RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : APRIL 2013

MATHEMATICS HONOURS

GEOMETRY (NAIVE LIE THEORY)

AVAILABLE MARKS : 55
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. (**25** marks)

(a) Define the term *matrix Lie group*. Given a matrix Lie group G, define its *tangent space at the identity*, $T_{\mathbf{1}}\mathsf{G}$. Hence prove that $T_{\mathbf{1}}\mathsf{G}$ is a Lie algebra.

(b) Define the matrix Lie group $\mathsf{U}(n)$, and then determine (its Lie algebra) $\mathfrak{u}(n) = T_{\mathbf{1}}\mathsf{U}(n)$.

[15,10]

Question 2. (**30** marks)

(a) Define what is meant by an *ideal* of a Lie algebra, and then prove that if H is a normal subgroup of a matrix Lie group G, then $T_{\mathbf{1}}\mathsf{H}$ is an ideal of the Lie algebra $T_{\mathbf{1}}\mathsf{G}$.

(b) Define the term *simple Lie algebra*, and then prove that the cross-product Lie algebra on $\mathbb{R}^3$ is simple.

(c) What is the relationship, if any, between the cross-product Lie algebra on $\mathbb{R}^3$ and the Lie algebra $\mathfrak{so}(3)$? Make a clear statement (but DO NOT prove it).

[15,13,2]