'Reasoning and Reflecting' in Mathematical Literacy

Hamsa Venkat, Mellony Graven,Erna Lampen, Patricia Nalube and Nancy Chitera
Marang Centre, Wits University

(Mellony Graven is currently Director of the Mathematics Centre at St Andrew's College and Diocesan School for Girls)

Introduction

In December 2008 we at the Marang Centre, Wits University decided to hold a two day workshop focused on a review of the recently completed Mathematical and Sciences matric examinations. Since this was the first set of matric examinations for the new National Senior Certificate for the Further Education and Training (FET) band, this seemed an important activity for academics, lecturers and researchers involved in Mathematical Sciences education research and development.

The discussions were lively and a range of interesting observations were made comparing the various levels of cognitive demand across various Mathematics, Science, Biology and Mathematical Literacy papers, as well as observations about the nature of contextualization in the papers. It is beyond the scope of this paper to share with you the various interesting insights, arguments and findings but we strongly encourage you to undertake a similar activity across these subjects in your schools.

Rather, in this paper we wish to share three key issues which emanated from our focus group discussions in relation to the Mathematical Literacy papers. In this focus group we attempted to analyse the questions in the two papers according to the provided taxonomy in the Subject Assessment Guidelines (SAG) (DoE 2007, p8). We write this paper largely to challenge the validity of this taxonomy and draw on some questions in the papers as examples to support our arguments.

The three arguments in focus in this paper are: firstly, the lack of a shared understanding of what it means to reason in the context of Mathematical Literacy; secondly (and following from the previous point), the separation of 'reasoning and reflecting' from 'doing' in the taxonomy structure; and lastly, the impact of what we view as 'over-scaffolding' on the openings available for reasoning in the papers.

Below, we introduce the taxonomy and then go on to discuss these issues.

The problematic taxonomy of cognitive levels

The SAG for Mathematical Literacy (DoE, 2008, 8) provides a taxonomy of 'cognitive demand' and a framework that details the proportion of questions that should fall within each level of the taxonomy in the Grade 12 Senior Certificate examinations. The levels, specified in terms of increasing cognitive demand, are detailed thus within the SAG document:

- Level 1: Knowing (30% of marks)
- Level 2: Applying routine procedures in familiar contexts (30%)
- Level 3: Applying multi-step procedures in a variety of contexts (20%)
- Level 4: Reasoning and reflecting (20%)

The percentages specify the proportions of marks across the two Mathematical Literacy papers that need to be assigned to each level.

One problem with the use of the taxonomy is the under-description of the term "reasoning". What does it mean to reason? Our previous experience of working with this taxonomy in various teacher workshops of the Mathematical Literacy thrust of Marang revealed immense differences in the way in which teachers, researchers, and teacher educators classified various activities and questions. There were instances when the same question or activity was classified at all four levels by different people. Coming to agreement on the most appropriate level of a question was seemingly futile and impassioned arguments followed with logical justifications following each person's classification. Similarly, in this December Marang workshop lively argument about 'the right level' of questions followed. In this paper we do not wish to make claims about the cognitive levels of various questions – indeed we would be unlikely, between the five of us, to come to agreement and this paper would never be completed. Rather it is the taxonomy itself and the inadequate definition of reasoning that makes radically contrasting classifications inevitable.

What does it mean to reason?

Our view is that reasoning is an inherent part of learning at all levels, and indeed, an inherent part of human activity. Reasoning is often described in terms of dichotomies. For example Vygotsky's (1987) distinction between spontaneous and scientific reasoning and Scribner's distinction between practical and theoretical reasoning (Scribner 1986). Dichotomies persist even within descriptions of theoretical reasoning. For example, our mathematical experiences focus on inductive and deductive reasoning. Inductive reasoning involves looking for patterns and regularities across specific cases in order to describe a rule based generalization; deductive reasoning on the other hand, involves reasoning from a mathematical rule or definition to find answers in specific cases. But we reason in our everyday lives as well, often quite unrelated to mathematics. Sfard (2007) views reasoning broadly in terms of 'thinking as communication' – with oneself or others – and argues that this thinking consists of acts such as asking questions, hypothesizing, finding counter-arguments and drawing conditional conclusions within a situation. This broad view of thinking is appropriate to Mathematical Literacy, located as it is at the intersection of everyday life and mathematics.

Rather than going deeply into the complete field of informal reasoning or everyday reasoning, we would like to describe aspects of reasoning of the kind we think are applicable to Mathematical Literacy. We hope that this will stimulate a debate that will lead to better clarification of what we define as reasoning in Mathematical Literacy, and how we can judge cognitive demands of tasks. Reasoning in everyday life involves making sense of a situation by scanning possibilities and deciding on those that fit the question or the argument best. This is called analysis. Reasoning also involves giving reasons - stringing together evidence and claims in an argument - I say the bus is late (claim), because none of the learners from that area is at school yet (evidence). Reasoning also involves judging statements or reasons, or asking for reasons. In practical situations (very often mathematical literacy problems involve reasoning about practical situations) reasoning means formulating problems in a situation, rather than solving already formulated problems; solving problems in flexible ways, rather than by following a pre-scribed and fixed procedure; incorporating the context as part of the problem-solving system, rather than stripping away the environment; and seeking modes of solution that require least effort or are most economical in terms of time, effort, cost, etc. (Scribner, 1986, p.25). These characteristics of practical reasoning, which people use in their everyday lives when they solve problems, alerted us to the fact that "doing" cannot be separated from reasoning and reflecting in a subject like Mathematical Literacy.

The problem of separating 'reasoning and reflection' from doing

In theoretical terms, placing 'knowing' at level 1 and 'reasoning and reflecting' at level 4 suggests a separation between a kind of knowing which cannot effect any action, and thinking about some completed action. To us, this taxonomy separates 'doing' and 'thinking', with thinking following doing at a respectful distance. To exacerbate the impression, the actions described in the middle levels isolate routine, albeit multi-level applications of procedures. Viewed outside a context and separated from a purpose, applying routine procedures of however many steps, seems to require very little reasoning (I can change a baby's nappy without even thinking about the baby or the nappy, exactly because it is a routine procedure – unless I lose a safety pin and have to make another plan). Where is the place for thoughtful action?

Deliberate action? Thinking and reasoning while doing? We argue that the contexts in which Mathematical Literacy play out, provide opportunities for reasoning which are overlooked if mathematical procedures are isolated for attention. Its placement at the highest level also suggests that 'reasoning' is something that only the highest attainers will be able to do. To separate reasoning from other aspects of mathematical literacy working is inherently problematic in our view of humans as reasonable beings.

'Reasoning and reflection' involves the whole spectrum of cognitive demand

Looking at the Mathematical Literacy papers we quickly came across questions that appeared to back our view that reasoning occurs at all levels of cognitive demand. An example is Q1.4 from Paper 1:

Nabila has moved to the city and bought her own house. She decides to compare her monthly budget to her parents' monthly budget, as illustrated in the charts below.

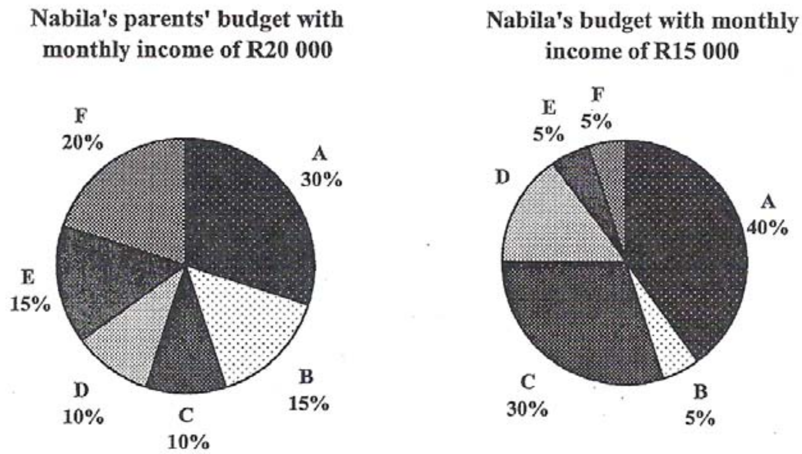


TABLE 1: Key for the pie charts

A	Food, bond, electricity
B	Insurance, medical aid
C	Clothing and personal care
D	Communication (cellphone, computer, internet service)
E	Savings
F	Other

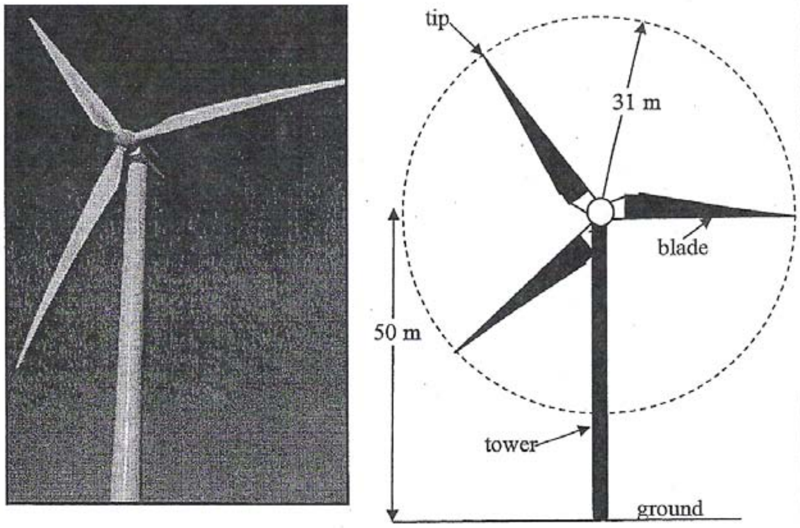
Q1.4.1 Write down TWO examples of monthly expenses that could be considered as 'Other' expenses. (2)

As a group, we felt that this question did involve reasoning – the candidate needs to make sense of the context of the question and think about what might reasonably be considered as other expenses within the contours of this situation. However, in the 'everyday' nature of the context, the reasoning called upon is relatively straightforward – essentially low level. We found it hard to classify this question as simply involving 'knowing' as we felt that reasoning was involved in deciding upon expenses that could be defined as 'other' within the situation. (For example, "hairdresser" would not be an acceptable answer, since that would fall under the category personal care).

At this stage we flag the difference between reasoning about the structure or nature of the context, as required by this question, and reasoning about the mathematics used. We do not want to suggest that all reasoning about the structure of the context is necessarily low level. Indeed, we argue that the cognitive demands of reasoning in context need to be incorporated in our understanding or reasoning in Mathematical Literacy.

A further example involving lower level reasoning, again from Paper 1, is seen in Q2.2:

Electricity has become a scarce resource in South Africa. As a result the country is investigating alternative sources of generating electricity. One alternative source of generating electricity is a wind turbine using rotating blades as shown in the picture and diagram below.



The wind turbine is mounted on the top of a 50 m high tower.
The length of each blade is 31 m.

2.2.2 Calculate the maximum height from the ground that the tip of a blade will be, if the turbine is rotating. (2)

Whilst in mathematical terms this is a relatively simple question, the question requires reasoning within the context of a rotating blade. This question requires the capacity to visualize the blades of the turbine rotating and understanding that the maximum height occurs when the tip is standing vertically above the tower. Reasoning about the structure of the situation is therefore involved, and it is unlikely to be entirely straightforward given that few learners are likely to "know" the context of wind turbines.

Opportunities for reasoning reduced by 'over scaffolding'

Another issue that emerged within our consideration of the examination papers related to our sense that opportunities for increased cognitive demand on reasoning were in some cases diminished by what we viewed as 'over-scaffolding' of questions. This is likely to be as a result of an attempt by examiners to adhere to the cognitive taxonomy by isolating "knowing" and "routine procedures" through scaffolding in the initial sub-questions. Our opinion is that such a practice might facilitate the marking process, but vitiates potential cognitive demands of the question.

In the educational literature, Bruner (1975) and Vygotsky (1978) have used notions of 'scaffolding' and 'mediation' as ways of supporting the development of thinking amongst learners during the learning process. However, the practice of scaffolding during assessment is problematic. Just as scaffolding around a building is removed in order to assess the completed building, so assessment of Mathematical Literacy should be unobstructed by scaffolding to allow candidates to display their full range of knowledge and reasoning abilities. In the papers, we came across examples that seemed to do the very opposite of this: they closed off openings for making choices (reasoning) about what information to select (reasoning), and decisions on how to represent and collate information most effectively and economically (reasoning). An example of this is given in Paper 2, within Q4.2 and the attached annexure on which learners had to fill in their answers. These are shown below:

4.2 **Lebo's family lives in the Eastern Cape. He now lives and works in Gauteng and earns a net salary of R10 625,00 per month. He sends home 35% of his net monthly salary every month and uses the rest for his own living expenses.**

Lebo's living expenses include the following:

- R3 500,00 per month for food and rental
- R18,00 per day for transport (He works for 21 days every month.)
- A cellphone contract of R135,00 per month
- Clothing accounts of R250,00 a month
- 10% of his net monthly salary for entertainment

4.2.1 Complete Lebo's monthly budget on ANNEXURE C. Show ALL the calculations on ANNEXURE C.

(a) Calculate the amount he sends home every month.
Fill in the answer at **A**. (2)

(b) Calculate the amount he has left for his own living expenses.
Fill in the answer at **B**. (1)

(c) Calculate his total monthly living expenses.
Fill in the answer at **C**. (6)

(d) Calculate the amount he has left after all his monthly living expenses have been paid.
Fill in the answer at **D**. (1)

ANNEXURE C, QUESTION 4.2.1

LEBO'S MONTHLY BUDGET			
		R	**c**
Net salary			
Amount sent home	A		
Amount for living expenses	B		
LIVING EXPENSES			
Food and rental			
Transport			
Cellphone contract			
Clothing account			
Entertainment			
TOTAL LIVING EXPENSES:	C		
AMOUNT REMAINING:	D		

The step-by-step nature of the question breakdown serves to direct candidates' attention to particular parts of the text in sequence. The provision of a sheet on which to fill in answers negates the need for candidates to take responsibility for organizing this information in ways that might facilitate combining the different parts to get to the answer for part d. The reasoning demands of the question, contextual as well as mathematical, are therefore reduced within this scaffolding structure.

This kind of step-by-step scaffolding was pointed out within the exemplar papers for Grade 12 Mathematical Literacy in 2008 (Prince, Frith et al. 2008) and has also been noted as a common feature of traditional mathematics classroom practices, sometimes described as 'funneling' (Bauersfeld 1980). Such scaffolding practices have been noted as problematic precisely because they reduce the need for learners to reason and reflect independently – to decide what information within a situation is relevant to the problem they want to solve, to think about how to organize, represent and process this information, and to reflect on how to interpret their solution within the contours of the situation. The South African Mathematical Literacy curriculum notes that a key aim of this new subject is to prepare learners to deal with the mathematical demands of everyday life:

'In everyday life a person is continually faced with mathematical demands which the adolescent and adult should be in a position to handle with confidence. […] Mathematical Literacy, should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy. Mathematical Literacy will ensure a broadening of the education of the learner which is suited to the modern world.' (DoE 2003, p9-10). Of course this is not to deny that scaffolding can be valuable in both teaching and assessment, but rather that over-scaffolding and inappropriate scaffolding can reduce opportunities for student reasoning in ways that undermine the development of learners as 'self-managing' persons.

In conclusion

Steen (2001), a key proponent of what he terms 'Quantitative Literacy' has pointed out that in people's daily lives, problems rarely occur in tidily packaged forms. Often, they require the need to make sense of available information, and then to identify and seek out supplementary information that is required to deal with the issue at hand, and then to work with this information in ways that help the individual to understand the issue. The mathematical work involved across these processes can range from relatively simple to highly complex; the context of the problem can similarly work across this range from simple to complex. Reasoning though, comes into play across all these levels and in both domains: context and mathematics. The scaffolding examples we have provided indicate a further important variable that can affect complexity in relation to both the mathematical and the contextual strands.

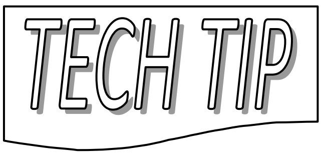
The fact that reasoning can occur at varying levels and be related to mathematical working and/or contextual complexity, and further, be affected by the degree of scaffolding that is built in, is unacknowledged in the taxonomy as it stands. Our view is that reasoning in many ways, is an overarching dimension that runs across the other three levels. Placing it at the highest level of the taxonomy tends to problematically emphasise 'unthinking' recall and procedural work as the entry level demand for doing Mathematical Literacy – an emphasis which we strongly disagree with.'

The taxonomy as it stands currently is therefore problematic. Further investigation is required if it is to better reflect the kinds of competence that are central to Mathematical Literacy. Through continued investigation of the practices of Mathematical Literacy assessment we hope to, in future writing, propose an alternative taxonomy. We encourage Mathematical Literacy teachers to engage with us on your experiences of using this taxonomy in your teaching and assessment and your ideas for its improvement.

References:

Bauersfeld, H. (1980). "Hidden dimensions in the so-called reality of a mathematics classroom." Educational Studies in Mathematics 11: 23-41.

Bruner, J. (1975). "The ontogenesis of speech acts." Journal of Child Language 2: 1-40.
DoE (2003). National Curriculum Statement Grades 10-12 (General): Mathematical Literacy, Department of Education.
DoE (2007). National Curriculum Statement Grades 10-12 (General): Subject Assessment Guidelines, Mathematical Literacy.
Prince, R., V. Frith, et al. (2008). Mathematical Literacy - Grade 11 and 12 exemplars. South African Mathematics Society Workshop. University of Pretoria.
Scribner, S. (1986). Thinking in action: Some characteristics of practical thought. Practical intelligence: Nature and origins of competence in the everyday world. R. J. Sternberg and R. K. Wagner. Cambridge, Cambridge University Press: 13-30.
Sfard, A. (2007). "When the rules of discourse change, but nobody tells you: making sense of mathematics learning from a commognitive standpoint." The Journal of the Learning Sciences 16(4): 567-615.
Steen, L. A. (2001). The case for quantitative literacy. Mathematics and Democracy. National Council on Education and the Disciplines and L. A. Steen. Washington D.C., The Mathematical Association of America: 1-22.
Vygotsky, L. (1987). Thinking and speech. The collected works of L.S. Vygotsky, Volume 1: Problems of general psychology, (Trans. N. Minick). R. W. Rieber and A. S. Carton. New York, Plenum.
Vygotsky, L. S. (1978). Mind in Society: The development of higher psychological processes. Cambridge, MA, Harvard University Press.



Is your Casio fx82ES screen too dark or light?

To change the density (or darkness) of you calculator screen follow these instructions.

Press **SHIFT + SETUP**	Scroll down (using the down arrow on the *replay* button); Press **5: ◀ CONT ▶**	Darken or lighten screen (using the right or left arrows on the *replay* button)
▼ 1:MthIO 2:LineIO 3:Deg 4:Rad 5:Gra 6:Fix 7:Sci 8:Norm	▲ 1:ab/c 2:d/c 3:STAT 4:Disp 5:◀CONT▶	CONTRAST LIGHT [◀] DARK [▶]