A Mathematical Research Programme on Gravitational Waves

Advanced Numerical, Dynamical, and Fractional Models for Einstein Systems

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Motivation and Context

- Gravitational waves probe strong-field General Relativity
- Detection demands robust mathematical models
- Numerical instability remains a key challenge

Research Positioning

- Mathematical focus
- Computational science focus
- Emphasis on structure-preserving numerical methods

Core Research Question

- How can advanced numerical and fractional methods improve stability and accuracy of gravitational-wave simulations?

Research Themes

1. Numerical evolution of Einstein equations

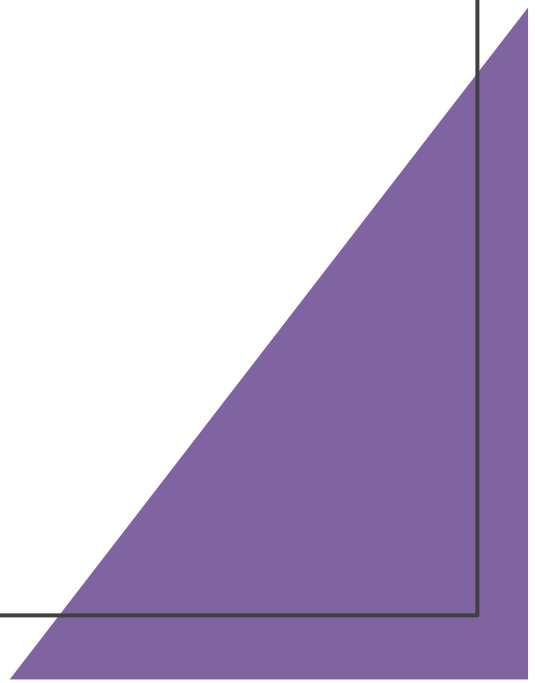
2. Dynamical systems and bifurcation analysis

3. Fractional and nonlocal gravitational models

4. Quasi-normal modes and waveform integrity

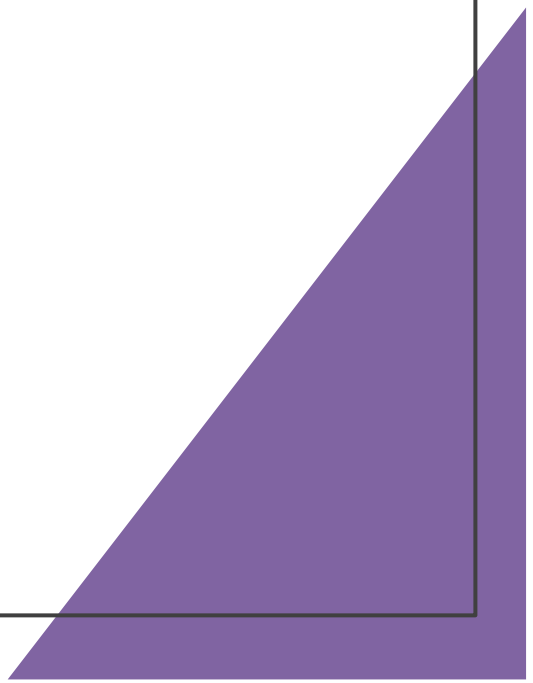
Numerical Evolution of Einstein Equations

- ADM/BSSN formulations
- Implicit and spectral schemes
- Long-time stability analysis



Dynamical Systems Perspective

- • Attractors and bifurcations
- • Numerical Floquet theory
- • Sensitivity of waveforms



Fractional and Nonlocal Models

- • GW memory effects
- • Caputo–Fabrizio operators
- • Non-singular kernels

Quasi-Normal Modes

- • Ringdown phase accuracy
- • Fractional perturbations
- • Improved frequency extraction

Conceptual Framework

Einstein Field Equations (PDE Systems)

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Formulations (ADM / BSSN / Harmonic)

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Advanced Numerical Schemes

(Spectral, Compact FD, Implicit Methods)

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Dynamical & Fractional Analysis

(Stability, Bifurcation, Memory Effects)

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Gravitational-Wave Signal Generation

(Inspiral – Merger – Ringdown Waveforms)

Fractional and Nonlocal Extensions

- Fractional derivatives model memory and hereditary effects
  - Caputo–Fabrizio derivative:
  - Applications: GW memory and waveform distortion

Quasi-Normal Mode Analysis

- Ringdown modeled as damped oscillations
  - Fractional perturbations modify decay rates and frequencies
  - Improved frequency extraction via spectral methods

Thank you