Interaction of Gravitational Waves with Matter

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Joint work with Nigel Bishop, Amos Kubeka, Monos Naidoo, and Petrus J. van der Walt

Outline

1. Bondi–Sachs formalism
2. Theoretical formulation
3. Gravitational-wave damping and heating
4. Recent results
5. Implications of results

Bondi–Sachs Formalism

The Bondi–Sachs metric in outgoing null coordinates takes the form

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\begin{aligned}
ds^2 = \; - \; & \left[e^{2\beta} \left(1 + \frac{W}{r} \right) - r^2 h_{AB} U^A U^B \right] du^2 - 2e^{2\beta} du dr \\
& -2r^2 h_{AB} U^B du dx^A + r^2 h_{AB} dx^A dx^B .
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Here $h^{AB} h_{BC} = \delta^A_B$ and $\det(h_{AB}) = \det(q_{AB})$, where q_{AB} is the unit-sphere metric and h_{AB} is the conformal angular metric. The spin-weighted field is defined as $U = U^A q_A$, with q_A the complex dyad.

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$W = \;\; \beta = \;\; J(= \frac{1}{2} q^A q^B h_{AB}) = \;\; U = 0.$

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Bondi–Sachs formalism, the Einstein field equations

$$R_{\alpha\beta} = 8\pi \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right), \tag{2}$$

are categorized as follows:

- **Hypersurface equations:** R_{rr}, $q^A R_{rA}$, $h^{AB} R_{AB}$ (for β, U, and W),
- **Evolution equation:** $q^A q^B R_{AB}$ (for J),
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The evolution equation governs GW propagation, while the constraints, if satisfied on the inner boundary Γ, hold throughout spacetime.

Theoretical Formulation

We consider a matter distribution in the form of a thin, low-density spherical shell surrounding a gravitational-wave source, as illustrated in Figure below.

The shell is located at $r =$ const., with density ρ allowed to vary angularly, so that the configuration is generally *non-spherically symmetric*.

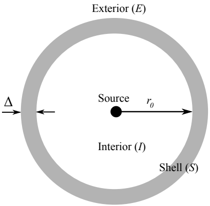

Figure: A GW source surrounded by a spherical shell of mass M_S, extending from $r = r_0$ to $r = r_0 + \Delta$.

Dust Shell $T_{\alpha\beta} = \rho V_\alpha V_\beta$

The introduction of a spherical dust shell of mass M_S, radius r_0, and thickness Δ modifies the GW strain as

$$H = -2\nu^2\sqrt{6}C_{40}\left(1 + \frac{2M_S}{r_0} + \frac{2iM_S}{r_0^2\nu} + \frac{iM_S e^{-2ir_0\nu}}{2r_0^2\nu} + \mathcal{O}\left(\frac{M_S\Delta}{r_0^2}, \frac{M_S}{r_0^3\nu^2}\right)\right), \tag{3}$$

where $\nu = 2\pi f$ and C_{40} is a constant.

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where $\nu = 2\pi f$ and C_{40} is a constant.

Each M_S-dependent term represents a correction to the vacuum waveform.

- The term $2M_S/r_0$ corresponds to gravitational redshift, leading to a frequency reduction. We assume this effect is already incorporated into observed GW waveforms.
- The term $iM_S/(\pi r_0 f)$ is out of phase with the leading contribution and induces a phase shift without changing the GW energy.
- The oscillatory factor $e^{4\pi i r_0 f}$ modifies the amplitude and arises from back-scattering effects that alter the near-source geometry and inspiral rate.

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The GW energy loss rate due to viscosity is

$$\left\langle \dot{E}_\eta \right\rangle = -12\eta C_{40}\nu^6 \delta r \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right), \qquad (4)$$

where δr is the shell thickness. The negative sign indicates energy transfer from GWs to matter.

The energy dissipated inside the viscous shell can be expressed in terms of the GW energy as

$$\left\langle \dot{E}_\eta \right\rangle = -16\pi\eta\delta r \left\langle \dot{E}_{GW} \right\rangle \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right). \qquad (5)$$

Due to the dissipation of energy caused by GWs, two phenomena may occur. The first is the damping of GWs, and followed by the heating of the viscous shell.

GW Heating

Energy dissipation inside the shell leads to heating. Assuming uniform heating,

$$\frac{\partial_u E_{shell}}{\Delta V} = \frac{\sqrt{\pi}}{6}\nu^2\eta\,\partial_u E_{GW}\,D_0, \tag{6}$$

with

$$D_0 = \frac{12(\nu^8 r^8 + 2\nu^6 r^6 + 9\nu^4 r^4 + 45\nu^2 r^2 + 315)}{\sqrt{\pi}\nu^{10} r^{10}}. \tag{7}$$

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The resulting temperature increase is

$$T - T_0 = \frac{\sqrt{\pi}G\eta}{6c^5 C\rho}\nu^2 \Delta E_{GW} D_0. \tag{8}$$

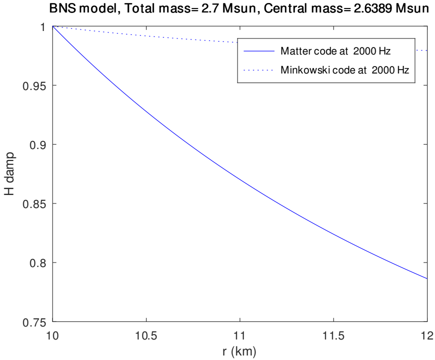
Recent results: BNS model

- The results are general, non-vacuum, static, spherically symmetric spacetime.
- A BNS merger is adopted as an astrophysical model, with parameters given below:

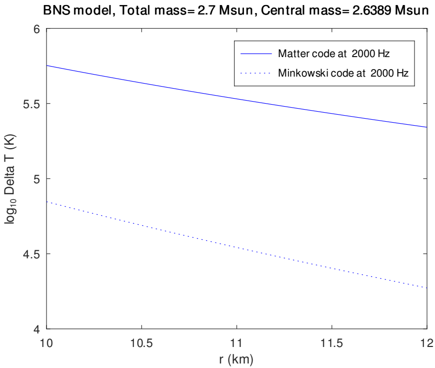
Quantity	**Typical Range or Value**
Core mass	2.4–$2.8\ M_{\odot}$
Core radius	10–$15\ \mathrm{km}$
Total bound mass (core + disk)	2.7–$3.1\ M_{\odot}$
Disk mass	5×10^{-4}–$0.3\ M_{\odot}$
Shear viscosity η	10^{24}–$10^{30}\ \mathrm{kg\,m^{-1}\,s^{-1}}$
Specific heat C	$5.84\ \mathrm{J\,kg^{-1}\,K^{-1}}$
GW frequency	1–$2\ \mathrm{kHz}$
GW energy release ΔE_{GW}	$10^{45}\ \mathrm{J}$
Density profile	$\rho(r) = \rho_0 e^{-r/r_0}$

η ($\mathrm{kg\,m^{-1}\,s^{-1}}$)	Frequency	Model	H_{damp} (outer)	ΔT (inner)
10^{24}	$\nu = 1\,\mathrm{kHz}$	Matter code	$< 10^{-3}$	1.2319×10^{8}
		Minkowski code	$< 10^{-2}$	1.6259×10^{7}
	$\nu = 2\,\mathrm{kHz}$	Matter code	0.78632	5.6650×10^{5}
		Minkowski code	0.97938	7.0194×10^{4}
10^{26}	$\nu = 1\,\mathrm{kHz}$	Matter code	$< 10^{-3}$	1.24165×10^{10}
		Minkowski code	$< 10^{-3}$	1.73900×10^{9}
	$\nu = 2\,\mathrm{kHz}$	Matter code	$< 10^{-3}$	5.4387×10^{7}
		Minkowski code	0.12466	6.8501×10^{6}
10^{30}	$\nu = 1\,\mathrm{kHz}$	Matter code	$< 10^{-3}$	1.2388×10^{14}
		Minkowski code	$< 10^{-3}$	1.6904×10^{13}
	$\nu = 2\,\mathrm{kHz}$	Matter code	$< 10^{-3}$	5.5976×10^{11}
		Minkowski code	$< 10^{-3}$	7.1121×10^{10}

GW damping in BNS case



GW heating in BNS case



Implications for Post-Merger BNS Systems

The ambient thermal state of a post-merger binary neutron star (BNS) system is expected to reach

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model show that *gravitational-wave–induced viscous heating alone* can raise the temperature to comparable values. This indicates that dissipation

of the post-merger gravitational-wave signal may contribute *non-negligibly* to the thermal budget of the remnant, in addition to conventional shock-heating mechanisms.

The talk is mainly based on the following papers: arXiv:2308.01615, arXiv:2405.07743, arXiv:2407.17143, arXiv:2512.16253

Thank You!