Gravitational waves originating from core-collapse supernovae

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Interact of GWs with matter

- The detection of gravitational waves (GWs) from distant events is possible because GWs are usually unaffected by matter, allowing them to traverse cosmological distances without significant attenuation.
- does not mean that GWs do not interact with matter
- travelling through a perfect fluid, GWs do not experience any absorption or dissipation;
- however, passing through a viscous fluid, GWs transfer energy to the fluid

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There are astrophysical scenarios which can be regarded as comprising a shell of matter around a GW source. We investigated in what way the GW signal would be affected in these scenarios.

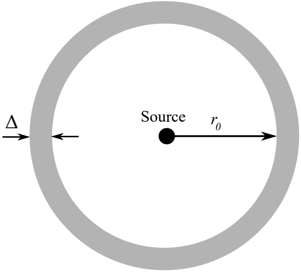

Figure: The problem of a GW source which is surrounded by a spherical shell of mass M_S located between $r = r_0$ and $r = r_0 + \Delta$, (r is the distance from the source).

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References: GW source surrounded by matter

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- N.T. Bishop, V. Kakkat, A.S. Kubeka, P.J. van der Walt, and M. Naidoo *Astrophysical and cosmological scenarios for gravitational wave heating* Phys. Rev. D. **110** 8 (2024) [**?**]

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Intro

- Core-collapse supernovae (CCSNe) represent one of the most violent astrophysical processes in the universe.
- As the iron core of a massive star exceeds the Chandrasekhar mass ($\sim 1.4\,M_{\odot}$), electron degeneracy pressure fails, triggering gravitational collapse.
- The collapse halts when nuclear densities ($\rho \sim 2 - 3 \times 10^{17}\,\mathrm{kg\,m^{-3}}$) are reached, leading to the formation of a proto-neutron star (PNS) and an outgoing shock wave.
- The post-bounce hydrodynamics and anisotropic mass motions produce gravitational waves expected to be detectable by current or planned observatories.
- For now, all detection of supernovae have been confined to electromagnetic detection.
- Photons originate at the outer edge of a star and hence provide only limited information on the interior regions. The detection of GWs which are the result of the aspherical motion of the inner regions will provide a wealth of information on these regions and the mechanism leading to the supernova explosion, where all the four fundamental forces of nature are involved.
- The PNS consists of a dense core surrounded by a less dense, hot mantle (or thin shell). Using a thin shell analysis, the effects of the outer core on GWs produced in the inner core are considered and we discuss the implications for GW detection from CCSNe.

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- For stars of mass larger than $8M_{\odot}$, their evolution proceeds to several stages of core burning.
- After each stage of burning, shells with lighter nuclei are forced outward as burning proceeds in the next heavier shell which in turn will be forced outward as a heavier shell develops in the subsequent burning phases.
- This leads to a series of concentric shells with the lightest shell, outermost at the edge of the envelop and shells arranged with progressively heavier burning components towards the core.
- At the C-O core left by He burning, the nuclear burning increases rapidly to go through the remaining stages to produce an Fe core.
- At the Fe core, nuclear fusion halts rendering the core inert. With no further burning processes to balance the gravitational attraction, the core begins to collapse.
- The core has the radius and density of an iron white dwarf, with the circumnuclear shells decreasing in density away from this core, with a density profile that is a power law going as $\rho \propto r^{-n}$.
- Typically n ranges from 1.5 to 3 from just above the core to the outer layers, with sometimes steep, shelf-like decline in the density between the shells such as between carbon and helium.

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The progenitor of a SN.

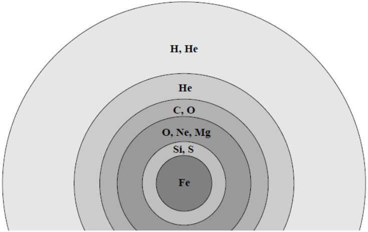

Figure: The 'onion-skin' layered concentric shells of nuclear burning remanants for a progenitor of a SN. The density distribution can be descibed by a power law of radius, going as $\rho \propto r^{-n}$, with n about 1.5 in the immediate layers above the core and increasing up to 3 in the outer layers.

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- The collapse of the core splits into two parts, leading to an inner and outer core.
- The inner core will halt collapse on reaching supranuclear densities where the nuclear matter stiffens.
- results in a bounce of the inner core with the resulting shock wave launched into the collapsing outer core. This *bounce* occurs within $\sim 1\,\mathrm{ms}$, producing a sharp burst of gravitational radiation
- the shock loses energy to dissociation of iron nuclei, stalling at $\sim 150\,\mathrm{km}$ within $\sim 10\,\mathrm{ms}$ after formation.
- The infalling shells of stellar matter then pile on the PNS, that may drive the evolution of the PNS to a BH.
- However, if there is a "revival" of the shock within a few hundred milliseconds, the resulting outward shock will lead to the expulsion of the infalling outer shells, leaving behind a stable neutron star.
- Following bounce, convection and the standing accretion shock instability (SASI) drive stochastic mass motions and asymmetric neutrino emission, both contributing to longer-duration GW signals lasting up to several hundred milliseconds.
- The frequency spectrum evolves from $\sim 100\,\mathrm{Hz}$ at early post-bounce phases to ~ 1–$1.5\,\mathrm{kHz}$ as the PNS contracts and oscillates in g- and f-modes.

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GWs generated in CCSNe

The GW strain amplitude from a CCSN at 10 kpc is expected to be $h \sim 10^{-22}$–10^{-21}, peaking near bounce and modulated by subsequent PNS oscillations Frequency bands depend strongly on the mass and rotation of the progenitor:

- Bounce signal: ~ 500–$1000\,\mathrm{Hz}$, duration $\sim 10\,\mathrm{ms}$.
- Convection/SASI: ~ 100–$300\,\mathrm{Hz}$, lasting $\sim 300\,\mathrm{ms}$.
- PNS g-mode oscillations: ~ 1–$2\,\mathrm{kHz}$.

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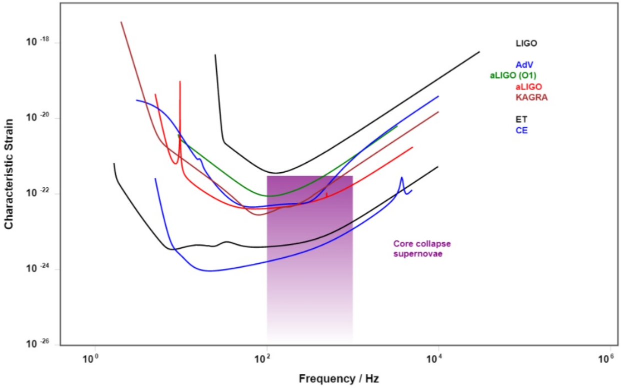

Figure: The signal range for CCSNe and sensitivities for various gravitational wave detectors, past, current and future for a source at 300kpc.

The problem is set up within the Bondi-Sachs formalism for the Einstein equations with coordinates based on outgoing null hypersurfaces [**?**, **?**].

- The null hypersurfaces are labelled by the coordinate $x^0 = u$;
- The angular coordinates are represented by x^A $(A = 2, 3)$;
- The surface area radial coordinate is represented by $x^1 = r$;
- The angular coordinates (e.g. spherical polars (θ, ϕ)) label the null ray generators of a hypersurface $u =$ constant.
- The Bondi-Sachs metric then describes a general spacetime, which may be written as:

$$ds^2 = -\left(e^{2\beta}(1 + W_c r) - r^2 h_{AB} U^A U^B\right) du^2 - 2e^{2\beta} du dr - 2r^2 h_{AB} U^B du dx^A + r^2 h_{AB} dx^A dx^B , \tag{1}$$

where, $h^{AB} h_{BC} = \delta^A_C$.
The condition that r is a surface area coordinate implies $\det(h_{AB}) = \det(q_{AB})$.
q_{AB} is a unit sphere metric (e.g. $d\theta^2 + \sin^2\theta d\phi^2$).

The Bondi-Sachs metric in Minkowski spacetime

In Minkowski spacetime $\beta = U^A = 0$, $h_{AB} = q_{AB}$, $W_c = 0$.
We introduce a complex dyad q_A (e.g. $q_A = (1, i\sin\theta)$).
Then h_{AB}, U^A can be represented by

$$J = h_{AB} q^A q^B / 2 \,, \quad U = U^A q_A \,,$$

with $J \neq 0$ characterizing a deviation from spherical symmetry.
In an appropriate gauge, J is directly related to the polarization states of a gravitational wave, $J = h_+ + i h_\times$.

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We make the ansatz of a small perturbation about Minkowski spacetime

$$\beta = \Re(\beta^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,, \quad U = \Re(U^{[2,2]}(r)e^{i\nu u})_1 Z_{2,2}\,,$$
$$W_c = \Re(W_c^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,, \quad J = \Re(J^{[2,2]}(r)e^{i\nu u})_2 Z_{2,2}\,,$$

The perturbations oscillate in time with frequency $\nu/(2\pi)$.
The quantities ${}_sZ_{\ell,m}$ are "real" spin-weighted spherical harmonic basis functions related to the usual ${}_sY_{\ell,m}$.
We consider a source that is continuously emitting GWs at constant frequency dominated by the $\ell = 2$ (quadrupolar) components.

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Solving the vacuum Einstein equations with no incoming radiation leads to

$$
\begin{aligned}
\beta^{[2,2]} =& b_0 \,, \\
W_c^{[2,2]} =& 4i\nu b_0 - 2\nu^4 C_{40} - 2\nu^2 C_{30} + \frac{4i\nu C_{30} - 2b_0 + 4i\nu^3 C_{40}}{r} \\
& + \frac{12\nu^2 C_{40}}{r^2} - \frac{12i\nu C_{40}}{r^3} - \frac{6C_{40}}{r^4} \,, \\
U^{[2,2]} =& \frac{\sqrt{6}(-2i\nu b_0 + \nu^4 C_{40} + \nu^2 C_{30})}{3} + \frac{2\sqrt{6} b_0}{r} + \frac{2\sqrt{6} C_{30}}{r^2} \\
& - \frac{4i\nu\sqrt{6} C_{40}}{r^4} - \frac{3\sqrt{6} C_{40}}{r^4} \,, \\
J^{[2,2]} =& \frac{2\sqrt{6}(2b_0 + i\nu^3 C_{40} + i\nu C_{30})}{3} + \frac{2\sqrt{6} C_{30}}{r} + \frac{2\sqrt{6} C_{40}}{r^3} \,.
\end{aligned}
$$

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Defining the rescaled GW strain by $\mathcal{H}_0 = r(h_+ + ih_\times)$, we find

$$\mathcal{H}_0 = \Re(H_0 \exp(i\nu u))\,{}_2Z_{2,2} \text{ with } H_0 = -2\sqrt{6}\nu^2 C_{40}\,.$$

C_{40} is determined by the physics, and b_0, C_{30} are gauge freedoms.

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The introduction of a spherical shell around the GW source of mass M_S, radius r_0 and thickness Δ modifies the wave strain to:

$$H = -2\nu^2\sqrt{6}C_{40}\left[1 + \frac{2M_S}{r_0} - \frac{2iM_S}{r_0^2\nu} + \frac{iM_S e^{-2ir_0\nu}}{2r_0^2\nu} + \mathcal{O}\left(\frac{M_S\delta}{r_0^2}, \frac{M_S}{r_0^3\nu^2}\right)\right]. \qquad (2)$$

Each of the terms containing M_S in Eq. (2) represents a correction to the wave strain in the absence of the shell [**?**].

- The first correction, $2M_S/r_0$, is part of the gravitational red-shift effect, the main consequence of which is a reduction in the frequency; this effect is well-known, and henceforth we will assume that GW waveforms to be considered have allowed for this effect.
- The second correction term, $iM_S/(\pi r_0^2 f)$, is out of phase with the leading terms $1 + 2M_S/r_0$ and hence represents a phase shift of the GW. This term has no effect on the energy of the GW.
- The presence of $e^{-4\pi i r_0 f}$ in the third correction term describes a change in the magnitude of $\mathcal{H}$, as verified in [**?**] and is due to an incoming wave modifying the geometry near the source and thus the inspiral rate.

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GWs are generated by aspherical motions in the inner core, commencing just after the bounce.
The inner core is surrounded by the outer core, treated as a thick matter shell, and we model its modifications to the GW signal.
The shell has an inner radius r_{in}, an outer radius r_{out}, and density at $r = r_{in}$ of ρ_0 with density fall-off $\rho \propto r^{-1-a}$ with $1/2 \leq a \leq 2$. The effect of the whole shell is obtained by decomposing it into thin shells and then integrating.

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Conclusions from [?]

- We investigated the effect of matter shells in various astrophysical scenarios (BBH, NSBH, CCSNe) which can be regarded as comprising a shell of matter around a GW source, where we investigated in what way the GW signal would be affected,
- Of the cases, the CCSN is that which yielded the largest shell modifications to the GWs, and for which the predictions are most reliable,
- The effects of matter shells are small but measurable if the SNR is sufficiently high,
- As GW observations become more accurate, through both hardware developments and, as time passes, increasing the chance of observing nearby events, these effects will need to be taken into account.

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Assuming geodesic motion, the velocity field is

$$V_0^{[2,2]}(r) = \frac{1}{r^3}\times$$
$$\left(3C_{40} - 2ir^3\nu C_{30} - 2ir^3\nu^3 C_{40} + 6i\nu C_{40} r - 2i\nu b_0 r^4 + \nu^2 r^4 C_{30} + \nu^4 r^4 C_{40} - 6C_{40}\nu^2 r^2\right)$$
$$V_1^{[2,2]}(r) = i\,\frac{9C_{40} + 12i\nu C_{40} r + 2i\nu b_0 r^4 - \nu^2 r^4 C_{30} - \nu^4 r^4 C_{40} - 6C_{40}\nu^2 r^2}{r^4\nu}$$
$$V_{ang}^{[2,2]}(r) = -\frac{i\sqrt{6}}{r^3\nu}\times$$
$$\left(3C_{40} - 2ir^3\nu C_{30} - 2ir^3\nu^3 C_{40} + 6i\nu C_{40} r - 2i\nu b_0 r^4 + \nu^2 r^4 C_{30} + \nu^4 r^4 C_{40} - 6C_{40}\nu^2 r^2\right). \quad (3)$$

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Suppose η is the coefficient of shear viscosity; θ is the fluid expansion, σ_{ab} is the shear tensor, and P_{ab} is the projection tensor:

$$\theta = g^{ab}\nabla_a V_b\,,\quad P_{ab} = g_{ab} + V_a V_b\,,$$
$$\sigma_{ab} = \frac{(P_{ac}\nabla_d V_b + P_{bc}\nabla_d V_a)g^{cd}}{2} - \frac{P_{ab}\theta}{3}\,. \tag{4}$$

We make the usual separation of variables, i.e.,

$$\sigma_{11} = \Re(\sigma_{11}^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,, q^A\sigma_{1A} = \Re(\sigma_{1U}^{[2,2]}(r)e^{i\nu u})_1 Z_{2,2}\,,$$
$$q^{AB}\sigma_{AB} = \Re(\sigma_{W}^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,, q^A q^B\sigma_{AB} = \Re(\sigma_{J}^{[2,2]}(r)e^{i\nu u})_2 Z_{2,2}\,.$$

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We find

$$\theta = \sigma_{00} = \sigma_{01} = \sigma_{0A} = 0\,,$$
$$-\sigma_{11}^{[2,2]} = \sigma_{W}^{[2,2]} = 12C_{40}\frac{3i - 3r\nu - ir^2\nu^2}{r^5\nu}$$
$$\sigma_{1U}^{[2,2]} = 2C_{40}\frac{6i - 6r\nu - 3ir^2\nu^2 + r^3\nu^3}{r^4\nu}$$
$$\sigma_{J}^{[2,2]} = C_{40}\frac{-3 - 3ir\nu + 3r^2\nu^2 + 2ir^3\nu^3 - r^4\nu^4}{r^3\nu}\,.$$

The expressions involve only the physical constant C_{40}, and not the gauge freedom constants b_0, C_{30}. Thus σ_{ab} is gauge independent. Also, the fluid expansion $\theta = 0$, so only the shear viscosity has an effect.

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We use the formula that the rate of energy loss per unit volume is $-2\eta\sigma_{ab}\sigma^{ab}$. This quantity is evaluated, then integrated over a shell of radius r and thickness δr; the integration is straightforward because of the orthonormality of the angular basis functions. We find that the average rate of energy loss to the shell is

$$\left\langle \dot{E}_\eta \right\rangle = -12\eta C_{40}^2 \nu^6 \delta r \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8} \right) , \qquad (5)$$

where $\langle f \rangle$ denotes the average of $f(u)$ over a wave period, i.e.

$$\langle f \rangle = \frac{\nu}{2\pi} \int_0^{\frac{2\pi}{\nu}} f dt \, ,$$

and where we have used $\left\langle \cos^2(\nu u) \right\rangle = \left\langle \sin^2(\nu u) \right\rangle = 1/2$ and $\langle \cos(\nu u) \sin(\nu u) \rangle = 0$.

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The rate of energy output as GWs is

$$\left\langle \dot{E}_{GW} \right\rangle = \frac{3C_{40}^2 \nu^6}{4\pi},$$

so that

$$\left\langle \dot{E}_{\eta} \right\rangle = -16\pi\eta\delta r \left\langle \dot{E}_{GW} \right\rangle \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right).$$

Energy conservation means that energy absorbed by the viscous fluid is balanced by a reduction in the GW energy. Thus

$$\begin{aligned}\left\langle \dot{E}_{GW} \right\rangle (r + \delta r) = \left\langle \dot{E}_{GW} \right\rangle (r) \times \\ \left[1 - 16\pi\eta\delta r \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right)\right].\end{aligned}$$

H represents the magnitude of the GWs, and $\left\langle \dot{E}_{GW} \right\rangle \propto H^2$

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$$H(r+\delta r) = H(r)\left[1 - 8\pi\eta\delta r\left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right)\right].$$

The resulting differential equation is solved to give

$$H(r) = C\exp\left(-8\pi\eta\left(r - \frac{2}{r\nu^2} - \frac{3}{r^3\nu^4} - \frac{9}{r^5\nu^6} - \frac{45}{r^7\nu^8}\right)\right), \qquad (6)$$

where C is a constant.

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There are two useful special cases. Let r_i, r_o be the inner and outer radii of the shell. If r_i, r_o are much larger than the wavelength λ of the GWs, then

$$H(r_o) = H(r_i)\exp\left(-8\pi\eta(r_o - r_i)\right) . \tag{7}$$

Equivalent results have been given before[1]. If r_i is much smaller than the wavelength of the GWs with $r_o \gg r_i$, then

$$H(r_o) = H(r_i)\exp\left(-\frac{360\pi\eta}{r_i^7\nu^8}\right) = H(r_i)\exp\left(-\frac{45\eta\lambda^8}{32r_i^7\pi^7}\right) . \tag{8}$$

To our knowledge, viscous damping of GWs with $r_i \ll \lambda$ has not been studied previously, and the result is new.

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[1] E.g., S.W. Hawking, Perturbations of an expanding universe, Astrophys. J. **145**, 544 (1966).

- The formulas above are in geometric units, and are converted to SI units on multiplication η by $G/c^3 = 2.477 \times 10^{-36}$s/kg. Reported values for η may be up to 10^{25}kg/m/s,
- A numerical relativity simulation with GW extraction far from the source would include the above effects. However, in situations such as CCSNe, GW extraction is calculated using a modified quadrupole formula,
- CCSNe includes processes with GW emission at 100Hz, so $\lambda =$3,000km, and $r_i \approx$ 15km. Thus $(\lambda/r_i)^7$ can be large, and GW damping may occur,

- CCSNe have been identified as candidates of sources of detectable GWs,
- Whilst binary black hole (BBH) and binary neutron star (BNS) mergers are currently the only GW events picked up by LIGO and VIRGO, supernovae are expected to produce, under certain conditions, GWs detectable by the current generation of interferometers or those on the horizon,
- For now, all detection of supernovae have been confined to electromagnetic detection. The GW signal from a supernova event would be different (but not altogether so) from the characteristic signal of a BBH merger or BNS merger,
- Photons originate at the outer edge of a star and hence provide only limited information on the interior regions. The detection of GWs which are the result of the aspherical motion of the inner regions will provide a wealth of information on these regions and the mechanism leading to the supernova explosion, where all the four fundamental forces of nature are involved.

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There has been a steady increase within the numerical relativity community of simulations of the evolution of CCSNe and we summarise some of the results of recent efforts in Table 1, giving some of the important parameter values.

Reference	r_i [m]	Frequency [Hz]
Andresen [?]	$10^4 - 2.8 \times 10^4$	100 - 700
Kuroda [?]	$10^4 - 2 \times 10^4$	100 - 671
Yakunin [?]	$10^4 - 2 \times 10^4$	200 - 600
Powell [?]	10^4	800 - 1000
Shibagaki [?]	10^4	200 - 800

Table: Parameter values from various references

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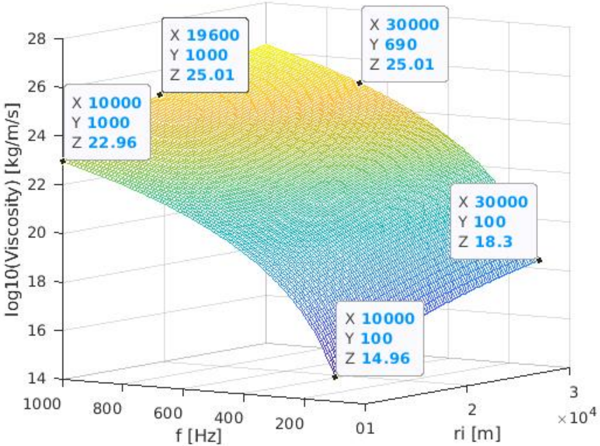

Figure: Viscous damping in CCSNe

- The figure plots the inner radius of the shell (r_i) on the x-axis (in m), the GW frequency (f) on the y-axis (in Hz), and $\log_{10}(\eta)$ on the z-axis, with η in kg/m/s,
- The surface plotted has damping factor 0.5, i.e. $H(r_o)/H(r_i) = 0.5$,
- Some points on the surface are labelled with their coordinate values, indicating which parts of the surface have $\eta > 10^{25}$ kg/m/s,
- Values of η a little above the surface would lead to (almost) complete damping, and those a little below would lead to (almost) no damping,
- The figure shows that, except in the case that both f and r_i are towards the top of their ranges, GW viscous damping is expected to be significant.

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- Extending to shell of viscous fluid : interaction may be astrophysically important when the distance between the matter and GW source is somewhat smaller than the GW wavelength.
- Further extention the considers damping contributing to the heating of the shell
- Early work mainly considered perturbations on a Minkowski background, but this background does not allow for the GW sources having strong gravitational fields.
- results are extended to the case that the background is a general, non-vacuum, static, spherically symmetric spacetime.
- Expressions are obtained for GW damping and the consequent heating of the fluid, and implemented in computer code.
- results are applied to astrophysical scenario of CCSNe.
- It is found that, compared to the Minkowski case, the damping and heating effects increase, in some cases by several orders of magnitude.
- It is possible for a GW signal to be completely damped, and for the heating to be such that a gamma-ray burst occurs.

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Extension of work on Thin Shell Analysis

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We obtain a formula for the rate of energy transfer from GWs to the fluid for a shell of radius δr

$$\dot{E} = \frac{16\pi\eta G}{c^3}\delta r f_E \,, \tag{9}$$

where the quantity f_E is evaluated numerically. In the case that the background is Minkowski, f_E has a simple form denoted by $f_{E,M}$

$$f_{E,M} = 1 + \frac{\lambda^2}{2\pi^2 r^2} + \frac{9\lambda^4}{16\pi^4 r^4} + \frac{45\lambda^6}{64\pi^6 r^6} + \frac{315\lambda^8}{256\pi^8 r^8} \,. \tag{10}$$

The GW damping effect is expressed as

$$H_o = H_i \exp\left(-\frac{8\pi G\eta}{c^3}\int_{r_i}^{r_o} f_E(r)dr\right) \,, \tag{11}$$

where η is the dynamical shear viscosity, and H_o, H_i are the rescaled gravitational wave strains at r_o, r_i respectively; i.e. $r(h_+ + ih_\times) = H\Re(e^{i\nu u})_2 Z_{2,2}$.

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The heating effect on a Minkowski background is described in [**?**, **?**]. In the case that it is appropriate to average over the spherical shell

$$\Delta T = \frac{2G\eta\Delta E_{GW} f_E}{C\rho c^3 r^2}\,, \tag{12}$$

where ΔE_{GW} is the energy transported through the shell in a given time-interval, C is the specific heat of the matter in the shell and ρ is its density. The above formula applies when the thermal diffusivity is high, or when there is no information about the orientation of the GW source.

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Approximate structural parameters of a proto-neutron star and its thin outer shell

Region	Radius (km)	Mass ($M_\odot$)	Density (kg m^{-3})
Inner core	10–15	0.5–0.7	$(2\text{–}3) \times 10^{17}$
Outer shell (mantle)	30–50	0.1–0.3	$(1\text{–}5) \times 10^{15}$

Table: Approximate structural parameters of a proto-neutron star and its thin outer shell.

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We evaluate Eqs. (11) and (12) using the parameters values in Table 2 and with the following considerations:

- the frequency varies over $100 - 1000$ kHz
- viscosity over 10^{24}–10^{30} $\mathrm{kg\,m^{-1}\,s^{-1}}$
- total mass is $1M_\odot$
- core mass is $0.5M_\odot$
- core radius is 10km
- GW damping factor is reported at the outer boundary
- maximum temperature rise at the inner boundary

The results are summarized in Table 3. For backgrounds with high values of viscosity and frequency, stronger damping and correspondingly larger heating is experienced than the Minkowski background. In Fig. 5, for example, taking $\eta = 10^{24}\mathrm{kg\,m^{-1}\,s^{-1}}$, we consider $\nu = 100$ Hz and $\nu = 1$ kHz, the damping factor at the outer boundary and the temperature rise at the inner boundary are as shown.

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Outer-boundary damping and maximum inner-boundary temperature rise for the CCSNe models

η	Frequency	Model	H_{damp} (outer)	ΔT_{max} (inner)
4*10^{24}	$\nu = 100\,\mathrm{Hz}$	Matter code	$< 10^{-3}$	3.6056×10^{25}
		Minkowski code	$< 10^{-3}$	1.7701×10^{25}
	$\nu = 1\,\mathrm{kHz}$	Matter code	$< 10^{-3}$	2.1173×10^{17}
		Minkowski code	$< 10^{-3}$	1.7783×10^{17}
4*10^{26}	$\nu = 100\,\mathrm{Hz}$	Matter code	$< 10^{-3}$	3.6056×10^{27}
		Minkowski code	$< 10^{-3}$	1.7701×10^{27}
	$\nu = 1\,\mathrm{kHz}$	Matter code	$< 10^{-3}$	2.1173×10^{19}
		Minkowski code	$< 10^{-3}$	1.7620×10^{19}
4*10^{30}	$\nu = 100\,\mathrm{Hz}$	Matter code	$< 10^{-3}$	3.6056×10^{29}
		Minkowski code	$< 10^{-3}$	1.7701×10^{29}
	$\nu = 1\,\mathrm{kHz}$	Matter code	$< 10^{-3}$	2.1173×10^{21}
		Minkowski code	$< 10^{-3}$	1.7783×10^{21}

Table: The table showing the outer-boundary damping and maximum inner-boundary temperature rise for the CCSNe models.

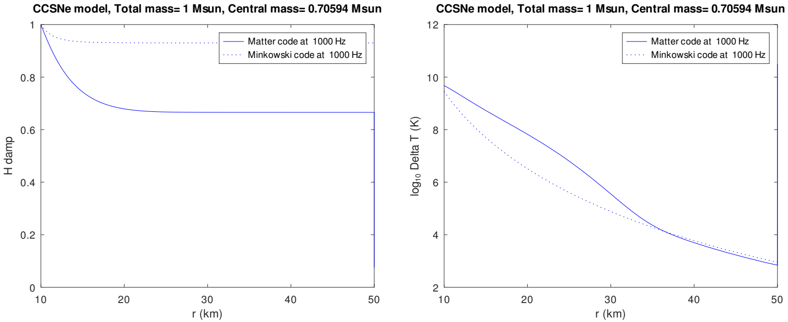

Figure: H damp (left) and $\log_{10}(\Delta T)$ (K) (right) plotted against distance (km) for the CCSN model.

Conclusions

- Changes to both phase and magnitude occur when GWs pass through a shell of matter, and these changes are large enough to be measurable in a GW signal from a close-by source,
- The effect of viscosity on GW propagation was calculated using energy considerations, and the known result for a viscous shell was recovered when $\lambda \ll r_i$,
- However, when $\lambda \gg r_i$, it was found that viscous damping of GWs can be a significant effect.
- A number of CCSNe models have been proposed, and it was found that in many cases significant viscous damping of GWs was predicted,
- GW generation in CCSNe involves a number of different physical processes, each with GW output at a different frequency. It may be that a GW observation of a CCSNe event would see only the higher frequencies, with the lower ones completely damped out.
- for GW signals completely damped, heating may be such that a gamma-ray burst occurs.

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Code

The code is available on:
https://arxiv.org/src/1912.08289v2/anc
https://arxiv.org/src/2102.00060v1/anc
https://arxiv.org/src/2206.15103v1/anc

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