RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 1 : SEPTEMBER 2009

MATHEMATICS HONOURS

GEOMETRIC CONTROL

AVAILABLE MARKS : 54
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. (**24** marks)

Let Z be a real, finite dimensional vector space.

(a) Explain what is meant by saying that $\Omega : Z \times Z \to \mathbb{R}$ is a *symplectic form* on Z. Hence, show that if Ω is a symplectic form on Z, then (the vector space) Z must be even-dimensional.

(b) Show that (the real vector space) $Z = W \times W^*$ admits a canonical *symplectic structure*.

(c) Given a smooth function $H : Z \to \mathbb{R}$, define the associated *Hamiltonian vector field* X_H on (Z, Ω) and the *Hamilton's equations* for H. Write these equations in *canonical coordinates*.

[12,8,4]

Question 2. (**30** marks)

Let (Z, Ω) be a *symplectic vector space* and let $C^\infty(Z)$ denote the *algebra* of smooth functions on Z.

(a) Given $F, G \in C^\infty(Z)$, define the *Poisson bracket* $\{F, G\}$, and then show that

$$X_{\{F,G\}} = -\left[X_F, \, X_G\right].$$

(b) Prove that the Poisson bracket

$$\{\cdot, \cdot\} : C^\infty(Z) \times C^\infty(Z) \to C^\infty(Z)$$

makes (the algebra) $C^\infty(Z)$ into a *Lie algebra*.

(c) State clearly concepts, results and facts/formulas used in (a) and (b).

[12,12,6]