Critiquing the Mathematical Literacy Assessment Taxonomy: Where is the Reasoning and the Problem Solving?

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In this paper we consider the ways in which the Mathematical Literacy (ML) assessment taxonomy provides spaces for the problem solving and reasoning identified as critical to mathematical literacy competence. We do this through an analysis of the taxonomy structure within which Mathematical Literacy competences are assessed. We argue that shortcomings in this structure in relation to the support and development of reasoning and problem solving feed through into the kinds of questions that are asked within the assessment of Mathematical Literacy. Some of these shortcomings are exemplified through the questions that appeared in the 2008 Mathematical Literacy examinations. We conclude the paper with a brief discussion of the implications of this taxonomy structure for the development of the reasoning and problem-solving competences that align with curricular aims. This paper refers to the assessment taxonomy in the Mathematical Literacy Curriculum Statement (Deparment of Education (DOE), 2007).

Mathematical Literacy was introduced as a new subject in the post-compulsory Further Education and Training (FET) curriculum in 2006. Its introduction made a mathematically-oriented subject – either Mathematics or Mathematical Literacy – compulsory for all FET learners. The curriculum statement for Mathematical Literacy defines the subject in the following terms:

> Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems. (DOE, 2003, p. 9)

This definition alongside the broader description of the new subject's aims in this document places emphasis on the need to develop life-oriented competences for a range of everyday situations in which mathematical reasoning and mathematical tools can be brought to bear productively to aid informed decision making and problem solving. Situational reasoning relating to the identification and selection of salient features of the context is therefore required alongside and integrated with mathematical reasoning.

1   Within this definition, we highlight two features that the broader literature suggests are central to the
2   notion of mathematical literacy[1] that is promoted in the South African rhetoric: firstly, the need for the
3   reasoning that is implicated within the need to "think numerically and spatially" and to "interpret" and
4   "critically analyse everyday situations"; and secondly, the need for "problem solving". Both of these
5   aspects have significant bodies of literature associated with them in the field of mathematics education –
6   some focused on the teaching and learning of mathematics, and some specific to discussions of
7   mathematical literacy. Olkin & Schoenfeld (1994), focusing on mathematics, describe problem solving in
8   terms of "confronting a novel situation and trying to make sense of it" (p. 43).

9   Steen (2001), a leading advocate of what he terms "quantitative literacy" comments that centrally what
10 quantitatively literate citizenship requires is "a predisposition to look at the world through mathematical
11 eyes … and to approach complex problems with confidence in the value of careful reasoning" (Steen, p.
12 2).

13 In both of these quotations – across mathematics and mathematical literacy – reasoning and problem
14 solving are seen to involve complex problems/novel situations. Further, Steen's viewpoint suggests that
15 "complex problems" are essential to developing mathematical literacy as a life-oriented competence.

16 In this paper our focus is on exploring the extent to which reasoning and problem solving as described in
17 the literature (Halmos, 1975; Polya, 1962; Steen, 2001) and in the rhetoric of international parallels to ML
18 (e.g., Functional Mathematics in England) figure within the assessment of ML in South Africa. The
19 results of the first ML examinations within the new FET National Senior Certificate (taken in 2008 in
20 which 79% of the cohort attained a 'pass' or above ($\geq$ 30%)) were met with some scepticism in public
21 commentaries (e.g., Jansen, 2009). Earlier in 2008, academic critiques had already raised concerns that
22 the exemplar papers for ML did not appear to align well with the taxonomy against which assessments
23 were supposed to be designed, with an overrepresentation of questions focused on the lower levels
24 (Prince, Frith, & Burgoyne, 2008).

25 This taxonomy, provided in the Subject Assessment Guidelines (SAG) document (DOE, 2007), is
26 outlined in the following terms, with guideline percentages given for the distribution of marks across its
27 levels in ML assessments:

28   Level 1:   Knowing (30% of marks)
29   Level 2:   Applying routine procedures in familiar contexts (30%)
30   Level 3:   Applying multi-step procedures in a variety of contexts (20%)
31   Level 4:   Reasoning and reflecting (20%)

32 Prior analyses have considered the 2008 ML examination papers in relation to this taxonomy (Umalusi,
33 2009), and pointed to an over-representation of lower level questions in Paper 2 specifically. Paper 2,
34 according to the specification in the SAG document, should focus predominantly on Level 3 and Level 4,
35 with a small proportion of marks allocated to Level 2 whilst Paper 1 focuses solely on Levels 1 and 2.

36 Given these analyses, our aim in this paper is to consider more specifically whether the taxonomy
37 provides sufficient openings for developing what the literature base identified above tells us are key
38 competences for becoming mathematically literate citizens – competences that are also highlighted in the
39 rhetoric of the South African ML curriculum statement.

40 In order to facilitate our consideration of this issue, we have structured this paper as follows. We begin
41 with a brief review of literature that point to reasoning and problem solving as critical to the notion of
42 mathematical literacy. We then go on to consider the taxonomy structure in some detail, and locate the
43 areas and levels within which reasoning and problem solving are assessed, and the ways in which other
44 competences figure alongside these. We argue that shortcomings in this structure in relation to the support
45 and development of reasoning and problem solving feed through into the kinds of questions that can be
46 asked within the assessment of ML. These shortcomings are exemplified through the questions that
47 appeared in the 2008 ML examinations. We conclude the paper with a brief discussion of the implications

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[1] We use the capitalised "Mathematical Literacy" to refer to the South African subject specification and its
enactments, and the small "mathematical literacy" to refer to a more generalised notion of a life-related competence
that has a significant literature base associated with it in the field of mathematics education.

1 of this taxonomy structure for the development of the reasoning and problem-solving competences that
2 align with curricular aims. Specifically, we relate this discussion to how we interpret students'
3 performance in the 2008 ML examination.

4

Essential competences for mathematical literacy

6 One of the difficulties in trying to identify essential competences for mathematical literacy is the fact that
7 a range of interpretations exist as to what mathematical literacy actually is. Jablonka (2003) notes, in her
8 analysis of a range of perspectives on mathematical literacy, that even the notion of essential competences
9 ("transferable methodological and process skills" (p. 79) in her terms) is only possible within particular
10 conceptions of what mathematical literacy is about. She argues that the centrality of problem solving and
11 reasoning are emphasised in the view of mathematical literacy embodied in the definition and problems
12 set within the international comparison oriented Program for International Student Assessment (PISA)
13 (OECD, 2003) tests for example. She points out the imperatives for standardisation across cultures and
14 measurement of competence within these tests, and argues that these imperatives take precedence over the
15 need for genuine relevance. Of interest in the South African ML context is the explicit acknowledgement
16 of the OECD conception of mathematical literacy and its associated view of development in mathematical
17 literacy competence within the development of the South African ML taxonomy. This suggests that
18 reasoning and problem-solving development will be central to the assessment framework.

19 Further insights on the nature of the problem solving needed for mathematical literacy is provided within
20 the documentation associated with the introduction of Functional Mathematics as a new subject in
21 England. The definition of Functional Mathematics in England overlaps in significant ways with the
22 South African definition of ML, containing phrases such as these:

23   Functional mathematics requires learners to be able to use mathematics in ways that make them
24   effective and involved as citizens, able to operate confidently in life and to work in a wide range of
25   contexts. (Qualification and Curriculum Authority, 2007, p. 19)

26 Problem solving is stressed as a key feature of being able to use mathematics in functional ways, with
27 some detail provided on the ways in which problems should be presented in order to aid the development
28 of a functional competence:

29   It is important that learners are not told, at the time a problem is set, which of the mathematical tools
30   they have at their disposal will actually be needed. Selecting the right tools is a core aspect of
31   becoming functional in mathematics. (Functional skills support programme, 2007, pp. 22-23)

32 And further:

33   It is very important for learners to experience the need to decide for themselves whether a problem can
34   be addressed using mathematics, what mathematics might help, and how the problem should be set out
35   mathematically. (ibid, p. 24)

36 The importance of the lack of "obvious-ness" of solution routes in order for genuine problem solving to
37 occur has also been repeatedly emphasised within the mathematics education literature:

38   ...to have a problem means: to search consciously for some action appropriate to attain a clearly
39   conceived, but not immediately attainable, aim. To solve a problem means to find such action. (Polya,
40   1962, p. 117)

41 A further feature of this kind of problem solving in which the solution route is not immediately obvious is
42 the importance of problem-posing, of asking questions that are appropriate to the context of the problem:

43   The hardest part of answering questions is to ask them. […] the chief problem is likely to be "what is
44   the problem?". Find the right question to ask, and you're a long way toward solving the problem
45   you're working on. (Halmos, 1975, pp. 466-467)

46

1   We argue that, both in mathematics and in mathematical literacy, the processes of problem-posing and
2   problem-solving require reasoning in a range of ways – in asking the right questions, in choosing the
3   mathematical tools that might assist in answering them, in selecting the most appropriate ways of
4   representing information using these tools, in working systematically, and in analysing and interpreting
5   results in context (with contexts usually drawn intra-mathematically in mathematics and from real life
6   situations in ML). An important feature of this view of reasoning is the way in which it *permeates* the
7   problem-solving process, rather than *following* the problem-solving process. We return to this point when
8   we consider the Mathematical Literacy taxonomy in the following section.

9   In the English Functional Mathematics specification, four features of "level differentiation" are outlined
10   for describing problem demand in the sphere of contextualised problems:

11   o   the complexity of the situation
12   o   the familiarity to the learner of the situation or problem
13   o   the technical demand of the mathematics required
14   o   the independence of the learner in tackling the situation or problem.
15   (Functional skills support programme, 2007, p. 25)

16   This model of complexity differs from the South African ML taxonomy in important ways. In the next
17   section, we consider the outline and specification of this taxonomy in some detail.

18

The Mathematical Literacy taxonomy

20   The headings of the four levels of the ML taxonomy, presented in the introduction, suggest two overt
21   strands that vary across the levels: a mathematical strand evident within the reference to "knowing"
22   (mathematical facts) at Level 1 and moving to the use of increasingly complex (or at least lengthy)
23   procedures (Levels 2 and 3); and a contextual strand that moves from "familiar" contexts (Level 2) to a
24   "variety of contexts" (Level 3).

25   Combining content (in terms of facts and procedures) and context oriented complexity within a single
26   hierarchy appears to suggest that both these aspects become more complex together. This contrasts with
27   the view presented in Functional Mathematics in the last section where the categories suggest that these
28   can vary independently of each other, with "technical demand of the mathematics" providing one avenue
29   for making problems more demanding, and "complexity of the situation" providing another avenue.

30   A further feature at the outline level is that recall of fact and engagement with mathematical procedures is
31   *followed* by "reasoning and reflecting" at Level 4. In the section above and in earlier writing (Venkat,
32   Graven, Lampen, Nalube, & Chitera, 2009), we have commented that this "deferral" of reasoning –
33   construed as following the "doing" of mathematics – is problematic from the perspective of the literature
34   outlined on what it means to be mathematically literate.

35   The Subject Assessment Guidelines document provides descriptions of each of the taxonomy levels in
36   Figure 1 (DOE, 2007, pp. 27-28).

37   Given our focus on reasoning and problem solving as central competences that need to be developed for
38   mathematical literacy, it was instructive to start by looking for where these features occurred within the
39   assessment taxonomy. In the terms outlined in our literature review, problem solving appears to figure
40   only at Level 3 (AMP1). Whilst there is reference to problem solving at Level 2 (ARP1), the emphases on
41   the "obvious-ness" of the procedure that is needed and the immediate availability of all the required
42   information would tend to disqualify this reference in the terms outlined in the literature on problem
43   solving. Problem-posing, in the ways referred to within our overview of the literature, figures only at
44   Level 4.

45

Level 1: Knowing (K)[2]	Level 2: Applying routine procedures in familiar contexts (ARP)	Level 3: Applying multi-step procedures in a variety of contexts (AMP)	Level 4: Reasoning and reflecting (RR)
Tasks at the *knowing* level of the ML taxonomy require learners to: **K1** Calculate using the basic operations including: ○ algorithms for +, -, ×, and ÷; ○ appropriate rounding of numbers; ○ estimation; ○ calculating a percentage of a given amount; and ○ measurement **K2** Know and use appropriate vocabulary such as equation, formula, bar graph, pie chart, Cartesian plane, table of values, mean, median and mode. **K3** Know and use formulae such as the area of a rectangle, a triangle and a circle where each of the required dimensions is readily available. **K4** Read information directly from a table (e.g. the time that bus number 1234 departs from the terminal).	Tasks at the *applying routine procedures in familiar contexts* level of the ML taxonomy require learners to: **ARP1** Perform well-known procedures in familiar contexts. Learners know what procedure is required from the way the problem is posed. All of the information required to solve the problem is immediately available to the student. **ARP2** Solve equations by means of trial and improvement or algebraic processes. **ARP3** Draw data graphs for provided data. **ARP4** Draw algebraic graphs for given equations. **ARP5** Measure dimensions such as length, weight and time using appropriate measuring instruments sensitive to levels of accuracy.	Tasks at the *applying multi-step procedures in a variety of contexts* level of the ML taxonomy require learners to: **AMP1** Solve problems using well-known procedures. The required procedure is, however, not immediately obvious from the way the problem is posed. Learners will have to decide on the most appropriate procedure to solve the solution to the question and may have to perform one or more preliminary calculations before determining a solution. **AMP2** Select the most appropriate data from options in a table of values to solve a problem. **AMP3** Decide on the best way to represent data to create a particular impression.	Tasks at the *reasoning and reflecting* level of the ML taxonomy require learners to: **RR1** Pose and answer questions about what mathematics they require to solve a problem and then to select and use that mathematical content. **RR2** Interpret the solution they determine to a problem in the context of the problem and where necessary to adjust the mathematical solution to make sense in the context. **RR3** Critique solutions to problems and statements about situations made by others. **RR4** Generalise patterns observed in situations, make predictions based on these patterns and/or other evidence and determine conditions that will lead to desired outcomes.

Figure 1: *Description of the Mathematical Literacy assessment taxonomy levels*

Given then that problem solving (using well-known procedures) makes an appearance only at Level 3, with reasoning deferred to Level 4, we then began to look across the levels at the ways in which complexity appeared to be conceptualised within the taxonomy. We were able to discern four threads relating to mathematical development within the structure, with other aspects appearing in much more localised ways within particular levels. The developmental threads we identified are as follows:

○ calculation/procedure
○ algebraic competence
○ dealing with tabulated information
○ knowledge and competence in data representation

In the following sections we discuss each of these developmental threads, and then go on to identify aspects that appear in more "singular", non-developmental ways within the taxonomy structure.

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[2] Abbreviation of level headings and numbering of aspects within levels added by us to facilitate reference.

1 ***Calculation/procedure***

2 Across the four levels of the taxonomy, there is recurring reference to calculations and procedures. At the
3 most basic level, the four taxonomy labels indicate that development within this thread proceeds along the
4 lines of knowledge/recall of procedure (K), application of routine procedures (ARP), application of multi-
5 step procedures (AMP), identifying the appropriate questions to ask in order to identify procedures to
6 solve problems using mathematics as a tool (RR).

7 Whilst this labeling indicates a rather mathematically-based progression across levels, closer reading of
8 the descriptions indicates that such mathematically-oriented progression is intertwined with the following
9 three sub-threads:

10 o The degree of "known-ness" of the procedure to be applied
11 o The degree of "immediate availability" of the information required to use the procedure to solve the problem
12 o The degree of "obvious-ness" around what procedure to use

13 We now discuss each sub-thread in turn.

14
15 *Degree of "known-ness" of the procedure to be applied*
16 The descriptions across the levels point to a progression from "basic operations" to "well-known
17 procedures" to "identification of" an appropriate mathematical procedure, which may also be well-known
18 or even basic. The listing of basic arithmetical operations under K1 suggests that a school mathematics
19 curriculum hierarchy of procedure may underlie the way progression within this sub-thread is interpreted.
20 As such, there is an overlap between this sub-thread and the notion of the "technical demand of the
21 mathematics" that is presented as one variable within problem demand in Functional Mathematics.

22
23 *Degree of "immediate availability" of the information required to use the procedure to solve the problem.*
24 The descriptions under ARP1 and AMP1 allude to this as a developmental sub-thread of application of
25 procedures. The lack of any allusion to contextualisation under K1 suggests that required information
26 may be directly given within decontextualised questions – a suggestion confirmed by the presence of such
27 questions in Paper 1 in the 2008 ML examination, in spite of advice to teachers to avoid this practice in
28 the SAG: "When teaching and assessing Mathematical Literacy, teachers should avoid teaching and
29 assessing mathematical content in the absence of context" (DOE, 2007, p7).

30
31 *Degree of "obvious-ness" around what procedure to use*
32 Comments within the descriptions for ARP1 and AMP1 suggest that this factor represents a key
33 discriminator between Level 2 and Level 3 questions. When combined with the lack of contextualisation
34 in some Level 1 questions, this sub-thread relates to the degree of explication of specific procedures in the
35 statement of the question. This suggests that at Levels 1 and 2, direct pointers towards the required
36 mathematics are acceptable, resulting in heavily scaffolded questions in the examination papers that
37 emphasise and test calculation skills, thus vitiating the assessment of problem solving.

38 Several interesting issues are raised within the calculation/procedure developmental thread and the sub-
39 threads within it in the context of the South African ML curriculum. First there is the mathematical
40 orientation of the progression – a feature that appears to contradict the curriculum specification within
41 which overt mathematical progression is limited, and explicitly acknowledged:

42 For Mathematical Literacy, the Assessment Standards do indicate progression from grade to grade.
43 However, this progression is not markedly evident in some of the Assessment Standards. The
44 complexity of the situation to be addressed in context, through using the mathematical knowledge and
45 ways of thought available to the learner, is where the extent of the progression needs to be ensured.
46 (DOE, 2003, p. 38)

47 The framing of progression in terms of mathematical procedure within the taxonomy is somewhat
48 problematic. We acknowledge and accept that this progression is both a necessary and useful *part* of the
49 frame of mathematical literacy development – that recognising and unpacking mathematical progression
50 within the context of problem solving provides both rationales for mathematical development and a range
51 of increasingly sophisticated tools for making sense of situations. However, to allow mathematical

1 progression to dominate the assessment of a curriculum that is oriented towards quantitative and
2 mathematical reasoning for life-situations appears akin to the "tail wagging the dog" and runs the risk of
3 diminishing the emphasis on understanding everyday contexts that is central to the curriculum rhetoric.

4 Secondly, the progression appears to move from an emphasis on basic arithmetical procedure into
5 directed straightforward procedures, and then onto less directed and more complex procedures. The
6 calculating thread therefore appears to build in procedural complexity alongside a "degree of explication"
7 strand, with procedures getting more complex *and* diminishing explication of the procedure to use
8 occurring together. This suggests that these two features cannot be varied independently. This view tends
9 to contradict Steen's position that quantitative literacy more often involves "real data and uncertain
10 procedures but require[s] primarily elementary mathematics" (Steen, 2001, p. 6).

11 Here, Steen's view suggests that a diminishing degree of explication – again related to the degree of
12 scaffolding - of specific procedures ought to be an overarching development thread in order to meet the
13 goals of mathematical literacy, with limited shifts towards more complex procedures.

14 The shift from a directed emphasis on basic calculation towards the selection and use of mathematical
15 tools within a problem-solving frame correlates with what has been presented by some writers as a
16 potential model of how sophistication builds up in mathematical literacy. Maguire and O'Donoghue
17 (2003) present an "organising framework" for numeracy within which numeracy builds from a "formative
18 phase" with an emphasis on arithmetic skills, into a "mathematical phase" in which the mathematics in
19 everyday situations is made explicit, and then into an "integrative phase" where mathematics,
20 communication and culture have to be synthesised in context. This view though, tends to be contradicted
21 by findings in mathematics education located within a situated learning perspective (Boaler, 1997;
22 Scribner, 1984). Both these studies suggest that people can become good (efficient and effective) at
23 activities that they practise – designing efficient distribution schedules in Scribner's case and non-routine,
24 mathematics problems in Boaler's case. Given that the taxonomy allows for 60% of the marks in ML
25 assessments, even at Grade 12 level, to come from basic arithmetic and simple calculation without
26 involving reasoning or problem solving in context, our concern is that there simply is not enough pressure
27 to infuse ML learning with these latter features. Instead, the mathematical calculation thread allows for
28 procedural mathematics to dominate in ways that often work against the aims of the curriculum.

29
30 ### *Algebraic competence*

31 This developmental thread appears somewhat incongruous when considered alongside the advocacy in
32 policy documents for work in ML to be led by the need to "engage with contexts rather than applying
33 Mathematics already learned to the context" (DOE, 2003, p. 42) – given that the emphasis appears to be
34 on mathematically focused progression located within one particular domain of mathematics – algebra.
35 Once again, whilst there are stronger indications here of an algebraically focused interpretation of
36 progression, i.e. knowledge and use of formulae (K3) to solving equations algebraically (ARP2) or
37 drawing algebraic graphs for equations (rather than situations) (ARP4) to generalising patterns in context
38 (RR4), statements incorporate the degree of immediacy of information needed to apply algebraic tools as
39 part of the increasing complexity of problems (e.g., K3).

40
41 ### *Dealing with tabulated information*

42 Information presented in tabular form curiously seems to receive special attention within the taxonomy,
43 rather than a more general competence in dealing with numerical information from a variety of sources
44 and representations (including graphs and written text) that is often highlighted in literature dealing with
45 the nature of mathematical or quantitative literacy (e.g., Steen, 2001). Progression within this
46 developmental thread appears to be delineated primarily in terms of the degree of interpretation needed to
47 "select" appropriate information from a table in order to answer a question or solve a problem. Thus, at
48 the knowing level, the information needed can be found "directly" from the question in the table provided
49 (K4), whilst a higher level competence in this strand would require some interpretation in order to select
50 the "most appropriate data from options in a table" (AMP2). The notion of immediacy is therefore present
51 within this thread as well.

Knowledge and competence in data representation

The hierarchy presented in this developmental thread appears to suggest that low level competence is evident in knowledge and use of some basic representations (bar charts, introduced in the foundation phase content mathematics curriculum), are mentioned specifically – K2). This proceeds into an unspecified drawing of "data graphs for provided data" (ARP3), and then into a more critical engagement with the kinds of data presentation that might support specific positions or arguments (AMP3). Critiquing provided data presentations can potentially fall within the remit of RR3, but is not indicated as the final progression of this thread. Within this thread too therefore, the "production" of mathematics prevails at Levels 1 and 2, with critical engagement appearing only at Level 3, implying that reasoning will not be assessed in the reading of tables or the drawing of graphs.

Non-progressing descriptors

This identification of developmental threads and some cross-cutting sub-threads leaves some aspects of the specification of the taxonomy as yet untouched. The "missing" descriptors appeared to us to stand in much more isolated ways within particular levels without a sense of progression attached to the skill or competence presented. Examples of this include the emphasis on measurement which appears in ARP5. Whilst this descriptor can be interpreted as following on from "given" dimensions which feature in K3, the activity of measuring appears at this point only and appears to qualify as a routine procedure regardless of the context of the actual measurement. Similarly, the need for "one or more preliminary calculations before determining a solution" (AMP1) appears to push a question into the third level irrespective of the actual calculations required. The practical consequence is that the stringing together of routine calculations is construed as indicative of a high level of mathematical literacy.

The key location for non-progressing descriptors though occurs within the "reasoning and reflecting" level. Problem posing and interpretation or adjustment of answers in context only figure at this level, with lower levels providing few, if any, handles for the development of this kind of reasoning and critique capacity. As stated earlier, we have expressed our concern that the taxonomy appears in this way to separate the "doing" of ML from the "reasoning" required for ML (Venkat et al., 2009). One consequence of this separation is that lower level reasoning skills are rendered invisible, for example in question 2.2.2 in Example 1 (see further on), which requires candidates to visualise where a maximum height occurs as the arms of a wind turbine circulate. Developing such visualisation skills is seen as an important component of reasoning for both mathematical literacy (de Lange, 1999) and for mathematics (Presmeg, 1986).

A second consequence is that, given that reasoning and reflecting is understood to "follow" the calculating of answers, reasoning questions tend to focus narrowly on rather limited interpretation and commentary on previously calculated answers. Question 2.4.3 in Paper 2, asking candidates to comment on whether a payment option involving a deposit and monthly instalments or one involving taking out a loan for an outright purchase is preferable, following step by step calculations for the previous steps provides a good example of this kind of limited interpretation demand. Essentially in this view, reasoning is reduced to a "reflection on prior calculation in context" – calculations that are so extensively scaffolded that the need for reasoning about the chain of steps required to make an informed decision is effectively removed.

In summary, two aspects occur recurrently in this analysis of the taxonomy – firstly, a tendency towards procedural orientations to progression in several of the threads, and secondly, the notion that the degree of "immediacy" of information availability and/or "explication" of the required mathematical tools provides a sub-thread contributing to mathematical progression. Key problems, in our view, relate firstly to the lack of a developmental thread across taxonomy levels related to openings for reasoning and problem solving, compounded by the interpretation of reasoning as a "deferred" activity understood in rather limited ways. Secondly, the emphasis on mathematically-oriented progression is problematic in assessing a curriculum that contains a limited degree of mathematical development in its specification. This tends to result in a somewhat arbitrary breakdown of procedures into taxonomy levels.

This breakdown is made much more explicit in the taxonomy version provided in the Examination Guidelines document which ML teachers are encouraged to use, in which, for example, calculating a

1 percentage of a given quantity qualifies for a Knowing (K) level competence, whilst calculating a
2 percentage increase or decrease of a given quantity falls with the Applying routine procedures (ARP)
3 level. Anomalies result from this kind of breakdown in mathematical terms – a percentage reduction
4 problem (e.g., to calculate the effect of reducing R400 by 12%), can be calculated as $0,88 \times R400$ for a
5 one-stage solution or as a two-stage solution by calculating the 12% amount and subtracting. The first
6 route in procedural terms carries exactly the same technical demand as a Level 1 question, but some
7 mathematical reasoning about the structure underlies this calculation. The second route makes the
8 procedure 'longer' – and the emphasis on moving from "routine procedures" to "multi-step procedures"
9 in the Level headings of the taxonomy suggests that it is this "length of procedure" orientation that
10 prevails in the taxonomy, rather than the reasoning underlying the more efficient procedure. As such, the
11 kind of reasoning that underlies the more efficient procedure in the first solution tends to remain invisible
12 in the taxonomy frame. In turn, this invisibility makes it unlikely that such reasoning will be encouraged
13 in ML classrooms.

14 In the next section, we compare three questions from the 2008 ML examinations which exemplify the
15 problems we have identified in this section. The first example is from Paper 1 which aims to assess ML at
16 Levels 1 and 2, while the other examples are from Paper 2, aimed at assessing ML at Levels 3 and 4.

17

18 **Questions from the 2008 Mathematical Literacy examinations**

19 *Example 1 – Paper 1, question 2.2*

2.2 Electricity has become a scarce resource in South Africa. As a result the country is investigating alternative sources of generating electricity. One alternative source of generating electricity is a wind turbine using rotating blades as shown in the picture and diagram below.
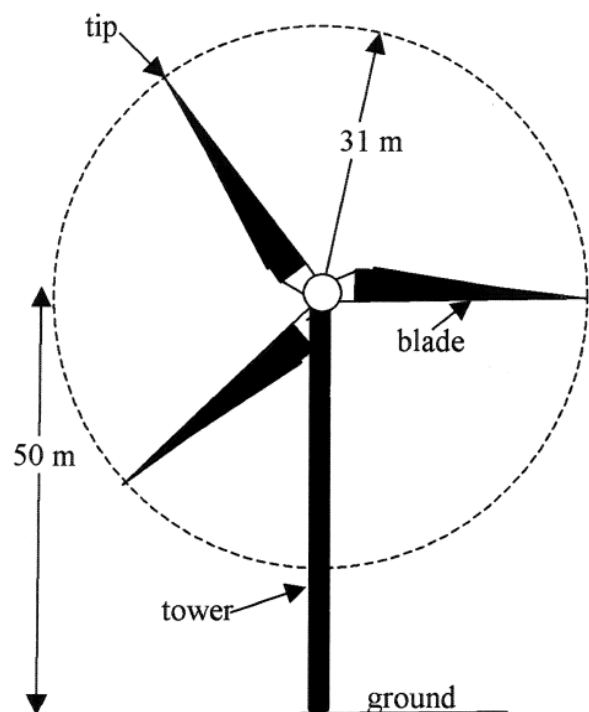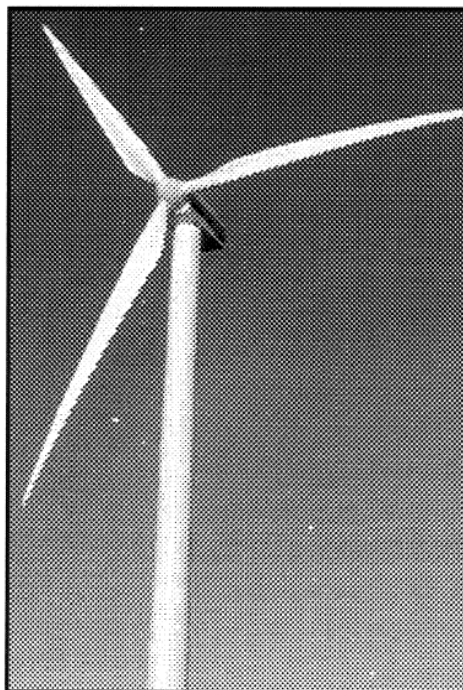
The wind turbine is mounted on the top of a 50 m high tower.
The length of each blade is 31 m.

20

1

> **2.2.1** What is the length of the diameter of the circle that the blades create as they rotate?
>
> **2.2.2** Calculate the maximum height from the ground that the tip of a blade will be, if the turbine is rotating.
>
> **2.2.3** Calculate the circumference of the circle made by the blades when they rotate.
>
> (**Circumference = 2** $\times \pi \times$ **radius**), using $\pi = 3,14$
>
> **2.2.4** Calculate the area of the circle made by the blades when they rotate.
>
> (**Area** = $\pi \times$ **(radius)^2**), using $\pi = 3,14$
>
> **2.2.5** Suppose each household requires 25 kW of electricity daily.
>
> If one wind turbine produces 1 750 kW of electricity daily, calculate how many households could be provided daily with electricity from one such turbine.

2

3 *Commentary*

4 We note that within the second diagram given, a mathematical representation of the wind turbine is
5 provided to candidates. This provision has several consequences. Firstly, the need to engage with the
6 context through making sense of the photo representation of the wind turbine, and connecting this
7 representation to the textual information given beneath it is diminished. The need to select the salient
8 features of the context in order to create a mathematical model is therefore completely removed. Further,
9 the reasoning load required to answer question 2.2.1 to 2.2.4 is also consequently reduced, although as we
10 have pointed out, low-level visualising is still required to "imagine" the location of a blade to get the
11 maximum height above ground (in spite of the fact that the taxonomy renders this invisible).

12 We argue that the lack of openings for problem representation, reasoning and problem solving in the
13 formulation of questions such as this one follow from the structure of the taxonomy. Paper 1 is structured
14 so that it contains only questions at Levels 1 and 2 of the taxonomy, and our analysis above indicates that
15 neither reasoning nor problem solving are represented at these levels. Essentially therefore, given the
16 mathematical model and the formulae needed for the circumference and area of a circle, the question
17 becomes a test of the candidate's ability to calculate – again, in line with the orientation of the taxonomy,
18 but much less in line with the spirit of the curriculum.

19

1 *Example 2 – Paper 2, question 2.4*

2.4   **Thandi decides to buy a dishwasher based upon the advertisement below.**

Come to **Best Buy Dealers** for the best deals in town.

Best Buy Dealers:

Dish Washer

Cash price: R2 699,00

OR

ONLY 10% deposit plus
24 equal monthly instalments
of R177,53

2.4.1   **Suppose Thandi decides to buy the dishwasher using the instalment option.**

    (a)   **What is the balance owing after paying the deposit?**

    (b)   **Calculate the total cost of the dishwasher.**

2.4.2   **Suppose Thandi takes a loan from ABC Bank for the full cash price of the dishwasher. She is charged interest of 18% p.a. compounded monthly and agrees to repay the loan over two years in equal monthly instalments.**

    Use the formula $A = P(1 + i)^{n}$ to calculate the total amount to be paid back, where:

    A = total amount to be paid back
    P = loan amount
    i = monthly interest rate
    n = number of months over which the loan will be taken

2.4.3   **Which method of payment would you advise Thandi to choose?   Give a reason for your answer.**

2

3 *Commentary*

4 Essentially this Paper 2 question asks candidates to calculate and then compare across the three options
5 available (with sub questions directing each calculation), and to then select one and provide some kind of
6 justification for it.

7 Questions such as 2.4.3, which asks candidates to look over a prior sequence of well structured
8 calculations, and to justify a selection in some way from these appears to be the most common way in
9 which "reasoning and reflecting" questions are incorporated into the ML examination. None of these
10 skills appears to us to merit the conferral of a high level of reasoning in relation to mathematical literacy.
11 However, given the lack of a developmental thread related to reasoning, and the scaffolding away of a
12 more genuine orientation to problem solving, such questions are by default classified at Level 4 within the
13 taxonomy.

14

1 *Example 3 – Paper 2, question 4.2*

4.2   Lebo's family lives in the Eastern Cape. He now lives and works in Gauteng and earns a net salary of R10 625,00 per month. He sends home 35% of his net monthly salary every month and uses the rest for his own living expenses.

Lebo's living expenses include the following:

- R3 500,00 per month for food and rental
- R18,00 per day for transport (He works for 21 days every month.)
- A cellphone contract of R135,00 per month
- Clothing accounts of R250,00 a month
- 10% of his net monthly salary for entertainment

4.2.1   Complete Lebo's monthly budget on ANNEXURE C. Show ALL the calculations on ANNEXURE C.

(a)   Calculate the amount he sends home every month.
Fill in the answer at **A**.

(b)   Calculate the amount he has left for his own living expenses.
Fill in the answer at **B**.

(c)   Calculate his total monthly living expenses.
Fill in the answer at **C**.

(d)   Calculate the amount he has left after all his monthly living expenses have been paid.
Fill in the answer at **D**.

2

3

4 *Commentary*

5 This question is also drawn from Paper 2, which should be primarily constituted by tasks at Levels 3 and
6 4 of the taxonomy. The phrasing of the initial context in question 4.2 draws students into the reading of a
7 relatively complex text, yet with all information explicitly provided. Much of the remaining complexity is
8 removed by the structure of the sub-questions in question 4.2.1. We point out here the step by step nature
9 of the scaffolding provided in question 4.2.1 in order to get to a value for how much Lebo has left in hand
10 after expenses each month. As in the previous example, problem representation skills are reduced
11 significantly through both the scaffolding and the provision of a table for collation, and each part of the
12 question requires relatively directed and relatively basic calculation skills. The provision of Annexure C
13 containing a table formatted to allow each of the expenses to be listed in rands and cents in a sensible
14 order, completely removes the Level 3 requirement to "decide on the best way to represent data to create
15 a particular impression."

16 As with the questions in Examples 1 and 2, we note the potential of this scenario to open up avenues for
17 complex problem solving, but in the over-scaffolding that follows, much of the potential for decision-
18 making and interpretation is removed. The difficulty essentially, therefore, lies not in the choice of
19 problem contexts, but in the design of the questions within them, which seem to be designed to mimic the
20 taxonomy in their unfolding.

21

22 **Conclusions**

23 Widespread evidence of assessment driving teaching (Clarke, 1996) indicates that the taxonomy upon
24 which ML assessments are designed has implications for teaching. Our analysis of the taxonomy reveals
25 that an emphasis on problem solving only really comes into play at Level 3 of the taxonomy and yet the

1   literature and curriculum documents suggest that this is the crux of the aim of ML. The emphasis on
2   routine calculations is marked overtly at Levels 1 and 2 – which by definition then, fall outside the realm
3   of problem solving. Thus immediately visible from this taxonomy is the way in which the key aims of
4   ML as given in the definition are located primarily in Levels 3 and 4. Indeed the definition of ML is that
5   it is a subject driven by life related applications which enables learners to think numerically and spatially
6   "in order to interpret [Level 3] and critically analyse [Level 4] everyday situations and to solve problems
7   [Level 3]" (DOE, 2003, p. 9, emphasis and levels added). In relation to the specification that Paper 1 only
8   includes questions at Level 1 and Level 2 of the taxonomy we have to question the validity of such a
9   paper in assessing the aims of ML as given by the definition. As we have pointed out, some aspects of the
10 literature point to difficulties with interpreting the basic calculation focus of Levels 1 and 2 as an
11 appropriate lead in to service the problem solving and reasoning demands of Levels 3 and 4.

12 Furthermore if across the two papers 60% of the marks can be obtained without any problem solving,
13 mathematical interpretation and reasoning, then again the validity of what a 'pass' in this subject means in
14 relation to the curriculum statement is called into question. In this respect the very high pass rate in
15 mathematical literacy is unsurprising. Perhaps what it indicates is that 79% of all learners who wrote the
16 examination were able to perform/demonstrate basic mathematical operations, calculations/ skills and
17 apply them to some extent in familiar situations where the necessary procedure is relatively obvious from
18 the information given (which is immediately available). Perhaps the proportion of learners who received
19 more than 60% for the examination is a better reflection of those who were able to demonstrate some
20 competence in meeting what the curriculum rhetoric suggests as requirements to be mathematically
21 literate.

22

23 **References**

24 Boaler, J. (1997). *Experiencing school mathematics: Teaching styles, sex and setting*. Buckingham: Open
25   University Press.
26 Clarke, D. (1996). Assessment. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.),
27   *International handbook on mathematics education* (pp. 327-370). Dordrecht: Kluwer Academic Publisher.
28 de Lange, J. (1999). *Framework for classroom assessment in mathematics*. Utrecht: Freudenthal Institute &
29   the National Center for Improving Student Learning and Achievement in Mathematics and Science.
30   Retrieved from http://www.fi.uu.nl/catch/products/framework/de_lange_frameworkfinal.pdf.
31 Functional skills support programme. (2007). *Resources to support the pilot of functional skills: Teaching and
32   learning functional mathematics*. London: Crown Copyright.
33 Halmos, P. (1975). The problem of learning to teach. *American Mathematical Monthly, 82*(5), 466-476.
34 Jablonka, E. (2003). Mathematical Literacy. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K.
35   S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 75-102). Dordrecht:
36   Kluwer Academic Publisher.
37 Jansen, J. (2009, 4 January 2009). Old school: New system produces the same results. *The Sunday Tribune,* p.
38   20.
39 Maguire, T., & O'Donoghue, J. (2003). Numeracy concept sophistication – An organizing framework, a useful
40   thinking tool. In J. Maaß & W. Schlöglmann (Eds.), *Proceedings of the 10th International Conference on
41   Adults Learning Mathematics* (pp. 154-161). Linz, Austria: ALM and Johannes Kepler Universität.
42 Department of Education. (2003). *National curriculum statement Grades 10-12. (General): Mathematical
43   Literacy*. Pretoria: National Department of Education.
44 Department of Education. (2007). *National curriculum statement Grades 10-12 (General): Subject Assessment
45   Guidelines. Mathematical Literacy*. Pretoria: National Department of Education.
46 Organisation for Economic Cooperation and Development. (2003). *The PISA 2003 assessment framework -
47   Mathematics, Reading, Science and problem-solving knowledge and skills*. Paris: Organisation for
48   Economic Cooperation and Development.
49 Olkin, I., & Schoenfeld, A. H. (1994). A discussion of Bruce Reznick's chapter. In A. H. Schoenfeld (Ed.),
50   *Mathematical thinking and problem solving* (pp. 39-51). Hillsdale, NJ: Lawrence Erlbaum Associates.
51 Polya, G. (1962). *Mathematical discovery: On understanding, learning and teaching problem-solving*. New
52   York: Wiley.
53 Presmeg, N. (1986). Visualisation in high school mathematics. *For the Learning of Mathematics, 6*(3), 42-46.
54

1 Prince, R., Frith, V., & Burgoyne, N. (2008, August). *Mathematical Literacy – Grade 11 and 12 exemplars*.
2   Paper presented at the South African Mathematics Society Workshop. Pretoria.
3 Qualification and Curriculum Authority. (2007). *Functional skills standards*. London: Qualification and
4   Curriculum Authority.
5 Scribner, S. (1984). Studying working intelligence. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its*
6   *development in social context*. Cambridge, MA: Harvard University Press.
7 Steen, L. A. (2001). The case for quantitative literacy. In National Council on Education and the Disciplines &
8   L. A. Steen (Eds.), *Mathematics and Democracy* (pp. 1-22). Washington, DC: The Mathematical
9   Association of America.
10 Umalusi. (2009). *2008 Maintaining standards report. From NATED 550 to the new national curriculum:*
11   *maintaining standards in 2008*. Pretoria: Umalusi.
12 Venkat, H., Graven, M., Lampen, E., Nalube, P., & Chitera, N. (2009). Reasoning and reflecting in
13   Mathematical Literacy. *Learning and Teaching Mathematics, 7*, 47-53.
14