Primordial gravitational waves : A cursory introduction

Petrus van der Walt

Department of Mathematics
Rhodes University
Grahamstown, South Africa

NITheCS Workshop 2026, Gqeberha

RELATED REFERENCES

Bishop, N. T., Kakkat, V., Kubeka, A. S., van der Walt, P. J., & Naidoo, M. (2024). *Astrophysical and cosmological scenarios for gravitational wave heating* Physical Review. D. (2016), 110(8)

Bishop, N. T., Kakkat, V., Kubeka, A. S., Naidoo, M., & van der Walt, P. J. (2024). *The interaction of gravitational waves with matter* International Journal of Modern Physics. D. Gravitation, Astrophysics, Cosmology

Bishop, N. T., van der Walt, P. J., & Naidoo, M. *Effect of a viscous fluid shell on the propagation of gravitational waves.* Physical Review. D. (2022), 106(8).

M. Naidoo, N.T. Bishop, and P.J. van der Walt, *Modifications to the signal from a gravitational wave event due to a surrounding shell of matter* Gen.Rel. Grav. **53** 77 (2021)

N.T. Bishop, M. Naidoo, and P.J. van der Walt, *Effect of a low density dust shell on the propagation of gravitational waves* Gen.Rel. Grav. **52** 92 (2020)

These originate from a paper published in 2005:

N.T. Bishop, *Linearized solutions of the Einstein equations within a Bondi–Sachs framework, and implications for boundary conditions in numerical simulations* Class. Quantum Grav. **22** 2393 (2005)

OVERVIEW

- Primordial gravitational waves
- Gravitational wave backgrounds
- Applying linearised models to an inflation scenario
- Applying linearised models to a first order phase transition scenario

PRIMORDIAL GRAVITATIONAL WAVES

Primordial gravitational waves (PGWs) are GWs that were generated by events in the very early epochs of the Universe.

Since PGWs are expected to only weakly interact with matter, it provides the only source of information on the early Universe, which is invaluable for validating theories of primordial cosmology. PGWs are further of interest to high-energy physics, since the energy levels in the early Universe cannot be reproduced in Earth-bound experiments.

PGWs are fundamentally stochastic due to sources originating from quantum effects as well as the inability to separate sources due to the number of sources over both space and time.

Analogous to the cosmic microwave background (CMB), PGWs also form a complete background. PGWs are, however, not the only contributors the GWB(s), since there are also astrophysical backgrounds, such as GWs originating from supermassive black holes.

Given that PGWs are of interest for fundamental cosmological and physical processes, the effects of matter interaction are imperative for interpreting observations.

PGWS : CHRONOLOGY OF THE EARLY UNIVERSE

Epoch	Time-span	Temperature	Properties
Planck epoch	$< 10^{-43}$s	$> 10^{32}$K	
Grand unification epoch	$< 10^{-36}$s	$> 10^{29}$K	
Inflationary epoch	$< 10^{-32}$s	10^{28}K to 10^{22}K	
Electroweak epoch	10^{-32}s to 10^{-12}s	10^{15}K	
Quark epoch	10^{-12}s to 10^{-5}s	10^{15}K to 10^{12}K	
Hadron epoch	10^{-5}s to 1s	10^{12}K to 10^{10}K	
Neutrino decoupling	1s	10^{10}K	
Lepton epoch	1s to 10s	10^{10}K to 10^{9}K	
Big Bang nucleosynthesis	10s to 10^{3}s	10^{9}K to 10^{7}K	
Photon epoch	10s to 370ka	4000K	
Recombination	18ka to 370ka	4000K	CMB

Table 1: Chronology of the early universe.

PGWS : SOURCES OF PGWS

- **Irreducible stochastic GW background (SGWB) from inflation**;
- Beyond the irreducible SGWB
  (Due to non-standard model effects of new species or symmetries);
- Preheating and similar non-perturbative phenomena
  (Particle creation due to non-perturbative as opposed to reheating);
- **First order phase transitions**; and
- Topological defects (solitons).
  (Degenerate vacuum states of the universe after a symmetry-breaking phase transition)

Reference:

Caprini, C., & Figueroa, D. G. (2018). Cosmological backgrounds of gravitational waves. Classical and Quantum Gravity, 35(16), 163001. `https://doi.org/10.1088/1361-6382/aac608`

PGWS : PGWS AND THE CMB

- The first expected observation of PGWs is B-modes on the CMB due to tensor perturbation;
- Announced in 2014 but then retracted after additional analysis on Planck data identified significant dust contamination in their observation;
- One of the main objectives of future CMB detectors, especially, ongoing projects at BICEP/Keck Array (BK).

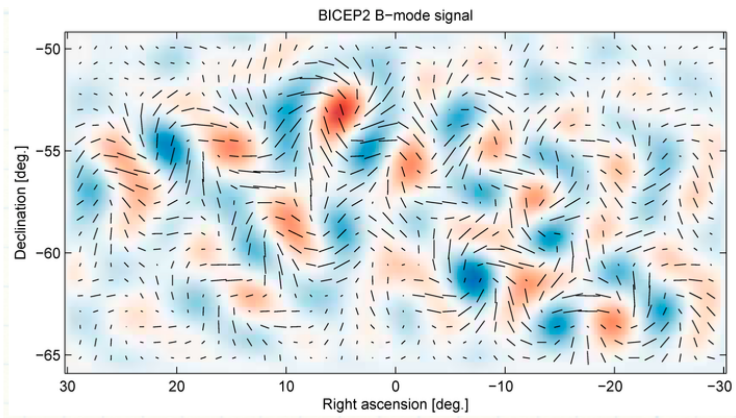

Figure 1: BICEP2 B-mode signal

GRAVITATIONAL WAVE BACKGROUNDS (GWBS) AND THE PTA

On 28 June 2023, the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) collaboration announced evidence for a GWB using observational data from an array of millisecond pulsars.

The sources of this gravitational-wave background cannot be identified without further observations and analyses, although binaries of supermassive black holes are leading candidates with the possibility of first order phase transitions as an alternative.

Reference: The NANOGrav 15-year Data Set: Search for Signals from New Physics
https://iopscience.iop.org/article/10.3847/2041-8213/acdc91

COSMOLOGICAL SCENARIO: PRIMORDIAL COSMOLOGY

Can we relate linearised results to cosmology? In the early Universe, we can consider viscous damping of PGWs in the primordial plasma.

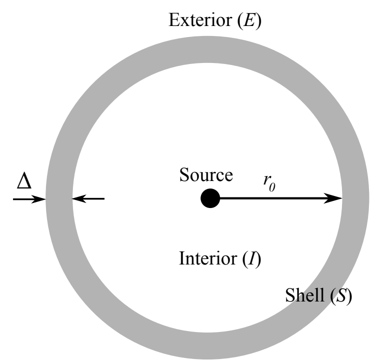

Figure 2: Thin shell

- The effect of viscosity on PGWs originating from a discrete source in a primordial plasma.
- A thin region of high viscosity.
- A very specific inflation scenario.

COSMOLOGICAL SCENARIO: PRIMORDIAL COSMOLOGY

Bearing in mind that,

- Epochs in the early Universe have different matter phases and properties.
- PGWs produce fundamentally stochastic background(s).
- Physical properties of the early Universe is note well understood.

MODELLING A HIGH VISCOSITY SCENARIO

At the present time, we have $t = 13.7$Gyr $(= 4.32 \times 10^{18} s)$, $T = 2.725$K and we normalise the *scale factor* to be $a_0 = 1$ at present.

Density of radiation and matter were approximately equal at $t = 56,000$yr $(1.7662 \times 10^{12}$s) and $T = 9,000K$

Consider an inflation scenario where the at end of inflation at $t = 10^{-32}s$, $T = 10^{27}$ to $10^{28}K$, $1/H = a/\dot{a} = 10^{-35}s$; thus *horizon scale* is $c/H = 3 \times 10^{-27}m$.

According to Misner, Thorne & Wheeler [1], eq, (28.1), T is proportional to $1/a$ Thus, at the end of inflation, $a = 10^{-27}$.

Now, the wavelength is proportional to a. GWs detectable now have frequency 10^{-9} to 10^{3}Hz, so wavelengths are in the range 3×10^{5}m to 3×10^{17}m. Thus at end of inflation wavelength is in the range 3×10^{-22} to 3×10^{-10}m.

[1] Misner, C. W., Thorne, K. S. & Wheeler, J. A., Gravitation, W.H. Freeman (1973).

MODELLING A HIGH VISCOSITY SCENARIO

From Weinberg 1971 [2] Eq. (3.20) rewritten to make the units consistent:

$$\tau = (16\pi G\eta/c^2)^{-1}$$

Eq. (3.21) rewritten to make the units consistent:

$$\eta = \frac{4}{15} a T^4 \tau / c$$

Eliminating τ from the two equations, we get

$$\eta^2 = \frac{aT^4 c}{60\pi G}$$

When $T = 10^{27}$K,

$$\eta = 3.68 \times 10^{58} kg/m/s$$

This value of η is very large, and even when multiplied by G/c^3, we get a value of 9.09×10^{22}

Using,

$$H(r_o) = H(r_i) \exp\left(-8\pi\eta(r_o - r_i)\right) .$$

with $r\nu \gg 1$ and rewritten in terms of t, we have

$$\frac{dH}{dr} = -8\pi\eta H \rightarrow \frac{dH}{dt} = -8\pi\eta c H$$

[2] Weinberg S., Entropy Generation and the Survival of Protogalaxies in an Expanding Universe, Astrophys. J. **168**, 175 (1971).

MODELLING A HIGH VISCOSITY SCENARIO

Now, η behaves as T^2, i.e. as $1/a^2$. Further, we are in the radiation dominated era, and a behaves as $t^{1/2}$. Thus η behaves as $1/t$ and we have $dH/dt = -8\pi\eta_i cHt_i/t$ with $\eta_i = 9.09 \times 10^{22}$ and $t_i = 10^{-33}$ Let $A = 8\pi\eta_i ct_i = 0.69$ Then integrating

$$\frac{dH}{H} = -A\frac{dt}{t} \quad \text{we get} \quad H_o = H_i \left(\frac{t_i}{t_o}\right)^A$$

Some example values:

- $t_o = 10\ t_i$, $H_o = 0.2\, H_i$
- $t_o = 100\ t_i$, $H_o = 0.04\, H_i$
- If $T_i = 10^{28}$K, then $t_o = 10\ t_i$, $H_o = 10^{-69}\, H_i$

Otherwise, considering that the wavelength is in the rage in 3×10^{-22} to 3×10^{-10}m while the horizon scale is $c/H = 3 \times 10^{-27} m$, can can be used to justify the case for $r_i \ll \lambda$, then

$$H(r_o) = H(r_i)\exp\left(-\frac{360\pi\eta}{r_i^7\nu^8}\right) = H(r_i)\exp\left(-\frac{45\eta\lambda^8}{32r_i^7\pi^7}\right) ,$$

in which case (λ/r_i) can be very large. Using the same formulation, significant damping can then occur for even small values of η_i.

COSMOLOGICAL SCENARIO: FIRST ORDER PHASE TRANSITIONS

Considering first order phase transitions (FOPTs), certain (beyond standard model) BSM scenarios propose that the electroweak (EW) FOPT overlaps in frequency and magnitude with the detection sensitivities of LISA. In fact, the design of LISA makes provision for detecting EW FOPTs Caprini et al. (2020).

In such scenarios, EW FOPT is a symmetry breaking process of nucleation where a scalar field, which can be a Higgs or Higgs-like field, is percolated as bubbles in the EW plasma consisting of standard model (SM) particles.

Whereas the SM predicts a smooth cross over between the EW epoch and the subsequent quark-gluon epoch, thermal FOPTs proposed by some BSMs can be rapid and highly energetic events where bubbles expand and collide to combine as larger bubbles, which interacts with the surrounding plasma to generate sound waves and turbulence.

The collision of bubbles itself, in the scalar field, along with both the propagating sound waves and turbulence form separate sources of GWs, with the sound wave propagation being the dominating effect. (See Weir (2018) and Ramsey-Musolf (2020).)

COSMOLOGICAL SCENARIO: FIRST ORDER PHASE TRANSITIONS

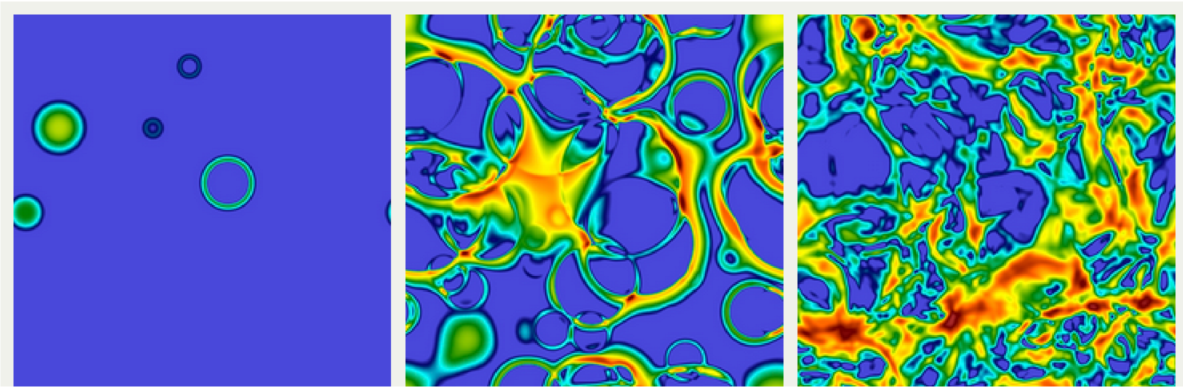

Figure 3: Electroweak FOPT process Weir (2018).

COSMOLOGICAL SCENARIO: FIRST ORDER PHASE TRANSITIONS – DAMPING

In order to make use of the damping equations, we need to understand what the frequencies, density and viscosity are for the EW FOPT.

From Schäfer & Teaney (2009), we use the viscosity as $\eta = 5 \times 10^{11}$ kg/m/s, which when multiplied by G/c^3 to convert to geometric units is: $\eta = 1.235 \times 10^{-24}$ m^{-1}. We get the density from the kinematic viscosity stated in Trachenko et al. (2021) as $\nu = \mu/\rho = 10^{-7} \Rightarrow \rho = \mu/\nu \sim 10^{18}$ kg/m^3.

We consider the expected measurable frequencies for GWs generated from the electroweak transition to the quark-gluon epoch. From Caprini et al. (2020), we take a representative peak observable frequency as $\sim 10^{-2}$ Hz. We then apply the same reasoning used earlier for inflation for the quark eopch with as $t = 10^{-12}$ s and $T = 10^{15}$ K, respectively, now obtaining the peak frequency at the time of generation is $f_{peak} \approx 10^{12}$ Hz, which amounts to a wavelength $\lambda_{peak} \approx 3 \times 10^{-4}$ m.

COSMOLOGICAL SCENARIO: FIRST ORDER PHASE TRANSITIONS – DAMPING

Using $\mathcal{H} = \dot{a}/a$ with $a(t) \propto t^{1/2}$, for a radiation Einstein-de Sitter model, we take $\mathcal{H} = 1/2 \cdot t^{-1} = 0.5 \times 10^{12}$. From $c/\mathcal{H}$, it then follows that the Hubble radius is $R_h = 6 \times 10^{-4}$ m.

As earlier, the GW damping effect is given by

$$\frac{dH}{dr} = -\,8\pi\eta H \left(1 + \frac{2}{r^2\nu^2} + \frac{9}{r^4\nu^4} + \frac{45}{r^6\nu^6} + \frac{315}{r^8\nu^8}\right)\,, \tag{1}$$

where r is the distance form the source and $\nu/(2\pi)$ is the GW frequency. With the R_h and λ being of similar order, i.e., 10^{-4} m, neither of the simplifying assumptions of $\lambda \ll r_i$ nor $\lambda \gg r_i$, presented earlier are applicable. It is therefore, necessary to solve (1) without simplification. For the cosmological scenario, we rewrite (1) in terms of t by introducing

$$r_{\nu 2} = \frac{\lambda^2\, t}{t_i\,(2\pi(r_i c t - c t_i))}\,. \tag{2}$$

Substituting (2) into (1) with some manipulation then gives

$$\frac{d\log H}{dt} = -\frac{8\pi\eta G}{ct}\left(1 + 2r_{\nu 2} + 9{r_{\nu 2}}^2 + 45{r_{\nu 2}}^3 + 315{r_{\nu 2}}^4\right)\,. \tag{3}$$

COSMOLOGICAL SCENARIO: FIRST ORDER PHASE TRANSITIONS – DAMPING

Equation (3) is solved using computer algebra and then the factor of damping is evaluated for different radii starting with R_h as maximum.

It is found that damping is negligible on the larger range of radii and more prominent on smaller scales.

This is an effect that exhibit a sudden transition from no damping at $r_i = 10^{-7}$ to sizeable damping at $r_i = 10^{-8}$, with a damping factor of 0.023, to complete damping at $r_i = 10^{-9}$.

COSMOLOGICAL SCENARIO: FIRST ORDER PHASE TRANSITIONS – HEATING

For experimental purposes, the physical process of nucleation, can be parametarised by model independent formulae making use of the following parameters. (See Weir (2018) and Caprini et al. (2020) for details.)

Parameter	Description
β^{-1}	Phase transition duration.
R_*	Size of the bubble towards the end of the PT.
v_w	Bubble wall speed.
$\mathcal{H}_*^{-1}$	Hubble time at the time of PT.
g_*	Degrees of freedom.
α	The GW energy budget ratio GWs generation vs. radiation (ρ_{vac}/ρ_{rad})

Table 2: Parameters used in nucleation formulae for the LISA design Caprini et al. (2020).

We can then proceed to align our linearised models for damping and heating with these parameters.

As a radius value, the bubble radius is calculated using

$$R_* = (8\pi)^{1/3} \frac{v_w}{\beta} \tag{4}$$

with $\beta = 50\,\mathcal{H}_*$, taking $\mathcal{H}_* = 0.5 \times 10^{12}$ s^{-1} as determined earlier, it follows that $R_* = 1.582 \times 10^{-5}$ m. As expected, the maximum bubble radius is somewhat smaller than the Hubble radius, determined earlier. The Hubble radius is therefore a better value as an extreme upper bound while not a complete over estimation.

In summary, the following parameter values align with detectable predictions while being useful for the damping and heat transfer models.

Parameter	Range
Bubble radius (R_*)	1.5818×10^{-5} m
Frequency peak (f_{peak})	10^{12} Hz
Density (ρ)	5×10^{18} kg/m³
GW energy, (ΔE_{GW})	$< 10^{26} - 10^{30}$ J
Viscosity (η)	5×10^{11} kg/m/s
Specific heat (C)	5.84 J/kg/K

Table 3: Parameter range for GWs generated by the EW FOPT and propagating through a quark-gluon plasma.

COSMOLOGICAL SCENARIO: FIRST ORDER PHASE TRANSITIONS – HEATING

The authors of Caprini et al. (2020) provided an online tool to test the overlap of a selection of parameters with the sensitivity of LISA. For the above parameters, the overlap is depicted in Figure 4.

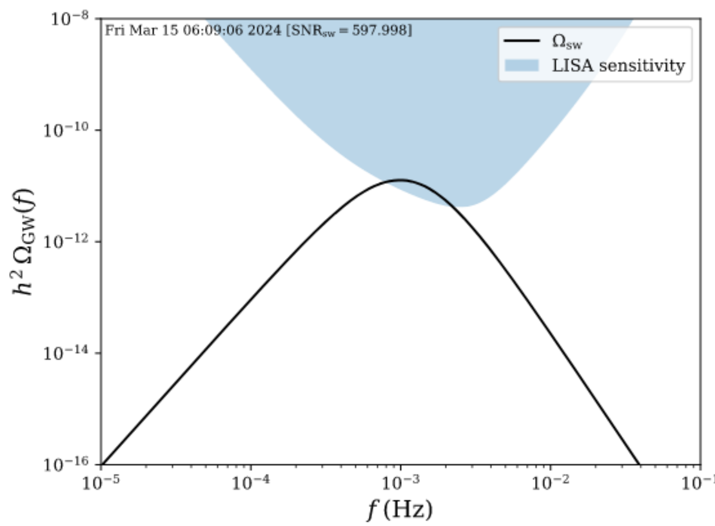

Figure 4: Using the POPT online tool, described in Caprini et al. (2020) with the parameter values $v_w = 0.9$, $\beta/\mathcal{H}_* = 50$, $\alpha_\theta = 0.2$, $T_* = 200\text{GeV}$, $g_* = 100$.

We then proceeded to combine the parameters in Table 3 with the heating equations, i.e.,

$$D_0 = \frac{12(\nu^8 r^8 + 2\nu^6 r^6 + 9\nu^4 r^4 + 45\nu^2 r^2 + 315)}{\sqrt{\pi}\nu^{10} r^{10}} \tag{5}$$

and

$$T - T_0 = \frac{\sqrt{\pi} G \eta}{6 c^5 C \rho} \nu^2 \Delta E_{GW} D_0 \,. \tag{6}$$

For the energy, the mass of the Hubble sphere is calculated as

$$M_{R_h} = \frac{4}{3}\pi {R_h}^2 = 4.524 \times 10^9 \text{kg} \,. \tag{7}$$

using $R_h = 6 \times 10^{-4}$ m, as previously determined. Using $E = mc^2$, this translates to a rest mass energy of $E_{R_h} = 4.072 \times 10^{26}$ J .

COSMOLOGICAL SCENARIO: FIRST ORDER PHASE TRANSITIONS – HEATING

As an extreme upper bound where the mass of the Hubble sphere is completely converted into energy, i.e., $\Delta E_{GW} = E_{R_h}$, substituting the values in Table 3 into Eqs. (5) and (6) yields the temperature increase as

$$\Delta T \sim 10^{-10}\,\mathrm{K}\;. \tag{8}$$

Alternatively, using a Hubble radius derived from the ΛCDM model, $R_h = 0.013m$, see Melia (2022), the corresponding energy follows as $\Delta E_{GW} \sim 10^{30}$J with the temperature increase resulting in

$$\Delta T \sim 10^{-6}\,\mathrm{K}\;. \tag{9}$$

In both cases the expected temperature increase is yielded as insignificant, given that a small portion of the mass is realistically expected to be converted to GW energy.

COSMOLOGICAL CONCLUSIONS

Current status:

- Primordial gravitational waves provide fundamental information about the early Universe.
- It should be taken into account that these GWs are stochastic in nature and observable only as a stochastic background.
- Taking damping effects into account can make significant changes to how PGW observations should be interpreted.
- There are some opportunities to make use of discrete models to investigate the effects of damping on PGWs, which are illustrated for:
  - Cosmological inflation under high viscosity scenarios, and
  - Electroweak first order phase transitions.
- Results can either be interpreted from a physical or an existential point of view.

COSMOLOGICAL CONCLUSIONS

Future developments:

- Extend the model to evaluate a stochastic background.
- Vary the viscosity with time evolution.
- Consider physical properties related to the various epochs.
- Relate results to observations.

ACKNOWLEDGEMENTS

This work was supported by the National Research Foundation, South Africa, under grant numbers 118519.

Additional funding was provided by NITHeCS including the current workshop, amongst others.

THANK YOU

Bibliography

Caprini, C., Chala, M., Dorsch, G. C., Hindmarsh, M., Huber, S. J., Konstandin, T., Kozaczuk, J., Nardini, G., No, J. M., Rummukainen, K., Schwaller, P., Servant, G., Tranberg, A. & Weir, D. J. (2020), 'Detecting gravitational waves from cosmological phase transitions with lisa: an update', *Journal of Cosmology and Astroparticle Physics* **2020**(03), 024.
URL: *https://dx.doi.org/10.1088/1475-7516/2020/03/024*

Melia, F. (2022), 'The electroweak horizon problem', *Physics of the Dark Universe* **37**, 101057.
URL: *https://www.sciencedirect.com/science/article/pii/S2212686422000565*

Ramsey-Musolf, M. J. (2020), 'The electroweak phase transition: a collider target', *J. High Energy Phys.* **2020**(9).

Schäfer, T. & Teaney, D. (2009), 'Nearly perfect fluidity: from cold atomic gases to hot quark gluon plasmas', *Rep. Prog. Phys.* **72**(12), 126001.

Trachenko, K., Brazhkin, V. & Baggioli, M. (2021), 'Similarity between the kinematic viscosity of quark-gluon plasma and liquids at the viscosity minimum', *SciPost Phys.* **10**(5).

Weir, D. J. (2018), 'Gravitational waves from a first-order electroweak phase transition: a brief review', *Philosophical Transactions: Mathematical, Physical and Engineering Sciences* **376**(2114), 1–15.
URL: *http://www.jstor.org/stable/44678718*