Approximation methods in general relativity for gravitational-wave astrophysics

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Exercise 2

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GW phasing formula

- The time evolution of the orbital phase can be computed by solving the following set of coupled ordinary differential equations, called the phasing formula.

$$G = c = 1$$

GW flux (quadrupolar)

$$\mathcal{F}_{\rm PN} \simeq \frac{32}{5}\left(\frac{\mu}{m}\right)^2 v^{10}$$

Orbital binding energy (Newtonian)

$$E_{\rm PN} \simeq -\frac{1}{2}\mu v^2$$

$$\mu = m_1 m_2/m$$

Reduced mass

$$\frac{dv}{dt} = -\frac{\mathcal{F}_{\rm PN}}{dE_{\rm PN}/dv}$$

Orbital phase

$$\varphi_{\rm orb} \equiv \varphi_{\rm GW}/2$$

$$v \equiv (m\omega_{\rm orb})^{1/3}$$

PN parameter

$$\frac{d\varphi_{\rm orb}}{dt} = \frac{v^3}{m}$$

$$m = m_1 + m_2$$

Total mass

Exercise: Compute and plot the GW polarizations

- Compute and plot the GW polarizations $h_+(t)$ and $h_\times(t)$ from an inspiralling binary black hole at a distance of $100\ \mathrm{Mpc}$ with $m_1 = m_2 = 10\ M_\odot$, starting from an orbital frequency of $20\ \mathrm{Hz}$.

$$h_+(t) = -\frac{4\mu}{d}\, v^2\ \cos\varphi_{\mathrm{GW}}(t)$$
$$h_\times(t) = -\frac{4\mu}{d}\, v^2\ \sin\varphi_{\mathrm{GW}}(t)$$

- **Note** All expressions are in geometric units: $G = c = 1$. Thus, masses and distance are given in seconds. In order to get physical units, all masses should be multiplied by G/c^3 and distance should be divided by c.