Approximation methods in general relativity for gravitational-wave astrophysics

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Exercise 1

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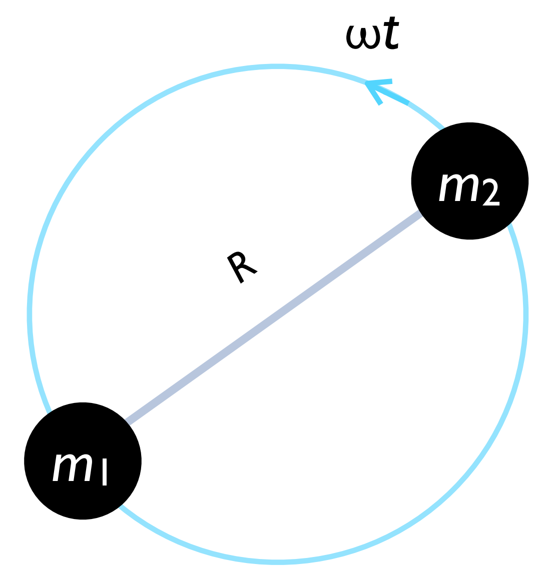
Gravitational waves in linearized GR

- In transverse-traceless gauge, the metric perturbation can be related to the quadrupole-moment of the source.

$$\omega = \sqrt{\frac{Gm}{R^3}}$$

$$h \sim \frac{2}{d}\ddot{Q}_{jk} \sim \left(\frac{G}{c^4}\right)\frac{4\mu}{d}\left(\frac{Gm}{R}\right)\cos 2\,\omega t$$

10^{-45}!



$M = m_1 + m_2$

Exercise: Compute GW amplitudes and frequencies

- **A “GW generator”** Two masses with 10^3 kg separated by 10m orbiting each other at a frequency of 10 Hz ($m_1 = m_2 = 10^3$ kg, $d = 10^4$ km).
- **Planets** A Jupiter-sized planet orbiting a Sun-like star in our own galaxy ($m_1 = 10^{-3} M_\odot$, $m_2 = 1 M_\odot$, $P_{\rm orb} = 12$ yrs, $d = 1$ kpc).
- **Binary stars** A main-sequence star binary in our own galaxy ($m_1 = m_2 = M_\odot$, $P_{\rm orb} = 1000$ yrs, $d = 1$ kpc).
- **White-dwarf binaries** A white-dwarf binary in our own galaxy ($m_1 = m_2 = 0.5\ M_\odot$, $P_{\rm orb} = 30$ mins, $d = 1$ kpc).
- **Binary pulsars** The Hulse Taylor binary pulsar ($m_1 = m_2 = 1.4\ M_\odot$, $P_{\rm orb} = 7.8$ hrs, $d = 6.4$ kpc).
- **Compact binary coalescence** Binary black-hole (BH) coalescence in Virgo cluster. ($m_1 = m_2 = 10\ M_\odot$, $P_{\rm orb} = 0.1$ sec, $d = 16.5$ Mpc)
- **Supermassive BH binaries** Merger of two galaxies each hosting a supermassive BH ($m_1 = m_2 = 10^6 M_\odot$, $P_{\rm orb} = 1$ yr, $d = 1$ Gpc)

Exercise: Plot the amplitude and frequency of GWs from different sources