RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : OCTOBER 2009

MATHEMATICS HONOURS

GEOMETRIC CONTROL

AVAILABLE MARKS : 50
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. (**24** marks)

Let $\Sigma = (\mathsf{G}, \Gamma)$ be a (right-invariant) *control system* with the understanding that the state space G is a matrix Lie group and that the class $\mathcal{U}$ of admissible controls consists of *piecewise-constant controls*.

(a) Define the terms *trajectory, attainable set* (from $g \in \mathsf{G}$) and *orbit* (through $g \in \mathsf{G}$).

(b) Explain what is meant by saying that a point (state) $y \in \mathsf{G}$ is *normally attainable* from $x \in \mathsf{G}$.

(c) State (but DO NOT prove) *Krener's Theorem.* Hence, prove that

$$\mathrm{Lie}\,(\Gamma) = \mathfrak{g} \iff \mathrm{int}\,\mathcal{A}(1) \neq \emptyset.$$

(Here, $\mathfrak{g}$ denotes the Lie algebra of G and $\mathcal{A}(1)$ denotes the attainable set from the identity $1 \in \mathsf{G}$.)

[9,3,12]

Question 2. (**26** marks)

Let $\Sigma = (\mathsf{G}, \Gamma)$ be a (right-invariant) *control system* with the understanding that the state space G is a matrix Lie group and that the class $\mathcal{U}$ of admissible controls consists of *piecewise-constant controls*.

(a) Explain what is meant by saying that Σ is *controllable*. Give necessary conditions for controllability.

(b) State the following controllability tests

i. Group Test.

ii. Local Controllability Test.

iii. Closure Test.

Give *detailed* proofs for any two of these tests.

[4,22]