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Using fluency activities in after school maths clubs to enhance learner performance in the primary grades

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ABSTRACT

A predominance of finger counting strategies in South Africa's primary learners indicates a need for learners to develop a flexibility and fluency of working with numbers. One aspect of my recent doctoral research study investigated learners' mathematical proficiency progress whilst participating in Grade 3 after school maths clubs over the course of a year. Drawing on literature, I designed a series of written activities aimed at supporting club learners to improve their fluency in a range of ways. These activities provided me as researcher and club facilitator with a fast way of evaluating and encouraging learner progress, whilst enabling the learners to practice the fluency they were developing through other activities of the club. Whilst the dual focus proved successful in the research clubs, an earlier publication investigated the emergent issues with the use of these activities in the clubs. This article extends that earlier work in that I explore the rationale for the design and development of these fluency activities by locating them in the broader literature, pointing to their usefulness as one way to enhance learner performance in early grade classrooms. Additionally, I report on findings of learner progress from learners in the two research clubs with regard to these activities.

Keywords: fluency, primary after school maths clubs, numeracy, research and development, fluency activities, mathematical proficiency

INTRODUCTION AND CONTEXT

National and international assessment results, research studies and reports point to South Africa as having educational challenges, specifically with mathematics, science and language (Fleisch, 2008; Reddy, 2006). Addressing some of these issues is a key aim for the South African Numeracy Chair (SANC) project, the context for the research detailed here. As a member of the SANC project, I co-ordinate and facilitate a number of primary after school mathematics clubs. During my doctoral research, two Grade 3 after school maths clubs formed the empirical field. As the maths club co-ordinator I am responsible for the club-learning programme design and related facilitator training of the maths club programme for the project and thus my work within the project is focussed on both development and research in the field of numeracy.

Learner-focused activities within the project foreground the importance of numeracy as well as to create a "maths is fun" ethos in schools. Learner after-school maths clubs are a key aspect of the project and provide a direct learner focused intervention. The SANC project clubs are defined as an extra curricula activity focused on developing a supportive learning community where learners' active mathematical participation, engagement and sense making are the focus. Individual, pair and small group interactions with facilitators are the dominant practices with few whole class interactions. Activities in the clubs are designed for both the recovery and extension of learners (Graven, 2011; Graven & Stott, 2012).

Working from a broad Vygotskian perspective of learning and development, the research that this study is based on had a dual focus and investigated how Grade 3 learners' mathematical proficiency progressed (or not) whilst participating in after school maths clubs over the course of a year, and additionally explored how the mediation offered in the clubs enabled or constrained the emergence of zones of proximal development (ZPD) and thus learning for the club learners. This article is based on data gathered with regard to the first part of the research aim in the two after school clubs.

Debbie: Using fluency activities in after school maths clubs

In running after school clubs since 2011, I have seen many instances where learners are not working efficiently or fluently because they are constrained by their lack of basic foundational mathematical knowledge. For example, it has been noted that basic number bonds of 10 and 20 are not recalled automatically for the majority of the learners. The learners rely on counting by ones or their fingers to work out, for example, the answers to $3 + 7$ or $3 + 97$. Their grasp of basic number sense and basic bonds is severely limited and few learners seem to have other strategies to solve problems or to work efficiently and fluently with numbers. This means that, although they do arrive at an accurate answer, they tend to be slow in getting there.

The issues with South African education and particularly its performance in mathematics are widely publicised in reports, papers, books and news articles (see for example Fleisch, 2008; Schollar, 2008; Spaul, 2013; Taylor, 2008). Fleisch (2008) focuses particularly on the literacy and numeracy 'crisis' in primary schools, whilst a paper commissioned by the Presidency (Taylor, Fleisch and Shindler, 2008) states that poor progress of students in high school and further education is due to the inadequate preparation of primary learners, particularly in literacy and numeracy.

The 2014 Annual National Assessment results (Department of Basic Education, 2014) reflect this poor performance. The 2014 results show that nationally, grade 3 results have increased by 15 percentage points since 2012, grade 4 results have stayed the same and grade 9 results have decreased down to 11%. There is an increase in the number of learners achieving 50% or more for both grades 3 and 6. However, only 3% of grade 9 learners achieve over 50% in the assessment and the report concludes "the performance of learners in Mathematics in the intermediate and senior phases is cause for concern" (p. 104).

Furthermore, Schollar (2008) points out that a 'cognitive backlog' compounds the problem of poor performance as many classes can be considered multi-grade classes. As a result, classrooms have a large range of learner levels making it difficult to "consistently teach to the required assessment standards for any particular grade" (ibid, p. 5) as teachers need to address the gaps in learners' mathematical knowledge which should have been grasped in earlier grades (Carnoy et al., 2011). Schollar (2008) concludes

the majority of South African learners are not developing any kind of understanding of the base-10 number system and the associated critical understanding of place value. They cannot mentally, or in writing, manipulate numbers, especially when they are large or contain fractions, do not readily understand the meaning of multiplication and division and cannot use the skills of borrowing and carrying upon which all more complex calculations depend (p.6).

Given these well-documented issues with learner mathematics achievement in South Africa, early intervention in the Foundation and Intermediate Phases is essential (Atweh, Bose, Graven et al., 2014). The SANC project as a whole therefore works towards improving mathematical proficiency in Foundation Phase learners. In this article, I explain timed fluency activities used in the two research clubs as both a complementary research and developmental tool for assessing the changing levels of learners' mathematical proficiency over time as well as supporting learners to improve their fluency, specifically in speed and accuracy of recall of the basic facts such as halving and doubling and point to their potential usefulness in early grade classrooms. Using ideas derived from literature, these activities were initially intended to be part of the range of mental activities promoted in the clubs. As they took only ten minutes and were easy to administer, it soon became clear that these were useful as a research data collection instrument and in the research study they were used to supplement the more detailed, rich one-to-one learner interview data which was the main data collection method.

My decision to include these as positive learning opportunities in the context of my maths clubs relied largely on facilitating the development of number sense in the club learners as well as developing the necessary club ethos in which to use them, ensuring that they did not create anxiety or stress for the

learners. As such, not only did the activities provide a mechanism for the learners to practice the fluency they were developing through other activities of the club but they also provided me, as researcher and facilitator, with a quick way of evaluating, encouraging and valuing learner progress. While these activities were useful in this form, in their initial use in the research clubs, ethical tensions arose for me. These activities are by design time pressured and can understandably be stress inducing for learners. Indeed, a sizable body of work puts forward an argument against timed activities in mathematics (see for example Boaler, 2012, Burns 2007). In this article, I briefly touch on how these were addressed, although elsewhere I have addressed these emergent ethical tensions in more detail and explored specifically how supplementary learner reflections made the use of these activities more holistic for the learners as they were able to reflect on their own progress in mathematics (Stott & Graven, 2013).

Theoretical / conceptual framework and literature review

Vygotsky's (1978) view of learning and development formed the broader theoretical framework for this doctoral study. His "general genetic law of cultural development" conceptualised development as the transformation of socially shared activities into internalised processes, arguing that higher mental functioning appears first on the social level and then on the individual level.

Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people ... and then inside the child. This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher [mental] functions originate as actual relations between human individuals" (ibid, p.57).

Vygotsky saw learning and development as dialectically linked and described the dialectical nature of learning and development thus:

learning awakens a variety of internal-development processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. ... learning is not development; however, properly organised learning results in mental development and sets in motion a variety of developmental processes that would be impossible apart from learning. Thus learning is a necessary and universal aspect of the process of developing culturally organised, specifically human, psychological functions (ibid, p. 90).

With the focus in the clubs on group and paired interactions as well as the promotion of mathematical talk between participants, the notion of learning through interaction and cooperation with peers was central to the study and to the design of any club activities. Although the broader research study worked with both the individual and social aspects of learning and development, the focus for this article and the research question under consideration, was on individual learners and their mathematical proficiency progression over time. This focus on the individual learner in the broader perspective of Vygotskian learning and development allowed me to describe each learner's mathematical proficiency progression (in terms of what they internalised) during his or her participation in the club. Through analysis of data collected via these fluency activities and more detailed learner interviews, over time I was able to see how learners evolved and whether learning was taking place for each individual learner i.e. how they were coming to know, acquire or internalise mathematical knowledge.

Mathematical proficiency was the key conceptual construct for research under review here, as well as a central construct for the broader SANC project. The 2001 report called '*Adding it Up: Helping Children Learn Mathematics*' (Kilpatrick et al., 2001) maintained that all students should and can be mathematically proficient as proficiency is an important foundation for further instruction in maths as well as for further education in fields that require maths. Successful mathematics learning ('*mathematical proficiency*') comprises five *interwoven* strands: *conceptual understanding* (comprehension of mathematical concepts,

operations, and relations), *procedural fluency* (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), *strategic competence* (ability to formulate, represent, and solve mathematical problems), *adaptive reasoning* (capacity for logical thought, reflection, explanation, and justification) and *productive disposition* (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy) (ibid).

These five strands are separate yet interwoven and interdependent. Mathematical proficiency is only present when all strands are developed and proficiency develops over time. These proficiencies, when taken together, can lead to progressive mathematics learning. For the purposes of this article I shall concentrate on unpacking procedural fluency as the foundation for the fluency activities described here.

Procedural fluency, number sense and fluency

In expanding the short definition of procedural fluency given above, Kilpatrick et al. (2001) state that procedural fluency is “knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently” (p. 121). In contrast, number sense is a broader notion that can be described as a ‘feel’ for numbers, an ability to “observe patterns and relationships and make connections between numbers” (Anghileri, 2006 p. 2) and is often referred to as flexibility in thinking and working with number.

While the promotion of all five strands of mathematical proficiency was the overarching developmental focus for the learners in these research clubs, specific emphasis was given to the development of number sense and fluency. Research carried out by Ensor, Hoadley, Jacklin et al. (2009) and my own experience working with the learners in my local context suggests that number sense is largely absent in many of our South Africa's learners and concrete one-to-one methods of calculation and finger counting strategies continue to dominate throughout the primary grades. Ensor et al.'s research and my observations, provided support and a rationale for an emphasis on development of number sense and fluency in the young club learners. I turn now to review the notion of fluency in the literature.

Mathematics educators have a diverse range of definitions for fluency: speed, accuracy, mastery of facts, rapid recall and computation skills are some examples. However, research has shown that learning number facts and number sense through engaging activities that focus on mathematical understanding, is preferable to learning through rote memorisation (Boaler, Williams & Confer, 2014). Russell (2000) concurs and suggests that fluency demands more than memorisation of procedures and rests upon understanding of numbers, operations, place value and their relationships; learners need to understand *why* they are doing what they are doing and *know when it is appropriate* to use different methods. In line with Kilpatrick et al. (2001), she indicates that fluency consists of three elements: efficiency, accuracy and flexibility (ibid). *Efficiency* means that children do not get caught up in too many steps or lose track of the logic of the strategy. *Accuracy* refers to aspects like careful recording, checking results as well as knowledge of number facts and other important number relationships. Finally, *flexibility* relates to choosing an appropriate strategy for the numbers involved and being able to use more than one method to solve a problem.

Askew (2012) also speaks of “elements of fluency” but his elements are about knowing both basic facts and basic methods such as recalling addition and subtraction number bonds with numbers up to 20, adding a multiple of 10 or 100 to any number, multiplying any number by 2 or 10 and multiplication up to 10 x 10. He also argues that when children are not fluent in knowing and using these basic facts and methods, their mathematical learning is impeded. Being fluent means that working memory is freed up when working with a “more interesting and engaging piece of mathematics” (p. 54). Being fluent allows one to think about the more interesting piece of mathematics rather than being preoccupied with how to work out the basic facts. He argues further that lack of fluency in basic facts and methods can impede conceptual understanding because constantly working out the basics before doing anything else can take

up too much working memory and attention is diverted from thinking about the bigger picture. This coheres with Russell's notion of efficiency.

This brief review reveals that whilst there are a range of definitions and opinions regarding fluency in the literature, the argument for developing fluency in basic facts is convincing, assuming that the development is grounded in understanding and flexible thinking. Learning to be fluent is intertwined and developed along with sense making, flexible thinking and should involve more than memorising facts and procedures (Askew, 2012; Boaler, 2012; Burns, 2007; Gojak, 2012).

The aspects highlighted in this review were important considerations in my decision to use these fluency activities in the clubs. My focus was on developing efficiency and accuracy as an alternative to finger counting strategies that are predominant approaches to solving problems with South African learners. This interwoven view of fluency was thus adopted for this study and formed the basis for the design of the fluency activities as both data collection instruments and club activities.

Earlier I spoke about the objections raised in the educational literature and the ethical tensions that emerged for me with the use of these timed activities. Notwithstanding these objections, many mathematics educators consider that fluency can be developed and improved with practice without the need for rote learning or drilling (see for example Kilpatrick et al., 2001). But how does one develop fluency without promoting rote memorisation or creating the stressful situations that are associated with timed activities? Askew (2009) submitted that *thoughtful* use of different types of practice can help develop fluency and suggests that the development of such should involve "practice and problem-solving, understanding and (appropriate) rapid recall" (p. 27).

Learners can be encouraged to participate in different types of practice to develop fluency. For example *guided practice* can assist learners who still need the help of a more experienced other to keep them on track; *over-practice* can assist learners to practice the same thing every day and finally *deliberate practice* can be used to make the practice harder by setting a time limit or by working with bigger numbers (Askew, 2002). Deliberate practice can be seen as continually stretching the learner to just beyond his or her current abilities (Askew, 2002; Pegg, 2010). The activities discussed here were a form of deliberate practice for the learners, as a time limit was applied.

Askew (2009) shared his ideas for using these controversial timed activities with a different focus and emphasis, arguing that they can be motivating for the learner if used and presented in thoughtful ways in the classroom and as a form of deliberate practice. This motivational aspect was an additional consideration in the design of these fluency activities as club activities and I included aspects of self-reflection into the design of the activities (see Stott & Graven, 2013 for further information).

Developing fluency in the research clubs

The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways (Boaler et al., 2014 p. 3)

This was the rationale for the promotion of a variety of fluency activities in the clubs. I have mentioned numerous times that fluency in the clubs was developed in many ways and that these fluency activities were just one way for the learners to practise this fluency. To provide a more complete picture of how fluency was developed in the clubs based on the ideas given in this review, I explain briefly some of the other approaches used. Askew (2009) advocates that "little and often" (p. 28) rather than less frequent, longer blocks of time, is a good approach to developing fluency in young learners. With only an hour at the most allocated to club time, many of the activities in the clubs that promote fluency work from this basis. The activities promoted are easy to set up, explain and the sessions are kept to 15 to 20 minute blocks.

Number talks were used as one strategy for simultaneously developing number sense and mathematics facts in the club learners (Boaler et al., 2014). Originally developed by Ruth Parker and Kathy Richardson

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in the early 1990s, number talks were conceptualised as short teaching activities that teachers can use as lesson starters. Boaler (2015) indicates that a number talk involves posing a maths problem such as 18×5 and asking learners to solve the problem mentally, individually and without pencils and paper. The teacher then asks the learners to share the different methods used to solve the problem and importantly discusses why they work. By using number talks, teachers can, in a short space of time, change learners' view of mathematics, teach them number sense, help them develop mental maths skills and engage them in creative, open mathematics (ibid). I adapted and used number talks with age/grade appropriate prompts to encourage these aspects and to encourage learner mathematical talk (Stott & Graven, 2015). Additionally, I found that by playing simple, short, focussed mathematical games with cards and dice with few rules aided the development of core number skills, mental agility and fluency in number.

Methodology

The study was a longitudinal, largely qualitative multi-site case study in two Grade 3 after school maths clubs during 2012. The Elmtree Prep club comprised of 7 learners and the Luhlaza Primary club comprised ten learners.

In order to track progress (or lack thereof) in mathematical proficiency of the club learners over time, I collected data in two forms: firstly from one-to-one Mathematical Proficiency interviews (adapted from Wright, Martland, Stafford et al., 2006 and Askew, Brown, Rhodes et al., 1997) and secondly from the fluency activities described here. The interviews were administered in March and November of 2012 and the fluency activities in March and July/August 2012. The nature of the one-to-one interviews and findings for these club learners have been published in Stott (forthcoming). I turn now to the design of the instrument, describe how it was administered in the clubs and how data was derived from it for analysis purposes. Although much of what is covered here appears in other publications work (Stott & Graven, 2013; Stott, 2013), I include the necessary details for an understanding of the data presented later.

The timed fluency activities consisted of six different individual activities, each of which was completed in a specified amount of time. Table 1 below describes and gives a sample of each activity, details the time allocation and total marks for each activity. These examples are cropped to save space.

Table 1: Description, details and sample of each of the 6 activities (Stott, 2013)

Activity type	Description & Sample	Time allocation	Activity out of ...																		
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Each of the six activities was printed onto a script and administered one at a time to the learners in both research clubs during club time in March and July / August of 2012. The learners named and dated the scripts and after the learners had completed the activities in the times detailed in Table 1, the scripts were collected. After the club, the scripts were used to work out the different scores detailed below. The scores were not shared with the learners or used to make public comparisons between learners. The resultant scores were then entered into a spreadsheet for analysis purposes.

The notions of efficiency and accuracy discussed earlier were used these as the analysis indicators for this fluency activity data. Each fluency activity provided *three* different scores:

1. **Actual score:** number of items the learners answered correctly in the activity. This is a simple overall mark out of total possible marks
2. **Completion %:** number of answers the learners actually completed
3. **Accuracy %:** number of correct answers out of those completed

These scores revealed the *speed* the learner worked at and how *accurate* that working was. Specifically, the *completion* and *accuracy* percentages allowed tracking of both efficiency and accuracy in the following

ways. A completion rate was derived from the *speed* (or how *efficiently* the learners were working) at which the learners answered in the given time period. An accuracy rate was derived from the number of *correct answers* completed in that time (Stott, 2013; Stott & Graven, 2013).

For each activity in March, each learner had three scores and another three for the activity in July/August. I compared the scores from each activity to see where the learners were specifically making progress. In doing this, it was essential to review both the *completion* and *accuracy* scores together to get a meaningful picture of the learner's progress over time. I illustrate this using an example of the doubling activity for a hypothetical learner called Vuyo.

Table 2: Hypothetical doubling activity administered to Vuyo in March and August (derived from Stott, 2013)

	Total marks for Doubling activity	Scores		
		Actual score	Completion %	Accuracy %
March	17	7 items correctly answered ∴ 7 out of 17 = 41%	10 items answered in total ∴ 10 out of 17 = 59%	7 out of 10 answered correctly 70%
August	17	12 items correctly answered ∴ 12 out of 17 = 71%	15 items answered in total ∴ 15 out of 17 = 88%	12 out of 15 answered correctly 80%

The doubling activity has a total of 17 possible marks, one mark per item/response. In March Vuyo answered 7 of the 17 items correctly, giving an actual score of 41%. However, in looking at the other scores, we see that Vuyo answered 10 of the possible 17 items (59% *completion*) and that of these 10 answers, 7 were correct (70% *accuracy*). This gives a significantly different perspective to the 41% overall score, revealing that even though he did not complete much of the assessment, 70% of his what he did complete was correct.

In August, Vuyo answered 12 of the 17 items correctly, giving an actual score of 71%. Looking at completion and accuracy we see that Vuyo answered 15 of the possible 17 items (88% *completion*) and that of these 15 answers, 12 were correct (80% *accuracy*). Comparing the second set of activities to the first, we can see that his *actual* score for this doubling activity increased by 30 percentage points from 41% to 71%, the *completion* score increased from 59% to 88%, and the *accuracy* score increased from 70% to 80% indicating that Vuyo made progress from March to August on all fronts.

Findings

In this section I present and discuss the results from these fluency activities for each learner in the two case study clubs. I review the results for four of these activities as not all were administered to all learners due to absenteeism at the required times. Using the scores described above, I present the results in terms of changes over time in scores with a specific focus on *completion* and *accuracy*, although for fullness, the *actual* score is also shown for each learner. Learner names have been changed for confidentiality reasons.

Club One: Elmtree Prep

Figure 1 shows that the overall club average for fluency activities administered in March 2012 had a 79% completion rate and a 94% accuracy score. In the activities administered in August 2012, the overall club average completion rate had increased by 18 percentage points to 97% and the accuracy score by 4 percentage points to 98% across the four activities for the six¹ club learners in Table 3 below. Thus overall, the learners increased their speed of completion in the intervening 5 months.

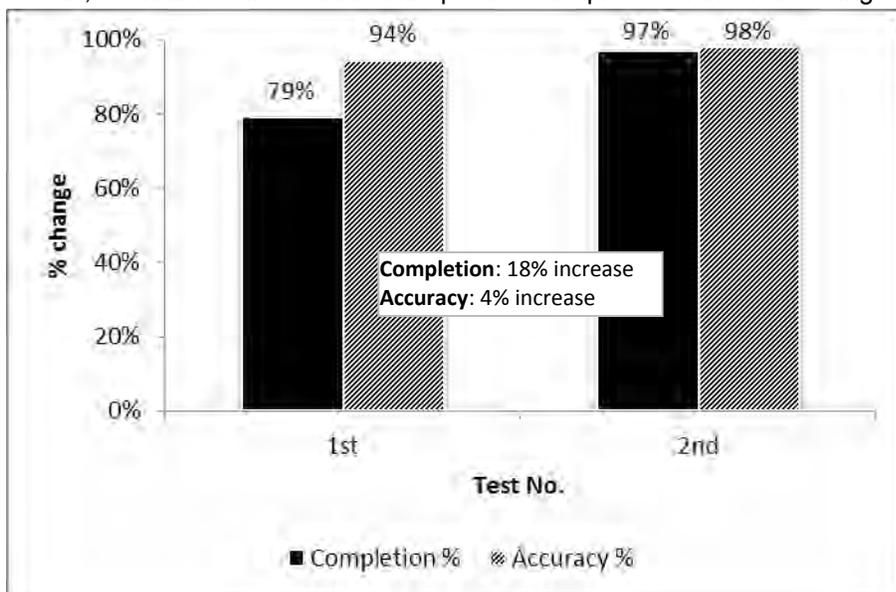


Figure 1: Overall club fluency activity averages: completion & accuracy - Elmtree Prep (six learners)

From the individual scores shown in Table 1, I highlight some results worth noting. For Zac, Themabela and Zintle accuracy reduced slightly in some of the second round activities. Anathi, Kholeka and Cebisa for the most part maintained and increased both their completion rates and accuracy over time. Zac's results are mixed. Having increased his completion rate in *add and subtract to 10 and 100*, his accuracy lowered on these. On the other hand, he maintained his completion rate in *doubling and halving* and achieved 100% accuracy for *halving*.

Themabela's results showed substantial increases in completion for *add and subtract to 10 and 100* (50 and 60 percentage points respectively), although this increased speed seemed to affect her accuracy for *add and subtract to 10*. However, she achieved 100% accuracy in the other three activities. Zintle also showed significant increases in the number of items answered in 3 activities and she made slight increases in accuracy in two activities. Her accuracy on *halving* however decreased slightly over time. The increase in completion and accuracy scores pointed to improved efficiency and fluency. These findings pointed to these learners becoming faster at both writing and recalling the basic facts with little compromise to accuracy as this mostly increased across the activities and learners as well.

Table 3: % change for each learner for each fluency activity in Elmtree Prep

Learner	% ANSWERED (COMPLETION)			% ACCURACY			ACTUAL SCORE		
	1 st	2 nd	%	1 st	2 nd	%	1 st	2 nd	%
/									

¹ Nate was absent when the second set of activities were administered, thus none of his scores are included and results are only reported for six of the seven club learners

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activity	activit y	activit y	Change	activit y	activit y	Change	activit y	activit y	change
Anathi	20%			3.75%			22%		
Add & subtract 10	35%	95%	60%	86%	100%	14%	30%	95%	65%
Add & subtract 100	80%	100%	20%	94%	95%	1%	75%	95%	20%
Doubling	100%	100%	0%	100%	100%	0%	100%	100%	0%
Halving	100%	100%	0%	100%	100%	0%	100%	100%	0%
Cebisa	6.5%			4.5%			10%		
Add & subtract 10	80%	95%	15%	88%	100%	13%	70%	95%	25%
Add & subtract 100	95%	100%	5%	95%	100%	5%	90%	100%	10%
Doubling	100%	100%	0%	100%	100%	0%	100%	100%	0%
Halving	94%	100%	6%	94%	94%	0%	88%	94%	6%
Kholeka	1.5%			5%			7%		
Add & subtract 10	95%	100%	5%	95%	100%	5%	90%	100%	10%
Add & subtract 100	100%	95%	-5%	85%	100%	15%	85%	95%	10%
Doubling	94%	100%	6%	100%	100%	0%	94%	100%	6%
Halving	100%	100%	0%	100%	100%	0%	100%	100%	0%
Zac	11.25%			-2.25%			11%		
Add & subtract 10	70%	100%	30%	100%	90%	-10%	70%	95%	25%
Add & subtract 100	85%	100%	15%	100%	95%	-5%	85%	95%	10%
Doubling	100%	100%	0%	100%	100%	0%	100%	100%	0%
Halving	100%	100%	0%	94%	100%	6%	94%	100%	6%
Themabela	27.5%			5%			25%		
Add & subtract 10	35%	85%	50%	100%	94%	-6%	35%	80%	45%
Add & subtract 100	35%	95%	60%	86%	100%	14%	30%	95%	65%
Doubling	100%	100%	0%	94%	100%	6%	94%	100%	6%
Halving	100%	100%	0%	94%	100%	6%	94%	100%	6%
Zintle	31%			1.25%			29%		
Add & subtract 10	45%	75%	30%	100%	100%	0%	45%	75%	30%
Add & subtract 100	50%	85%	35%	90%	94%	4%	45%	80%	35%
Doubling	47%	100%	53%	88%	94%	7%	41%	94%	53%
Halving	88%	94%	6%	100%	94%	-6%	88%	88%	0%

Club Two: Luhlaza Primary

Figure 2 shows that the club average for the first set of fluency activities (administered in March 2012) had a 46% completion rate and 64% accuracy score. In the second set of activities (administered in July 2012), overall the club average completion rate increased by 22 percentage points to 68% and the accuracy score by 19 percentage points to 83%.

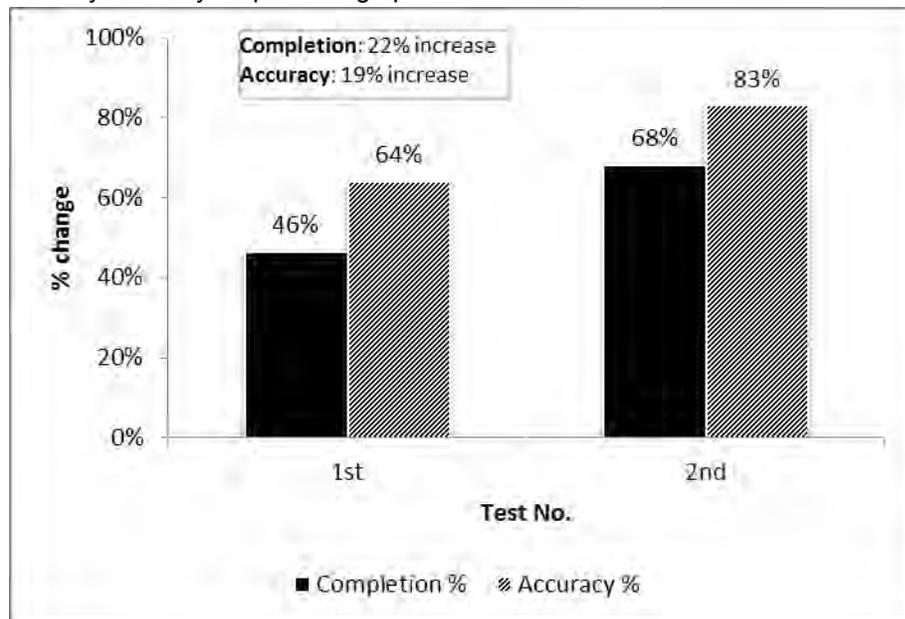


Figure 2: Overall club fluency activity averages: completion & accuracy - Luhlaza Primary (ten learners)

From the progress for each of the ten learners in March and July 2012 shown in Table 4 below, I highlight some notable results. Although all learners increased their average speed of completion across all activities in the second round, only two learners (Kuhle and Siphon C) completed any activities (as shown by 100% completion for *halving* and *add & subtract 10*). Other learners such as Chebe, Khule and Siphon N made noteworthy improvements in completion with an average completion rate across all activities of 31, 35 and 30 percentage points respectively.

Looking at individual activities, different learners made different gains in completion speed. For example Khule showed a 60% improvement in the *add and subtract 100* test. However, some learners completed less than they did in the first round of activities. See for example Akhona, Emi, Oma, Siphon C, Aphwe and Apolo.

With regards to accuracy, all learners apart from Siphon N and Apolo showed positive progress on average across their four activities. Noteworthy examples are Chebe and Akhona with an average accuracy percentage change of 40% or more. However, Siphon N's substantive increases in completion for the *halving* and *add and subtract 100* activities had a detrimental effect on his accuracy which showed negative change. The only activity where he shows positive progress for both completion and accuracy is for the *add and subtract 10* test.

Other learners for example Akhona, Oma and Siphon C showed regression with regards to accuracy in at least one test. For example Siphwe and Emi in *add and subtract 10*. Apolo's results are worth looking at. He showed a 65% improvement in his *add and subtract 10* activity but in both rounds of activities, he failed to achieve accurate answers for any activity except *halving* and his actual scores additionally showed that he made no progress over time in these fluency activities.

As with Elmtree Prep, the increases in completion and accuracy scores pointed to improved efficiency and procedural fluency for the majority of learners. With some individual exceptions (such as Apolo), the data

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pointed to learners becoming faster at both writing and recalling the basic facts with little compromise to accuracy as this mostly increased across the activities and learners as well.

Table 4: % change for each learner for each fluency activity in Luhlaza Primary

Learner activity	% ANSWERED (COMPLETION)			% ACCURACY			ACTUAL SCORE		
	1st activity	2nd activity	% Change	1st activity	2nd activity	% Change	1st activity	2nd activity	% change
Akhona			18%			40%			36%
Add & subtract 10	30%	25%	-5%	50%	100%	50%	15%	25%	10%
Add & subtract 100	40%	95%	55%	25%	100%	75%	10%	95%	85%
Doubling	59%	71%	12%	90%	100%	10%	53%	71%	19%
Halving	76%	88%	12%	69%	93%	24%	53%	82%	29%
Chebe			31%			44%			36%
Add & subtract 10	15%	60%	45%	0%	83%	83%	0%	50%	50%
Add & subtract 100	20%	70%	50%	0%	93%	93%	0%	65%	65%
Doubling	71%	94%	23%	100%	94%	-6%	71%	88%	17%
Halving	82%	88%	6%	93%	100%	7%	76%	88%	12%
Khule			35%			3%			34%
Add & subtract 10	35%	85%	50%	86%	94%	8%	30%	80%	50%
Add & subtract 100	40%	100%	60%	100%	80%	-20%	40%	80%	40%
Doubling	76%	88%	12%	85%	100%	15%	65%	88%	23%
Halving	82%	100%	18%	93%	100%	7%	76%	100%	24%
Emi			22%			14%			30%
Add & subtract 10	10%	30%	20%	100%	17%	-83%	10%	5%	-5%
Add & subtract 100	45%	35%	-10%	11%	86%	75%	5%	30%	25%
Doubling	41%	94%	53%	29%	100%	71%	12%	94%	82%
Halving	47%	71%	24%	100%	92%	-8%	47%	65%	18%
Oma			15%			16%			19%
Add & subtract 10	40%	55%	15%	63%	100%	37%	25%	55%	30%
Add & subtract 100	45%	85%	40%	67%	71%	4%	30%	60%	30%
Doubling	59%	71%	12%	70%	92%	22%	41%	65%	24%
Halving	82%	76%	-6%	100%	100%	0%	82%	76%	-6%
Sipho C			20%			15%			29%

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Add & subtract 10	65%	70%	5%	54%	100%	46%	35%	70%	35%
Add & subtract 100	45%	95%	50%	89%	95%	6%	40%	90%	50%
Doubling	82%	76%	-6%	93%	100%	7%	76%	76%	0%
Halving	71%	100%	29%	92%	94%	2%	65%	94%	29%
Sipho N			30%			-2%			21%
Add & subtract 10	25%	55%	30%	60%	82%	22%	15%	45%	30%
Add & subtract 100	45%	95%	50%	78%	63%	-15%	35%	60%	25%
Doubling	59%	65%	6%	100%	91%	-9%	59%	59%	0%
Halving	41%	76%	35%	100%	92%	-8%	41%	71%	30%
Aphiwe			13%			36%			33%
Add & subtract 10	10%	50%	40%	100%	80%	-20%	10%	40%	30%
Add & subtract 100	40%	80%	40%	0%	63%	63%	0%	50%	50%
Doubling	41%	47%	6%	86%	88%	2%	35%	41%	6%
Halving	82%	47%	-35%	0%	100%	100%	0%	47%	47%
Apolo			11%			-3%			-12%
Add & subtract 10	15%	80%	65%	0%	0%	0%	0%	0%	0%
Add & subtract 100	10%	35%	25%	0%	0%	0%	0%	0%	0%
Doubling	65%	41%	-24%	91%	71%	-20%	59%	29%	-30%
Halving	82%	59%	-23%	93%	100%	7%	76%	59%	-17%
Siphiwe			22%			27%			22%
Add & subtract 10	5%	35%	30%	100%	57%	-43%	5%	20%	15%
Add & subtract 100	25%	30%	5%	0%	50%	50%	0%	15%	15%
Doubling	6%	59%	53%	0%	100%	100%	0%	59%	59%
Halving	47%	47%	0%	100%	100%	0%	47%	47%	0%

Summary across both clubs

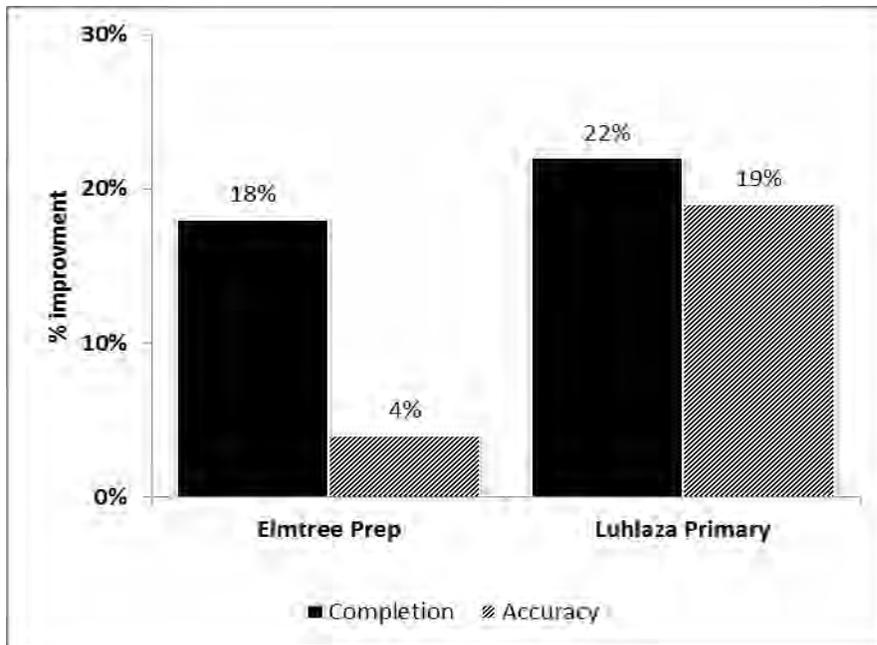


Figure 3: Fluency activities club comparison: overall % change for completion & accuracy

Discussion

From the data shown here on the fluency activities, we see that across both clubs learners on average completed more in the specified time in the July/August activities and were more accurate. As shown in Figure 3, significant increases in speed (completion) for both clubs were 18% and 22% respectively. Luhlaza Primary had a noteworthy increase in accuracy (19%). This was due to several learners making substantial improvements in accuracy in the July/August activities, as discussed above.

I have argued that while these fluency activities measured speed and accuracy, these improvements in fluency were not developed by doing these activities *per se*, but more likely as a result of using other sense making and flexible thinking activities promoted in the clubs, such as the number talks (Stott & Graven, 2015) and card and dice games (Stott, Graven, Koliti et al., 2013) discussed previously. Earlier, I spoke of *deliberate practice* (Askew, 2002) as a way of developing fluency. The central idea of deliberate practice is to continually stretch the learner to just beyond his or her current abilities and this can be done by setting a time limit is set or by using bigger numbers. The full range of activities promoted and the games played in the clubs aimed to facilitate deliberate practice in developing fluency and as such, these timed fluency activities were a form of deliberate practice for the learners.

CONCLUDING REMARKS

When these activities were first used in the clubs during 2012, Graven and I wrote about some of the emergent ethical tensions and how we dealt with these (see Stott & Graven, 2013 for more detail). As a way to address the initial ethical concerns that I had with the administration of these activities, after the second round of activities were re-administered and the scores captured for research purposes, I gave both the March and July scripts back to the learners in a subsequent club session. Facilitating with some structured prompts, the learners were given time to look at both scripts and were able to compare and reflect on their own progress in these activities over time. Sample prompts included: which activity did you do best in; which activity gave you your lowest marks; did you do better in March or July and why do you think you have made progress? In this way the use of these activities was more holistic for the learners as they were able to reflect on their own progress in these specific aspects of mathematics.

Having used these fluency activities over the past three years in other clubs and engaging more deeply with the conceptual frameworks that inform their use, in this paper I have located their design and use in the broader literature on mathematical proficiency, number sense and fluency, have described their design and presented data from my two doctoral research clubs.

The data from the fluency activities presented here provided a useful picture of learner mathematical progress with regard to different aspects of fluency, particularly efficiency and accuracy and supplemented more detailed learner interview data. These activities also enabled the learners to practice the fluency and number sense they were developing through other activities of the club. As such I suggest that they are a useful complementary tool for both research and development in the foundational aspects of mathematics. I continue to use these activities in subsequent non-research clubs as both activities and data collection instruments. I have recently developed other activities for the development and assessment of multiplication facts for example. The activities continue to be used in the manner described here with the same supporting ethos and learner self-reflection. In 2016, one of our project schools will be using the activities in all of their Grade 3 to 7 maths classrooms as one way of gauging learner progress. Based on the ideas put forward in this article, and provided a suitable classroom ethos is established, I argue that they could be one useful way for educators to quickly gauge learner progress in various aspects of number and for learners to practise fluency.

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