	2	3	4	5	6	7	8	9
C	0	1	2	3	4	5	6	7	8	9
D	0	1	2	3	4	5	6	7	8	9

A				
B				

C				
D				

Place Value Spin Board

- This game is for 2 or 4 players.
- Each player gets a game board like this.
- Take turns to spin the place value spinner.
- Choose one of the digits 0 – 9
- Choose an empty square to write down the digit.
- The digit must be in the place shown on the spin board, e.g.
if the spinner lands on Tens, write the digit in the 3rd square.
- When all the squares are filled, calculate the sum of your numbers.
- The one with the highest number is the winner.