Gravitational waves linearized about a spherically symmetric background

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GWs and matter: Key points from earlier work

- A non-dissipative fluid does not absorb energy
- If a GW propagates from r to $r + \Delta$ through a dissipative medium with shear viscosity η

$$(h_+, h_\times)(r + \Delta) = (h_+, h_\times)(r)\exp(-8\pi\eta\Delta)$$

  in geometric units. Translating to SI units,

$$8\pi\eta\Delta = \frac{8\pi\eta\Delta G}{c^3} = 8\pi\eta\Delta \times 2.477 \times 10^{-36}$$

- Viscosity of common fluids range from 10^{-3}kg/m/s (water) to 10kg/m/s (syrup)
- Astrophysical viscosity values can be as high as 10^{25}kg/m/s at the core of a supernova, but this occurs in a region of size at most 10^5m.
- GWs have been observed from large distances $\sim$ 1Gpc $= 3.0857 \times 10^{25}$m, but even so, an unreasonably large viscosity would be needed for significant damping.
- Thus, matter is commonly regarded as being transparent to GWs.

Coordinates are based on outgoing null hypersurfaces, (u, r, x^A); u labels the null hypersurface, r is a radial coordinate, and x^A are angular coordinates θ, ϕ. Then

$$ds^2 = -\left(e^{2\beta}\left(1 + W_c r\right) - r^2 h_{AB} U^A U^B\right) du^2 - 2e^{2\beta} du dr - 2r^2 h_{AB} U^B du dx^A + r^2 h_{AB} dx^A dx^B .$$

Minkowski spacetime has $W_c = \beta = U^A = 0$, $h_{AB} = q_{AB}$ with q_{AB} a unit sphere metric, $q_{AB} dx^A dx^B = d\theta^2 + \sin^2\theta d\phi^2$. We define perturbations

$$\beta = \Re\left(\beta^{[2,2]}(r) e^{i\nu u}\right) Z_{2,2}\,, W_c = \Re\left(W_c^{[2,2]}(r) e^{i\nu u}\right) Z_{2,2}\,,$$
$$U = \Re\left(U^{[2,2]}(r) e^{i\nu u}\right) {}_1Z_{2,2}\,, J = \Re\left(J^{[2,2]}(r) e^{i\nu u}\right) {}_2Z_{2,2}\,,$$

where $U = U^A q_A, J = h^{AB} q^A q^B/2$ with $q_A = (1, i\sin\theta)$ and where ${}_sZ_{2,2}$ are spin-weighted spherical harmonics. This "separation of variables" ansatz leads to the linearized Einstein equations reducing to ODEs in r.

Procedure for solving the linearized Einstein equations

- Solving $R_{11} = 0$ gives $\beta^{[2,2]}(r) = b_0$ constant.
- Eqs. $q^A R_{1A} = 0$ and $q^A q^b R_{AB} = 0$ are ODEs in $U^{[2,2]}(r)$ and $J^{[2,2]}(r)$, and can be combined into a single master equation

$$x^4 \frac{d^2 J_2(x)}{dx^2} + 2x(2x^2 + i\nu x)\frac{dJ_2(x)}{dx} - 2(2x^2 + i\nu x)J_2(x) = 0\,,$$

where $x = 1/r$ and $J_2(x) = d^2 J(x)/dx^2$. The solution is

$$J_2(x) = C_1 x + C_2 \exp\left(\frac{2i\nu}{x}\right)\left(3x - 6i\nu - 6\frac{\nu^2}{x} + 4\frac{i\nu^3}{x^2} + 2\frac{\nu^4}{x^3}\right)\,.$$

- Integrating $J_2(x)$ twice leads to expressions for $J^{[2,2]}(r)$, $U^{[2,2]}(r)$ involving 4 arbitrary constants.
- Integrating $h^{AB} R_{AB} = 0$ leads to $W_c^{[2,2]}(r)$ and introduces another integration constant.
- The solution involves 6 arbitrary constants, 2 of which are fixed using the constraint equations $R_{00} = 0, q^A R_{0A} = 0$. Then the solution is[1]

[1] N.T. Bishop, Linearized solutions of the Einstein equations within a Bondi–Sachs framework, and implications for boundary conditions in numerical simulations, Class. Quant. Grav. **22** 2393–2406 (2005)

$$\beta^{[2,2]} = b_0\,,$$

$$W_c^{[2,2]} = 4i\nu b_0 - 2\nu^4 C_{40} - 2\nu^2 C_{30} + \frac{4i\nu C_{30} - 2b_0 + 4i\nu^3 C_{40}}{r}$$
$$+ \frac{12\nu^2 C_{40}}{r^2} - \frac{12i\nu C_{40}}{r^3} - \frac{6C_{40}}{r^4} - C_{in0}\exp(2ir\nu)\frac{3}{r^4}\,,$$

$$U^{[2,2]} = \frac{\sqrt{6}(-2i\nu b_0 + \nu^4 C_{40} + \nu^2 C_{30})}{3} + \frac{2\sqrt{6}b_0}{r} + \frac{2\sqrt{6}C_{30}}{r^2}$$
$$- \frac{4i\nu\sqrt{6}C_{40}}{r^4} - \frac{3\sqrt{6}C_{40}}{r^4} - C_{in0}\exp(2ir\nu)\sqrt{6}\left(i\frac{\nu}{r^3} - \frac{3}{2r^4}\right)\,,$$

$$J^{[2,2]} = \frac{2\sqrt{6}(2b_0 + i\nu^3 C_{40} + i\nu C_{30})}{3} + \frac{2\sqrt{6}C_{30}}{r} + \frac{2\sqrt{6}C_{40}}{r^3}$$
$$+ C_{in0}\exp(2ir\nu)\sqrt{6}\left(\frac{1}{r^3} - 2i\frac{\nu}{r^2} - \frac{\nu^2}{r}\right)\,.$$

If there are no incoming GWs, $C_{in0} = 0$. Defining the rescaled GW strain by $\mathcal{H}_0 = r(h_+ + ih_\times)$, we find

$$\mathcal{H}_0 = \Re(H_0\exp(i\nu u))\,{}_2Z_{2,2} \text{ with } H_0 = -2\sqrt{6}\nu^2 C_{40}\,.$$

C_{40} is determined by the physics, and b_0, C_{30} are gauge freedoms.

The model: GW source surrounded by a spherical matter shell

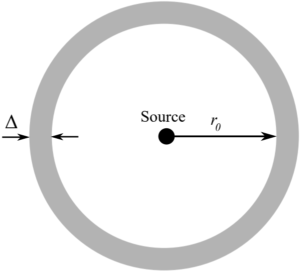


Figure: GW source inside a spherical shell of mass M_S between $r = r_0$ and $r = r_0 + \Delta$.

- Construct solutions (a) Exterior to the shell, (b) Within the shell, and (c) Interior to the shell.
- Enforce continuity of the metric and first derivatives at the boundaries of the shell, and so construct a complete solution.

The modified GW strain due to a dust shell

$$H = H_0 \left[1 + \frac{2M_S}{r_0} - \frac{iM_S\lambda}{r_0^2\pi} + \frac{iM_S\lambda e^{-4ir_0\pi/\lambda}}{4r_0^2\pi} + \cdots \right] .$$

Each of the terms containing M_S represents a correction to the wave strain in the absence of the shell.

- The first correction, $2M_S/r_0$ represents the (well-known) gravitational red-shift effect.
- The second correction term, $iM_S\lambda/(\pi r_0^2)$, is out of phase with the leading terms $1 + 2M_S/r_0$ and hence represents a phase shift of the GW without changing the GW energy.
- The third correction term represents a partial reflection of the GW by the shell, and leads to the possibility of a matter shell causing a GW echo.

- There has been previous work on whether the astrophysical environment might modify a GW signal including the possibility of an echo[2]
- It has been proposed that GW echoes are observed in LIGO data of GW150914[3], but this suggestion is not widely accepted.
- Could a matter shell have produced the suggested echo? It was at 0.3s after merger, so the shell would have a radius of 45,000km. The magnitude of the echo was about 0.0992 times the original signal. Using 132Hz for the frequency, gives $M_S \approx 740,000 M_\odot$. Such a mass within a radius of 45,000km would constitute a black hole, so the scenario of an echo caused by a shell can be discounted for GW150914.
- A much smaller shell radius would avoid an unreasonably large shell mass, but the effect would be seen as modifying the original signal, rather than an echo.
- The magnitude of the echo term can be written

$$\frac{2M_S}{r_0} \times \frac{\lambda}{8\pi r_0}$$

where λ is the wavelength

[2] E.g., Barausse, E., Cardoso, V., Pani, P.: Can environmental effects spoil precision gravitational-wave astrophysics? Phys. Rev. D **89**(10), 104059 (2014). Konoplya, R.A., Stuchlík, Z., Zhidenko, A.: Echoes of compact objects: new physics near the surface and matter at a distance. Phys. Rev. D **99**, 024007 (2019)

[3] Abedi, J., Dykaar, H., Afshordi, N.: Echoes from the Abyss: tentative evidence for Planck-scale structure at black hole horizons. Phys. Rev. D **96**(8), 082004 (2017)

Astrophysical example: Shell around BBH merger

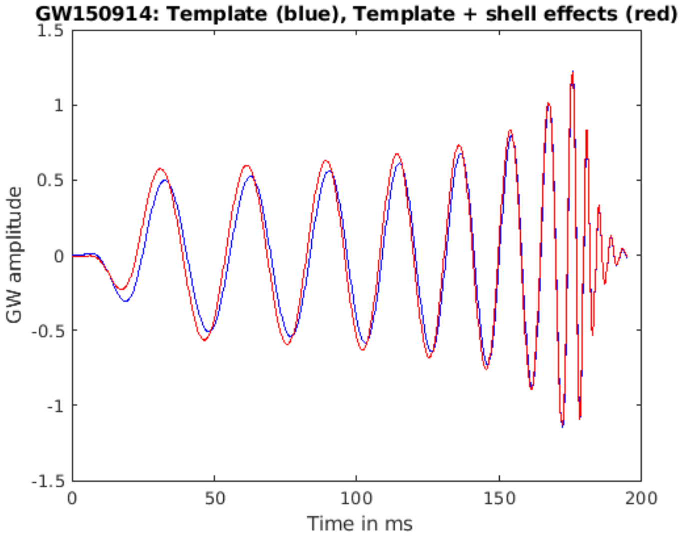


Figure: The effect of a matter shell of radius 900km and mass $60M_\odot$ on the signal of GW150914. The original signal is in blue, and the original signal plus modifications is in red.

Making the usual separation of variables, i.e.,

$$\sigma_{11} = \Re(\sigma_{11}^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,,\, q^A\sigma_{1A} = \Re(\sigma_{1U}^{[2,2]}(r)e^{i\nu u})_1 Z_{2,2}\,,$$
$$q^{AB}\sigma_{AB} = \Re(\sigma_{W}^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,,\, q^A q^B\sigma_{AB} = \Re(\sigma_{J}^{[2,2]}(r)e^{i\nu u})_2 Z_{2,2}\,.$$

we find

$$\begin{aligned}
\theta = \sigma_{00} = \sigma_{01} = \sigma_{0A} &= 0\,, \\
-\sigma_{11}^{[2,2]} = \sigma_{W}^{[2,2]} &= 12C_{40}\,\frac{3i - 3r\nu - ir^2\nu^2}{r^5\nu} \\
\sigma_{1U}^{[2,2]} &= 2C_{40}\,\frac{6i - 6r\nu - 3ir^2\nu^2 + r^3\nu^3}{r^4\nu} \\
\sigma_{J}^{[2,2]} &= C_{40}\,\frac{-3 - 3ir\nu + 3r^2\nu^2 + 2ir^3\nu^3 - r^4\nu^4}{r^3\nu}\,.
\end{aligned}$$

Energy loss due to viscosity

We use the formula that the rate of energy loss per unit volume is $-2\eta\sigma_{ab}\sigma^{ab}$. This quantity is evaluated, then integrated over a shell of radius r and thickness Δ; the integration is straightforward because of the orthonormality of the angular basis functions. We find that the average rate of energy loss to the shell is

$$\left\langle \dot{E}_\eta \right\rangle = -12\eta C_{40}^2 \nu^6 \Delta \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right) ,$$

where $\langle f \rangle$ denotes the average of $f(u)$ over a wave period, i.e.

$$\langle f \rangle = \frac{\nu}{2\pi} \int_0^{\frac{2\pi}{\nu}} f dt \, ,$$

and where we have used $\left\langle \cos^2(\nu u) \right\rangle = \left\langle \sin^2(\nu u) \right\rangle = 1/2$ and $\langle \cos(\nu u) \sin(\nu u) \rangle = 0$.

The rate of energy output as GWs is

$$\left\langle \dot{E}_{GW} \right\rangle = \frac{3C_{40}^2 \nu^6}{4\pi},$$

so that

$$\left\langle \dot{E}_\eta \right\rangle = -16\pi\eta\Delta \left\langle \dot{E}_{GW} \right\rangle \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right).$$

Energy conservation means that energy absorbed by the viscous fluid is balanced by a reduction in the GW energy. Thus

$$\left\langle \dot{E}_{GW} \right\rangle (r + \Delta) = \left\langle \dot{E}_{GW} \right\rangle (r) \times \left[1 - 16\pi\eta\Delta \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right)\right].$$

H represents the magnitude of the GWs, and $\left\langle \dot{E}_{GW} \right\rangle \propto H^2$

$$H(r+\Delta) = H(r)\left[1 - 8\pi\eta\Delta\left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right)\right].$$

The resulting differential equation is solved to give

$$H(r) = C\exp\left(-8\pi\eta\left(r - \frac{2}{r\nu^2} - \frac{3}{r^3\nu^4} - \frac{9}{r^5\nu^6} - \frac{45}{r^7\nu^8}\right)\right),$$

where C is a constant.

There are two useful special cases. Let r_i, r_o be the inner and outer radii of the shell. If r_i, r_o are much larger than the wavelength λ of the GWs, then

$$H(r_o) = H(r_i) \exp\left(-8\pi\eta(r_o - r_i)\right) .$$

Equivalent results have been given before[4]. If r_i is much smaller than the wavelength of the GWs with $r_o = \alpha r_i$ with $\alpha > 1$ then

$$H(r_o) = H(r_i) \exp\left(-\frac{360\pi\eta}{r_i^7 \nu^8}\left(1 - \alpha^{-7}\right)\right) = H(r_i) \exp\left(-\frac{45\eta\lambda^8}{32 r_i^7 \pi^7}\left(1 - \alpha^{-7}\right)\right) .$$

As $r_o \to \infty$, this reduces to

$$H(r_o) = H(r_i) \exp\left(-\frac{45\eta\lambda^8}{32 r_i^7 \pi^7}\right) ,$$

but the damping effect is reduced by only a little for a shell of finite thickness. For example, $(1 - \alpha^{-7})$ is 0.99 for $\alpha = 2$ and is 0.5 for $\alpha = 1.104$.

[4] E.g., S.W. Hawking, Perturbations of an expanding universe, Astrophys. J. **145**, 544 (1966).

GW Heating

- Energy lost by GWs = Energy gained by viscous fluid.
- The calculation of changes to the temperature T needs following physical parameters
  - Specific heat capacity C
  - Density ρ
  - Energy of GWs ΔE_{GW}
- Let

$$\psi = \frac{2\pi r}{\lambda}$$

- Then

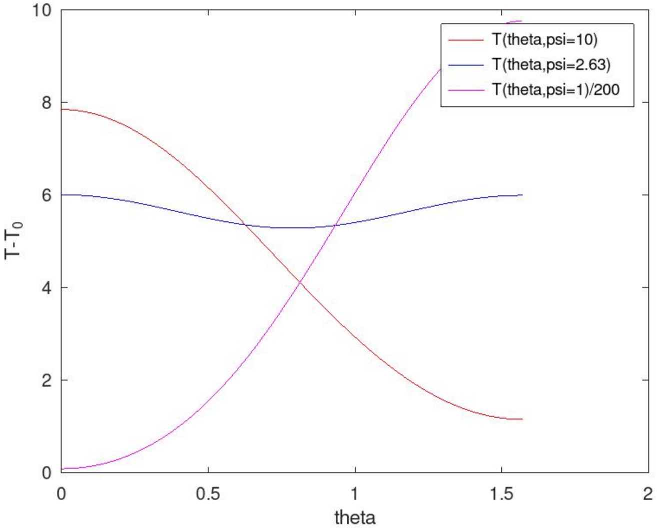
$$T - T_0 = \frac{2G\eta\Delta E_{GW}}{r^2 c^3 C\rho} \left[D_0 Y_{0,0} + D_2 Y_{2,0} + D_4 Y_{4,0}\right]$$

  with D_0, D_2, D_4 having the same funcional form
- When there is good heat flow in the shell, or if the GW sgnal is on average isotropic, then

$$T - T_0 = \frac{2G\eta\Delta E_{GW}}{r^2 c^3 C\rho} D_0 Y_{0,0} = \frac{2G\eta\Delta E_{GW}}{r^2 c^3 C\rho} \left(\frac{315}{\psi^8} + \frac{45}{\psi^6} + \frac{9}{\psi^4} + \frac{2}{\psi^2} + 1\right)$$

- For large r, $T - T_0$ falls off as r^{-2}, but for small r/λ fall-off goes as r^{-10}.

The heating effect as a function of θ for different $\psi = 2\pi r/\lambda$



- The de Sitter metric is $ds^2 = -dt^2 + A^2 e^{2\alpha t}(d\rho^2 + \rho^2 d\Omega^2)$, where $\alpha = \sqrt{\Lambda/3}$ with Λ being the cosmological constant.
- In Bondi-Sachs coordinates,

$$ds^2 = -du^2(1 - r^2\alpha^2) - 2dudr + r^2 d\Omega^2 \,.$$

- Construction of the Einstein equations leads to a master equation (when $\ell = 2$)

$$x^2(x^2 - \alpha^2)\frac{d^2 J_2(x)}{dx^2} + 2x(2x^2 + i\nu x + \alpha^2)\frac{dJ_2(x)}{dx} - 2(2x^2 + i\nu x + \alpha^2)J_2(x) = 0 \,,$$

  where $x = 1/r$ and $J_2(x) = d^2 J(x)/dx^2$.
- This equation has a simple solution representing outgoing GWs, $J_2(x) = x$, which also applies in the Minkowski case. The solution for incoming GWs is a complicated function involving α.
- Modifications to the GW field are significant when $\alpha r \gtrapprox 1$, i.e. $r \gtrapprox 5$Gpc.

The damping/heating effects on a Schwarzschild background

Making the ansatz of a small perturbation about Schwarzschild spacetime leads to

$$\beta = \Re(\beta^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,, \quad U = \Re(U^{[2,2]}(r)e^{i\nu u})_1 Z_{2,2}\,,$$
$$W_c = -\frac{2M}{r^2} + \Re(W_c^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,, \quad J = \Re(J^{[2,2]}(r)e^{i\nu u})_2 Z_{2,2}\,.$$

Manipulation of the Einstein vacuum equations leads to a master equation

$$-2J_2(2x + 8Mx^2 + i\nu) + 2\frac{dJ_2}{dx}\left(2x^2 + i\nu x - 7x^3 M\right) + x^3(1 - 2xM)\frac{d^2 J_2}{dx^2} = 0$$

where $J_2(x) \equiv d^2 J^{[2,2]}/dx^2$ and $x = 1/r$. In the Minkowski background case $M = 0$, there is a simple analytic solution

$$J_2(x) = b_1 x + b_2 \exp\left(\frac{2i\nu}{x}\right)\left(3x - 6i\nu - \frac{6\nu^2}{x} + \frac{4i\nu^3}{x^2} + \frac{2\nu^4}{x^3}\right)\,,$$

where b_1, b_2 are integration constants; further, if there are no incoming GWs, then $b_2 = 0$ and the various metric terms $J^{[2,2]}(r)$ simplify to polynomials in $1/r$. **However**, in the Schwarzschild case $M \neq 0$, it is not possible to solve the master equation analytically, and construction of a numerical solution is not straightforward since the differential equation has an essential singularity at $x = 0$.

- Let $S_N = \sum_{n=1}^{N} a_n x^n$ denote a partial series solution about $x = 0$, then it can be shown that, given $\epsilon > 0$, values for x_0, N can be determined such that[5]

$$\left| J_2(x) - \sum_{n=1}^{N} a_n x^n \right| < \epsilon \ \ \forall x \ \ \text{with } 0 < x \leq x_0 \, .$$

- Choose $\epsilon = 10^{-16}$ (machine precision), and construct boundary data $J_2(x_0)$, $\partial_x J_2(x_0)$ for the numerical integration of the master equation in $x_0 \leq x \leq 0.25$.
- Construct metric quantities $J^{[2,2]}(r), U^{[2,2]}(r), W_c^{[2,2]}(r), \beta^{[2,2]}(r)$ with all integration constants set to 0. Let $R^{[0]}_{00}, R^{[0]}_{0A}$ be the resulting components of the Ricci tensor. Then we use the constraint equations to write

$$R_{00} : \ C_{30} 6i\nu \left(\frac{2}{r^2} + \frac{3M}{r^3} \right) + C_{50} \left(\frac{i\nu}{r^2} - \frac{3}{r^3} \right) + R^{[0]}_{00} = 0$$

$$q^A R_{0A} : \ C_{30} \frac{3i\nu M \sqrt{6}}{r^2} - C_{50} \frac{\sqrt{6}}{2r^2} + q^A R^{[0]}_{0A} = 0 \, ,$$

and find values of C_{30}, C_{50} that minimize the square of the error over the numerical grid in the range $x_0 \leq x \leq 0.25$.

[5] F. Olver, *Asymptotics and special functions* (Academic Press, New York, 1974)

In the $M = 0$ case, there are analytic formulas for the metric coefficients. The maximum of the absolute value of the error for each of these quantities is

Quantity	$\|\|\text{Error}\|\|_\infty$
J	5.3×10^{-10}
U	1.6×10^{-9}
W_c	1.7×10^{-9}
f_E	2.5×10^{-8}

In the $M = 1$ case, the numerical solutions obtained are substituted into the left hand side of the Einstein equations, and we evaluate the maximum of the absolute value

Quantity	$\|\|\text{Error}\|\|_\infty$
$q^A R_{1A}$	7.8×10^{-15}
$h^{AB} R_{AB}$	7.5×10^{-15}
$q^A q^B R_{AB}$	4.1×10^{-14}
R_{01}	4.7×10^{-16}
R_{00}	6.8×10^{-10}
$q^A R_{0A}$	6.4×10^{-10}

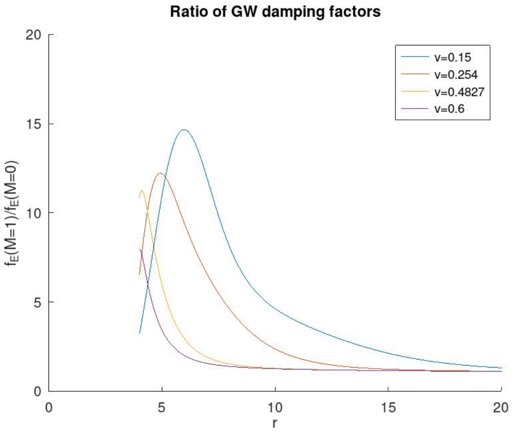
- Having found the metric, it is then straightforward to find the velocity field and then the shear tensor σ_{ab}. The rate of energy loss per unit volume $-2\eta\sigma_{ab}\sigma^{ab}$ is found, leading to

$$\left\langle \dot{E}_S \right\rangle = -16\pi\eta\frac{G}{c^3}\delta r \left\langle \dot{E}_{GW} \right\rangle f_E(r,\nu,M)\,,$$

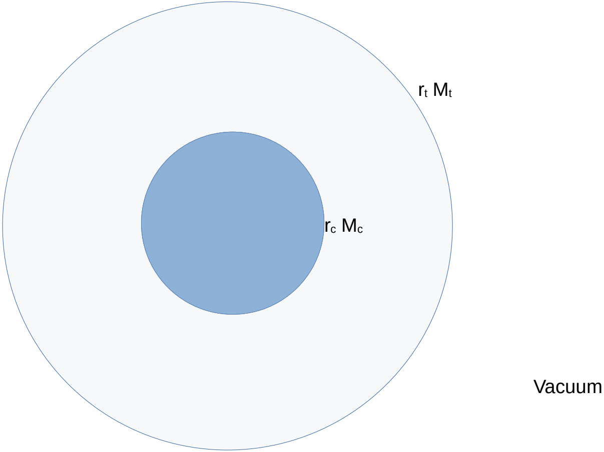
where $\left\langle \dot{E}_S \right\rangle$ is the time-averaged rate of energy increase to a spherical shell of radius r and thickness δr. We compute

$$\frac{f_E(r,\nu,M)}{f_E(r,\nu,0)}$$

with $f_E(r,\nu,0)$ known analytically and $f_E(r,\nu,M)$ evaluated numerically.



Using a Minkowski background underestimates the damping/heating effect.



rt Mt
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Vacuum

- The model comprises a core of radius r_c and mass M_c inside a matter distribution of variable density. The total mass is M_t and $r > r_t$ is vacuum.
- The background metric is

$$ds^2 = -(1 + W_c^{[B]} r)\exp(2B^{[B]})du^2 - 2\exp(2B^{[B]})du\,dr + q_{AB}dx^A\,dx^B$$

  and the background matter fields are the density $\rho^{[B]}$ and the pressure $p^{[B]}$, with the density being regarded as given. Then the Einstein equations together with the conservation conditions $\nabla_a T^{ab} = 0$ lead to a system of ODEs in $\partial_r \beta^{[B]}$, $\partial_r W_c^{[B]}$ and $\partial_r p^{[B]}$
- The system is integrated numerically inwards from r_t to r_c using the Schwarzschild metric as boundary condions at $r = r_t$ (i.e., $B^{[B]} = 0$, $W_c^{[B]} = -2M_t r_t^2$, $p^{[B]} = 0$).

General spherically symmetric, static matter distribution

- The ansatz of metric perturbations on the background metric in the Einstein equations does not lead to a master equation, but instead to a coupled system of 5 first-order ODEs in the variables $J^{[2,2]}(r)$, $\partial_r J^{[2,2]}(r)$, $U^{[2,2]}(r)$, $\partial_r U^{[2,2]}(r)$ and $W_c^{[2,2]}(r)$.
- The system is solved numerically using the perturbed Schwarzschild solution to provide boundary data at $r = r_t$ to find the solution in $r_c \leq r \leq r_t$.
- Then, as before, we find the velocity field, shear σ_{ab} and the quantity f_E.

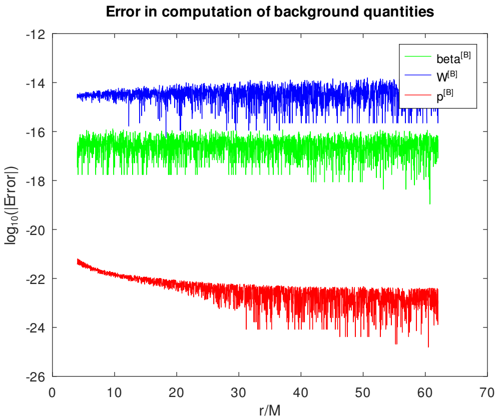

Figure: The matter code is run with $\rho^{[B]}$ =constant and then, using the exact Schwarzschild interior solution, the erors in $\beta^{[B]}$, $W^{[B]}$, $p^{[B]}$ are plotted on a $\log_{10}$ scale against r/M.

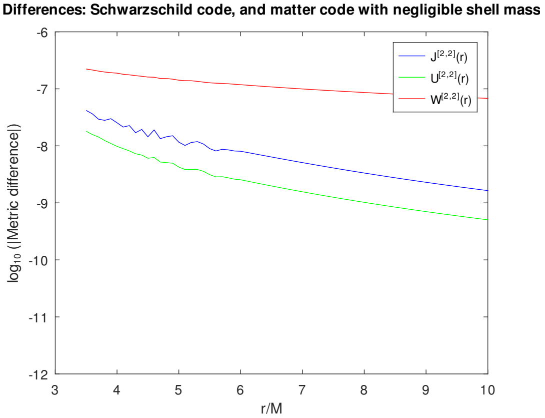

Figure: The differences on a $\log_{10}$ scale in $J^{[2,2]}$, $U^{[2,2]}$, $W^{[2,2]}$ as calculated using the Schwarzschild code and the matter code with negligible shell density, $\nu = 0.2549$ (which is 132Hz for $M = 62M_\odot$)

Comparing Schwarzschild and matter codes: f_E

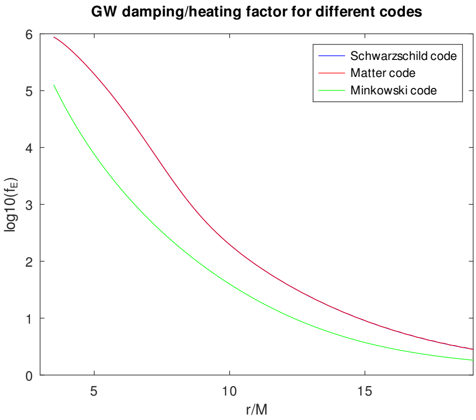

Figure: Plots of f_E on a $\log_{10}$ scale with $\nu = 0.2549$ computed by Schwarzschild code, matter code with negligible shell density, and Minkowski code.

Conclusions

When the GW wavelength $\gg$ shell radius

- Measurable changes occur when GWs pass through a dust shell.
- If the shell is viscous, then GW damping and heating occur.
- The effects are enhanced when the background is Schwarzschild-like and r/M is not large.
- GW damping and heating can be astrophysically significant for
  - Core collapse supernova
  - Matter around merging black holes
  - Binary neutron star system post-merger
  - Primordial GWs from the inflationary era.

Key point: GW interactions with matter have long been regarded as being too small to be significant, but that may not be the case when $\lambda \gg r_i$.

THANK YOU

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