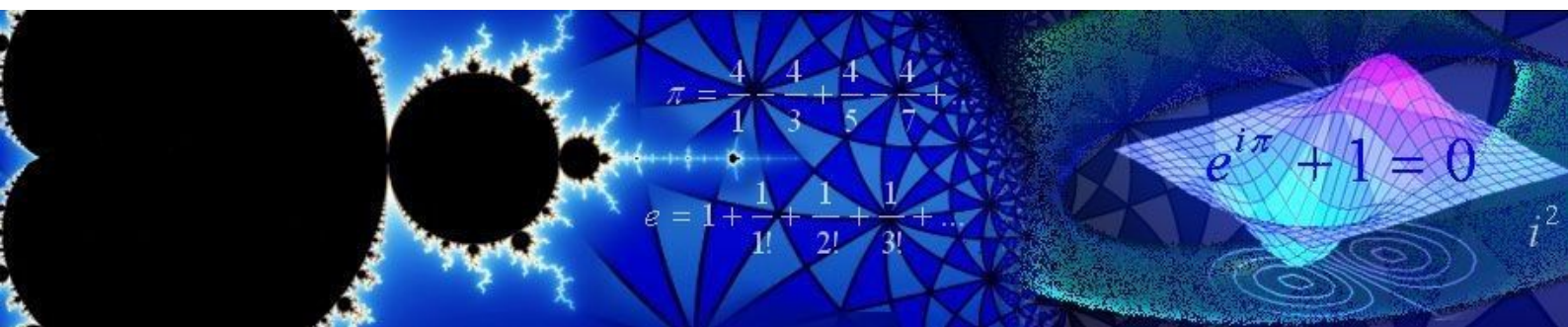




61st SAMS Congress

3-5 Dec 2018, Rhodes University

Programme and Abstracts



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Programme Overview

SAMS Congress 2018 – Conference Programme

Overview

Sunday

17:00-19:00 Registration and Welcome

Monday

8:00-9:00 Registration

9:00-9:30 Opening

9:30-10:30 Plenary: Prof Mosimege Mogege (UFS)

10:30-11:00 Coffee

11:00-13:05 Parallel Sessions

13:05-14:30 Lunch

14:30-15:30 Plenary: Prof Keshlan Govinder (UKZN)

15:30-16:00 Coffee

16:00-18:05 Parallel Sessions

Tuesday

9:30-10:30 Plenary: Dr Julien Larena (UCT)

10:30-11:00 Coffee

11:00-13:05 Parallel Sessions

13:05-14:30 Lunch

14:30-15:30 Plenary: Dr Anneliese Schauerte (UCT)

15:30-16:00 Coffee

16:00-18:05 Parallel Sessions

19:00 Conference dinner (Steve Biko dining hall)

Wednesday

9:30-10:30 Plenary: Prof Willem Conradie (Witwatersrand)

10:30-11:00 Coffee

11:00-13:05 Parallel Sessions

13:05-14:30 Lunch

14:30-15:30 AGM

Conference Programme

	Sunday 2018/12/02				
17:00-19:00	Registration and Welcome (Eden Grove)				
	Monday 2018/12/03				
8:00-9:00	Registration (Eden Grove)				
	Eden Grove Blue				
9:00-9:30	Opening				
9:30-10:30	Plenary: Prof Mosimege Mogege (UFS)				
10:30-11:00	Coffee				
Parallel Sessions	Eden Grove Blue	Eden Grove Red	Seminar Room 1	Seminar Room 2	Seminar Room 3
	Functional Analysis and Operator Theory	Categories, Algebra and Topology	Graph Theory and Combinatorics	Session in Honour of Jim McKenzie	Lie Symmetries and ODEs
	Chair: M. Messerschmidt	Chair: A. Goswami	Chair: Stephan Wagner	Chair: DP Mason	Chair: BP Ntsime
11:00-11:25	R. Brits	G. Janelidze	J.P. de Wet	Appreciation of Jim McKenzie	L.D. Moleleki
11:25-11:50	F.P. Schulz	G. Janelidze	P. Dankelmann	M.K. Dougherty	B.P. Ntsime
11:50-12:15	R. Benjamin	S. Moshokoa	I. Allie		M.D. Mhlongo
12:15-12:40	H. Raubenheimer	G.L. Booth	S. Mafunda	A.J. John	P.A. Razafimandimby
12:40-13:05	E.D. Zeekoei	T. Dube	E. Andriantiana	G.Y. Weldegiyorgis	R. Biggs
13:05-14:30	Lunch (Eden Grove)				
14:30-15:30	Plenary: Prof Keshlan Govinder (UKZN)				
15:30-16:00	Coffee				
Parallel Sessions	Eden Grove Blue	Eden Grove Red	Seminar Room 1	Seminar Room 2	Seminar Room 3
	Functional Analysis and Operator Theory	Categories, Algebra and Topology	Graph Theory and Combinatorics	Session in Honour of Jim McKenzie	Applications to the sciences
	Chair: S ter Horst	Chair: W Conradie	Chair: E Andriantiana	Chair: SD Maharaj	
16:00-16:25	R. Willie	A. Goswami	S.J. Selkirk	M. Amir	L.C. Galane
16:25-16:50	W-S. Lee	J.R.A. Gray	CK Mokalapa	B.P. Brassel	S.M. Kgomo
16:50-17:15	B. Zinsou	S. Veldsman	D. Ralaivaosaona	A.V. Nikolaev	F.J. Mhlanga
17:15-17:40	R. Pant	S.S. Mthethwa	A.A.V. Dossou-Olory	Z.I. Ali	R.L. Monaledi
17:40-18:05	F.U. Ogbuisi		S. Wagner		S.C. Nkosi
	Tuesday 2018/12/04				
	Eden Grove Blue				
9:30-10:30	Plenary: Dr Julien Larena (UCT)				
10:30-11:00	Coffee				
Parallel Sessions	Eden Grove Blue	Eden Grove Red	Seminar Room 1	Seminar Room 2	Seminar Room 3
	Functional Analysis and Operator Theory	Categories, Algebra and Topology	PDEs and ODEs	Computational methods	Applications to the Sciences and Mathematical Physics
	Chair: R Heymann	Chair: T Dube			
11:00-11:25	L.E. Labuschagne	P.P. Ghosh	T. Chinyoka	R.K. Mahlakwana	S.L. Tilahun
11:25-11:50	P. de Jager	Z. Janelidze	M. Labuschagne	S.D. Olonijju	K.S. Tshivhi
11:50-12:15	C. Steyn	J.H. Meyer	C.C. Okeke	B. Stapelberg	A.S. Kubeka
12:15-12:40	M. Weigt	Y. Hardy	N.F.J. van Rensburg	J.A.C. Weideman	L. Rundora
12:40-13:05	J.J. Conradie	L. van Wyk	H.M. Tenkam	A.L. Prins	C. Stevens
13:05-14:30	Lunch				
14:30-15:30	Plenary: Dr Anneliese Schauerte (UCT)				

15:30-16:00	<i>Coffee</i>			
Parallel Sessions	Eden Grove Blue	Eden Grove Red	Seminar Room 1	Seminar Room 2
	Functional Analysis and Operator Theory	Categories, Algebra and Topology	Graph Theory and Combinatorics	Session in Honour of Jim McKenzie
	Chair: P de Jager	Chair: A Schauerte	Chair: P Dankelmann	Chair: D.P. Mason
16:00-16:25	R. Heymann	T. Janelidze-Gray	E. Jonck	C.T. Duba
16:25-16:50	S. ter Horst	F.S. Assfaw	E.G. Mphako-Banda	O.A.I. Noreldin
16:50-17:15	E. Klem	J.C. Mba	J. H. Hattingh	H. Ogunseye
17:15-17:40	M. van Straaten	Announcement	A. Roux	I.S. Oyelakin
17:40-18:05	W. van Amstel		R. J. Maartens	D.P. Mason
19:00	<i>Conference dinner (Steve Biko dining hall)</i>			

Wednesday 2018/12/05

	<i>Eden Grove Blue</i>			
9:30-10:30	Plenary: Prof Willem Conradie (Witwatersrand)			
10:30-11:00	<i>Coffee</i>			
Parallel Sessions	Eden Grove Blue	Eden Grove Red	Seminar Room 1	Seminar Room 2
	Functional Analysis and Operator Theory	Categories, Algebra and Topology	Probability theory	Finite Groups and Topology
	Chair: R Brits	Chair: T Janelidze-Gray		
11:00-11:25	E. Kikianty	M. Iragi	C. Makasu	T. Malatji
11:25-11:50	M. Messerschmidt	A. Craig	A.J. van Zyl	M.J. Motalane
11:50-12:15	M. Wortel	E.B. Inyangala	H. Jafari	K. Sebogodi
12:15-12:40	J.J. Grobler	C.A. Agyingi		M.O. Rapudi
12:40-13:05	J.H. van der Walt			
13:05-14:30	<i>Lunch</i>			
14:30-15:30	AGM			

Plenary Sessions

Second-order quantifier elimination: a non-classical perspective

Prof Willem Conradie

School of Mathematics, University of the Witwatersrand

Second-order logic extends first-order logic with the ability to quantify over subsets, relations and functions on the domain of an interpretation. It is expressive enough to encode virtually any mathematical statement, and most well-known logics embed into it via relatively straightforward translations. This luxurious expressivity comes at great cost: second order-logic is algorithmically undecidable (in fact it has no recursively enumerable axiomatization) and lacks important meta-logical properties like compactness. Depending on which of these aspects — expressive power or lack of desirable computational and other properties — you choose to emphasise, it is either the ultimate logic or, as some philosophers contend, not logic at all, but rather set theory in disguise.

The question therefore naturally arises: given a second-order formula, is it possible to reduce it to a an equivalent formula in a simpler, better behaved logic, like first-order logic? This is known as the problem of Second-Order Quantifier Elimination (SOQE). The SOQE problems crops up in many and varied settings, including artificial intelligence, where it plays a role in modelling non-monotonic and approximate reasoning and in solving problems arising from natural language processing; in the theory of computational complexity where it is has connections with the $P=NP$ problem; in the correspondence and completeness theory of non-classical logics; in the computation of uniform interpolants of logical consequences which, in turn, is of great importance in fields of deductive databases and formal knowledge representation.

In this talk I will give an introduction to SOQE and then focus on its role in the correspondence and completeness theory of non-classical logics. Many non-classical propositional logics (including modal logics and their extensions, intuitionistic logic, relevance logic, many-valued logics etc.) naturally embed into second-order logic, since their formulas can be read as making statements about relational structures (like directed graphs) that involve quantification over subsets. I will outline the search for sufficient conditions for the equivalent elimination of the second-order quantifiers arising in this way and highlight some recent general results in this area. I will conclude with some open problems.

A Group Theoretic Approach to Differential Equations

Prof Keshlan Govinder

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Durban, South Africa

Sophus Lie's theory of extended groups applied to differential equations is one of the more successful methods of analysis of differential equations. The method requires the invariance of a system of equations under a family of transformations. We will provide an overview of the method indicating how it can be used to solve ordinary differential equations as well as find group invariant solutions to ordinary and partial differential equations. In our exposition of the method, we will indicate how we have extended the applicability of the original approach to explain new phenomena observed in the reduction of differential equations. In particular, we will touch on the concept of hidden symmetries. These are symmetries that arise 'unexpectedly' in the reduction and increase of order of ordinary differential equations. We will also show how these symmetries arise in partial differential equations and the impact this analysis has on some 'well-known' results.

Finite Beams in Cosmology

Dr Julien Larena

Department of Mathematics and Applied Mathematics, University of Cape Town

Weak gravitational lensing is now a firmly established tool to understand structure formation in our Universe. Standard weak lensing is based on the approximation that light beams can be treated as infinitesimal. Through a formalism recently developed, I will show that the finite size of sources and associated beams leads to corrections to this approach that remain observationally small for finite but small beams, thus confirming the validity of the standard formalism. However, I will show that this new formalism allows one to estimate new weak lensing observables highly sensitive to very small scale properties of the matter distribution, not accessible to the standard approach, but which might become detectable in future galaxy surveys.

Reflection on the Role of Mathematics Education Departments and Mathematics Departments in the Training and Development of Mathematics Teachers in South Africa

Prof Mogege Mosimege

School of Mathematics, Natural Sciences and Technology Education University of the Free State

The Revised Policy on the Minimum Requirements for Teacher Education Qualifications (MRTEQ) was published by the Department of Higher Education and Training in February 2015. One of the major issues that the MRTEQ is intended to address is the critical challenge of the poor content and conceptual knowledge found amongst teachers. This challenge is clearly noted in the review of the teacher qualifications that was conducted by the Higher Education Quality Committee (HEQC) of the Council on Higher Education (CHE) in 2010. The training and development of mathematics teachers in South Africa is currently provided for by Mathematics Departments (content) and Mathematics Education Departments (pedagogy) at different universities. In line with the requirements of the MRTEQ, the University Departments that play a role in the preparation of mathematics teachers need to reflect on this role and determine the extent to which they need to review their activities to produce mathematics teachers who can help address the crisis in the mathematics classroom in the schools. This Plenary Address considers the implications of the MRTEQ for Mathematics Departments and Mathematics Education Departments in the preparation of mathematics teachers.

Partial frames – a journey into the world of topology without points

Dr Anneliese Schaurte

Department of Mathematics and Applied Mathematics, University of Cape Town

The fundamental viewpoint of pointfree topology is simple: when considering a topological space, look at the open sets only, ignore the points. These open sets are closed under finite intersection and arbitrary union, so what you see is a frame, a certain kind of algebraic structure – to be precise, a complete lattice in which finite meet distributes over arbitrary join. This approach can obviously not distinguish between points which have exactly the same open neighbourhoods, but that is not a serious problem for a topologist. Of more interest is the fact that there are many frames that do not arise from spaces, making the new theory interesting and substantial. We refer to [3] below for a brief history of the subject, where connections with theoretical computer science, with sheaf, topos and category theory, with topological groups and with constructive mathematics are discussed. The article [1] deals in detail with notions at the uniform and metric level, [2] will give a taste of recent work for those interested in rings and ideals, while the text [4] gives a broad overview of the current state of the art. In this talk, I will be considering partial frames: these differ from frames in that not all subsets need have joins. More precisely: a partial frame is a meet-semilattice in which certain designated subsets are required to have joins, and finite meets distribute over these. We have found this a very illuminating context in which to do pointfree topology; apart from being a generalization of frame theory, it can serve to bring into relief what is generic and what is intrinsic to specific contexts and examples. I will give an overview, and some new results, which illustrate the algebraic as well as the topological nature of the subject. This is joint work with John Frith, also of the University of Cape Town.

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Parallel Sessions

Special Session: Categories, Algebra and Topology

λ -hyperconvexity in quasi-metric spaces.

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SAMS Classification: 05

Special Session: Categories, Algebra, and Topology

Very recently, Kemajou et al. started studying the notion of hyperconvexity (which they called q -hyperconvexity) that is suitable in the category of (T_0) -quasi-metric spaces and nonexpansive maps. Soon after other authors investigated some properties of Q_X . For instance very recently Agyingi et al. proved that Q_X is a tight extension of X and it is maximal among the tight extensions of X . Furthermore Agyingi et al. used the results of their studies of Q_X during their investigations about endpoints in quasi-metric spaces in which they could characterize Q_X in terms of the endpoints and startpoints of X . Recently Agyingi studied Q_X by viewing it as a space of minimal function pairs.

In this talk, the concept of λ -hyperconvexity suitable for T_0 -quasi-metric spaces will be presented. This is a generalization of the notion of q -hyperconvexity studied by Kemajou et al. It was necessary to introduce this generalization since as we will show the Sorgenfrey T_0 -quasi-metric space is not q -hyperconvex but it is λ -hyperconvex for $\lambda = \sqrt{2}$. We will also show that for $\lambda < 2$ the fixed point property holds for nonexpansive self maps in bounded λ -hyperconvex quasi-metric spaces. We also establish that the fix point set of nonexpansive maps on bounded λ -hyperconvex T_0 -quasi-metric spaces is itself λ -hyperconvex.

Compactness with Respect to an Interior Operator

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SAMS Classification: 05

Special Session: Categories, Algebra, and Topology

Categorical closure operators in the sense of [1] have played a vital role in the development of Categorical Topology. They have been used to study topological concepts like connectedness, compactness and separation on an arbitrary category; see in particular the monographs [2,4]. Recently, S.J.R. Vorster introduced a categorical notion of interior operators in [3]. These operators were employed to study the notions of connectedness and separation in the category of topological spaces and continuous maps; see [5,6]. In this presentation, for a given interior operator i on a category C in which pullbacks and suprema commute, we study a notion of compactness relative to i . It is shown that under appropriate hypotheses, most classical results about topological compactness can be generalized to this setting.

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 - [3] MR1810290 S.J.R. Vorster. Interior operators in general categories. *Quaest. Math.*, 23(4):405-416, 2000.
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-

Embedding Topological Near-rings

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SAMS Classification: 02

Special Session: Categories, Algebra, and Topology

It is well known that an arbitrary ring can be embedded in a ring $\text{End}(G)$ of endomorphisms of an abelian group. Similarly, it has been shown that an arbitrary near-ring can be embedded in a near-ring of self-maps of an additive (but not necessarily abelian) group. In this talk, we will consider what happens when we topologise this problem. Given an arbitrary topological near-ring R , it is possible to embed it in a near ring $N(G)$ of continuous functions of a topological group G ? It seems that there is more to this problem than initially meets the eye. What is the most appropriate topology on $N(G)$? Will the embedding be homeomorphic, or just continuous or just open? We will explore some of these intriguing questions.

Dualities for varieties of default bilattices

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SAMS Classification: 01, 02

Special Session: Categories, Algebra, and Topology

Bilattices provide an algebraic method for simultaneously modelling knowledge and truth. The four-element bilattice was first described by Belnap [1] before bilattices were studied in greater generality by Ginsberg [3]. We introduce a new family of bilattices that provide a hierarchy of values for different levels of truth and falsity. We describe the varieties generated by our new bilattices and make use of natural duality theory [2] to give dualities for these varieties. Lastly, we make some remarks about dualities for the quasivarieties generated by the same bilattices.

References

- [1] N.D. Belnap, How a computer should think, In: Contemporary Aspects of Philosophy, Oriel Press Ltd., 1977, pp. 30–56.
- [2] D.M. Clark, B.A. Davey, Natural Dualities for the Working Algebraist, Cambridge University Press, Cambridge, 1998.
- [3] M.L. Ginsberg, Multivalued logics: a uniform approach to reasoning in artificial intelligence, Computational Intelligence 4 (1988) 265–316.

Rings in which sums of d -ideals are d -ideals

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SAMS Classification: 01, 02, 05

Special Session: Categories, Algebra, and Topology

An ideal of a commutative ring is called a d -ideal if it contains the annihilator of the annihilator of each of its elements. Examples are minimal prime ideals. Denote by $\text{DId}(A)$ the lattice of d -ideals of a ring A . In the talk I will show that $\text{DId}(A)$ is an algebraic frame. Call a ring homomorphism compatible if it maps equally annihilated elements in its domain to equally annihilated elements in the codomain. Denote by $\mathbf{SdRng}_c$ the category whose objects are rings in which the sum of two d -ideals is a d -ideal, and whose morphisms are

compatible ring homomorphisms. I will show that $\text{DId}: \mathbf{SdRng}_c \rightarrow \mathbf{CohFrm}$ is a functor ($\mathbf{CohFrm}$ is the category of coherent frames with coherent maps), and I will give a construction of a natural transformation $\text{RId} \rightarrow \text{DId}$, in a most natural way, where RId is the functor that sends a ring to its frame of radical ideals. I will give an outline of a proof that A is a Baer ring if and only if it belongs to the category $\mathbf{SdRng}_c$ and $\text{DId}(A)$ is isomorphic to the frame of ideals of the Boolean algebra of idempotents of A . Time allowing, I will present some categorical properties of $\mathbf{SdRng}_c$.

Pseudotopological Spaces: A Report

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SAMS Classification: 05-01, 02-01

Special Session: Categories, Algebra, and Topology

The category of topological spaces can be extended to the category of pretopological spaces, which in turn extends to the category of pseudotopological spaces, the latter being a quasi-topos hull of the category of topological spaces.

The first extension of the category of topological spaces to pretopological spaces have now been obtained in any finitely complete category with finite coproducts and a proper factorisation system such that the admissible subobjects make a distributive complete lattice. This generalises the classical construction to a wider context.

The purpose of this talk is to provide a report of the progress of this project with the second extension, namely the extension to the category of pseudotopological spaces in the general context explained in the second paragraph.

An approximation scheme for cohomologies of filtered complexes

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SAMS Classification: 02

Special Session: Categories, Algebra, and Topology

In this talk we discuss about an approximation scheme for cohomologies of filtered complexes and relate it with spectral sequences of filtered complexes. We give an example of this approximation scheme with respect to double complexes.

On categorical algebraic properties of the category of Heyting semi-lattices

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SAMS Classification: 02

Special Session: Categories, Algebra, and Topology

Recall that a Heyting semi-lattice is a meet semi-lattice (with 1) equipped with a binary operation $\rightarrow$, called implication, satisfying:

$$w \leq x \rightarrow y \text{ if and only if } w \wedge x \leq y.$$

In this talk we consider the category of Heyting semi-lattices and describe some of its categorical algebraic properties, whose group theoretic counterparts are standard and well known. In particular we will explain that this category has centralizers and, furthermore, centralizers of normal monomorphisms are normal monomorphisms.

Determinant preservers of Kronecker products

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SAMS Classification: 02

Special Session: Categories, Algebra, and Topology

This talk will consider linear preservers of the trace of Kronecker sums and their connection with preservers of determinants of Kronecker products. The partial trace and partial determinant play a fundamental role in characterizing the preservers of the trace of Kronecker

sums and preservers of the determinant of Kronecker products respectively. The connection between the block/partial trace and block/partial determinant is given by the exponential map in the same way that the trace and determinant are connected by the exponential map.

Joint work with A. Fošner (University of Primorska).

On actions of crossed modules and Cat^1 -objects in
varieties of loops

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SAMS Classification: 02

Special Session: Categories, Algebra, and Topology

Crossed modules and Cat^n -algebra structures have been studied by several authors in the past because of their relation to homotopy theory (see for instance [1], [5] and [6]). It is well known that the category of crossed modules is equivalent to that of Cat^1 -groups.

In this talk, we review the notion of a Cat^1 -loop (see [3]) as a generalisation of a Cat^1 -group. Then, applying the theory of internal actions in a semiabelian category ([2], [4]), we describe actions and semidirect products of crossed modules in the category of Cat^1 -loops.

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Quasi-uniform structures and adjoint functors

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SAMS Classification: 5

Special Session: Categories, Algebra, and Topology

A notion of categorical quasi-uniformity introduced and studied in [2] has been shown to be equivalent to a family of categorical closure operators. Inspired by the behaviour of categorical closure operators along functors, we define the *continuity of a functor* between categories supplied with fixed quasi-uniformities and show that for a functor $F : \mathcal{A} \longrightarrow \mathcal{C}$ with a right adjoint $G : \mathcal{C} \longrightarrow \mathcal{A}$ and a quasi-uniformity $\mathcal{V}$ on $\mathcal{C}$, there is a coarsest quasi-uniformity on $\mathcal{A}$ for which F is continuous. With the help of categorical co-perfect syntopogenous structures, we shall demonstrate that this coarsest quasi-uniformity induced by F allows us to obtain a concrete description of the largest idempotent closure operator making F continuous.

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Towards pointfree model theory II

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SAMS Classification: 01, 02, 03

Special Session: Categories, Algebra, and Topology

Classically, a model of a first order theory $\mathcal{T}$ is a set M equipped with a collection of finitary relations corresponding to the collection of predicate symbols in $\mathcal{T}$. Such finitary relations being elements of Boolean algebras of the form $P(M \times \dots \times M)$ suggests seeing a model of $\mathcal{T}$ as nothing but a morphism $B_{syn}(\mathcal{T}) \rightarrow B_{sem}(M)$ of so-called Boolean algebras with quantifiers and equality, associated to $\mathcal{T}$ and to M , respectively. Here “syn” and “sem” stand for “syntax” and “semantics”, and those algebras are in fact not algebraic but rather categorical structures. What we call pointfree model theory deals with structures intermediate between $B_{syn}(\mathcal{T})$ and $B_{sem}(M)$, essentially in the same way, as complete Boolean algebras are intermediate between general Boolean algebras and complete atomic Boolean algebras. Explaining it we will also recall a talk on ‘Part I’, given several years ago on a meeting in Cape Town dedicated to Bernhard Banaschewski’s 85th Birthday, but unpublished. And, as well as in Part I, we shall mention the intuitionistic context, where Boolean and complete Boolean algebras are replaced with Heyting algebras and frames respectively.

On relative categorical algebra and Yoneda quasi-abelian categories

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SAMS Classification: 02

Special Session: Categories, Algebra, and Topology

Several types of categories studied in categorical algebra, including semi-abelian and homological ones, admit natural “relative” counterparts presented as pairs (C, E) , where E is a class of epimorphisms in a category C (see [1] and references there). In this talk we analyze the relationship between the algebra of such pairs (C, E) and the classical relative homological algebra, involving the Yoneda theory of quasi-abelian categories [2].

References:

[1] T. Janelidze, Relative semi-abelian categories, Applied Categorical Structures, Vol. 17, 373-386, 2009

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Ranks of elements of lattices

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SAMS Classification: 02

Special Session: Categories, Algebra and Topology

Consider a class $\mathcal{C}$ of (bounded) lattices. For a natural number n , denote by $\mathcal{C}_n$ the class of all lattices L in which there is a chain

$$0 = a_0 \leq a_2 \leq \cdots \leq a_n = 1$$

such that each interval

$$[a_i, a_{i+1}] = \{x \in L \mid a_i \leq x \leq a_{i+1}\}$$

belongs to $\mathcal{C}$. For an element x in a lattice L , write $\mathcal{C}_L(x)$ (and call it the $\mathcal{C}$ -rank of x) for the smallest natural number n such that

$$\{y \in L \mid x \leq y\} \in \mathcal{C}_n.$$

We explore some properties of the function $\mathcal{C}_L$ for finite lattices L , in the case when $\mathcal{C}$ is the class of complemented lattices and when $\mathcal{C}$ is the class of Boolean algebras. This talk is based on the research of my student, Francois van Niekerk.

Non-cancellation group of a direct product

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SAMS Classification: 02

Consider the semidirect product $G_i = \mathbb{Z}_{n_i} \rtimes_{\omega_i} \mathbb{Z}$. Methods for computation of the non-cancellation group of the direct product of G_1 and G_2 was developed [5]. The non-cancellation group of G_i under a finite group F was derived and computed in [3]. This study develops a general method of computing the non-cancellation group of the direct product of G_1 and G_2 under F , extending the result in [3].

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Congruence preserving functions on special p -groups

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SAMS Classification: 02

Special Session: Categories, Algebra, and Topology

It is determined, in several cases, when a special p -group G is 1-affine complete. This, in turn, helps to determine when the near-ring $C_0(G)$ of congruence preserving functions on G is actually a ring.

On completeness in dislocated symmetric spaces and some applications

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SAMS Classification: 05

Special Session: Categories, Algebra, and Topology

The main objective of this presentation is to study completeness of dislocated symmetric spaces. Properties of dislocated symmetric spaces are also presented. Finally, an analogue of the Banach Contraction Theorem for these spaces will be discussed.

Perfectness of N -star compactifications

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SAMS Classification: 05

Special Session: Categories, Algebra and Topology.

In 1965, K.D Magill [3] introduced the notion of the N -point compactification for topological spaces as any compactification $\zeta(X)$ such that $\zeta(X) \setminus X$ consists of exactly N points, where N is a positive integer. Magill defined the notion of an N -star for topological spaces and showed that a topological space has an N -point compactification if and only if it is locally compact and possesses an N -star. Motivated by the N -star notion for spaces, in 2014, D. Baboolal [1] introduced the analog notion of an N -star for frames. He then proved that, for any non-compact regular continuous frame L with an N -star, there is a strong inclusion associated with this N -star. The compactification corresponding to such a strong inclusion is called the N -star compactification. Of note, Baboolal showed that the least compactification of a non-compact regular continuous frame is one of the N -star compactifications (namely the 1-star compactification). Some conditions under which the latter is perfect were studied in [2]. With this at hand, we would like to explore the conditions under which an N -star compactification is perfect. We will focus mainly on the 2-star case. We believe that the results can be generalised to any $N > 1$. Some results on the connectedness of the remainder of a non-compact regular frame in its least compactification will also be discussed.

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Lie solvability and a related polynomial identity in matrix algebras

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SAMS Classification: 02

We consider the polynomial identity (PI) $[x_1, y_1][x_2, y_2] \cdots [x_q, y_q] = 0$, which has played an important role in many papers. The interplay between the mentioned PI and the PI of Lie solvability is explored.

Topological congruences with an application to radical theory

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SAMS Classification: 05, 02

Special Session: Categories, Algebra, and Topology

A fundamental notion in algebra is that of a congruence. In this talk we define a congruence on a topological space and show that it leads to topological versions of the typical algebraic notions associated with a congruence: kernels, quotients, the isomorphism theorems and subdirect representations.

As an application of the theory developed, the Hoehnke radical of a topological space can be defined as Hoehnke did for universal algebras in terms of congruences. It is shown how this ties in with the existing radical theory of topological spaces; i.e. the connectednesses and disconnectednesses as defined and developed by Arhangel'skiĭ and Wiegandt.

Special Session: Functional Analysis and Operator Theory

The lower Weyl spectrum of a positive element

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Special Session: Functional Analysis and Operator Theory

One of the central problems investigated in [1] and [2] is that of providing conditions under which the spectral radius of a positive operator T on a complex Banach lattice lies outside the lower Weyl spectrum of T given that it is not an element of its essential spectrum.

In this talk the lower Weyl spectrum of an arbitrary positive element of a general ordered Banach algebra is introduced, and work done in the aforementioned papers are extended to general ordered Banach algebras [3].

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A spectral characterization of isomorphisms on C^* -algebras

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Special Session: FAOT

Following a result of Hatori, Miura and Tagaki we give here a spectral characterization of an isomorphism from a C^* -algebra onto a Banach algebra. We then use this result to show that a C^* -algebra A is isomorphic to a Banach algebra B if and only if there exists a surjective function $\phi : A \rightarrow B$ satisfying (i) $\sigma(\phi(x)\phi(y)\phi(z)) = \sigma(xyz)$ for all $x, y, z \in A$ (where σ denotes the spectrum), and (ii) ϕ is continuous at $\mathbf{1}$. In particular, if (in addition to (i) and (ii)) $\phi(\mathbf{1}) = \mathbf{1}$, then ϕ is an isomorphism. Simple examples show that (i) cannot be relaxed to products of two elements, as is the case with commutative Banach algebras. The results presented here also elaborate on a paper of Brešar and Špenko.

The Lebesgue dominated convergence theorem revisited

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SAMS Classification: 09

Special Session: Functional Analysis and Operator Theory

The Lebesgue dominated convergence theorem states that a bounded (in the sense of being dominated by an integrable function) sequence of integrable functions that converge almost everywhere to an integrable function in fact converges in the L_1 -norm to that function. There is also a version in which almost everywhere convergence is replaced by convergence in measure. We review the various ways in which the theorem has been generalized, in particular by using different forms of boundedness. This leads naturally to the question: Is there, in a particular context, a “best possible” theorem of this type?

Composition operators between quantum symmetric spaces

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SAMS Classification: 09

Special Session: FAOT

Composition operators have a rich history and have numerous applications in classical mechanics [3], ergodic theory [2] and the characterization of isometries [1]. They have been extensively studied in the settings of spaces of measurable functions, analytic functions and continuous functions. The development of non-commutative analogues of composition operators [4,5] leads to the natural and intriguing possibility of extending results contained in the commutative (classical) setting to the non-commutative (quantum) setting. In this talk we present a brief background to the quantum setting and the development of quantum composition operators before describing some new results regarding the existence, boundedness and compactness of composition operators between quantum symmetric spaces.

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Maps on Banach Algebras which Spectrally Preserve Algebraic Operations

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Special Session: Functional Analysis and Operator Theory

Surjective linear maps between Banach algebras which preserve spectral properties have been extensively studied. Another direction is where linearity is dropped but some algebraic aspects are preserved spectrally. For instance, for a surjective map ψ between Banach algebras A and B one can assume that ψ is “spectrally-additive”, that is, $\sigma(x + y) = \sigma(\psi(x) + \psi(y))$ for all $x, y \in A$. Similarly, one may assume that ψ is “spectrally-multiplicative”. Do any of these restrictions on ψ force it to be linear or perhaps even an isomorphism? In general this problem seems highly non-trivial. In fact, we have only managed to find some affirmative answers for the multiplicative case. Let A and B be complex Banach algebras and let ϕ, ϕ_1 and ϕ_2 be surjective maps from A onto B . Denote by $\partial\sigma(x)$ the boundary of the spectrum of x . If A is semisimple, B has an essential socle and $\partial\sigma(xy) = \partial\sigma(\phi_1(x)\phi_2(y))$ for each $x, y \in A$, then we prove that the maps $x \mapsto \phi_1(1)\phi_2(x)$ and $x \mapsto \phi_1(x)\phi_2(1)$ coincide and are Jordan isomorphisms. Moreover, if A is prime with nonzero socle and ϕ_1 and ϕ_2 satisfy the aforementioned condition, then we show once again that the maps $x \mapsto \phi_1(1)\phi_2(x)$ and $x \mapsto \phi_1(x)\phi_2(1)$ coincide. However, in this case we conclude that the maps are either isomorphisms or anti-isomorphisms. Finally, if A is prime with nonzero socle and ϕ is a peripherally-multiplicative map, we prove that ϕ or $-\phi$ is an isomorphism or an anti-isomorphism.

Markov processes and Brownian filtrations in Riesz spaces

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SAMS Classification: 09,10

Special Session: Functional analysis and Operator Theory

We consider some properties of Markov and Strong Markov stochastic processes. We show that a Brownian motion is a strong Markov process and use this to show that a Brownian filtration is right continuous. These results are stated and proved in the abstract setting of ordered vector lattices and will be needed in the representation theory of Brownian motion.

Multiplication Fibre Operators: Exploring the Unbounded Case

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Special Session: Functional Analysis and Operator Theory

Multiplication operators on function spaces are of interest in various settings. Matrix multipliers are studied in control theory. Multiplication operators on L^p -Spaces have elegant properties related to the inducing function and provide valuable examples and counterexamples. The generalisation to Bochner Spaces and then to so-called Banach fibre spaces follow naturally, for instance from examples in ergodic theory. In this talk we recall the notion of so-called multiplication fibre operators and some of its properties. We then explore the generalisation to the case where the "pointwise" operators are unbounded as well as the relation to bi-continuous semi-groups of operators. Part of this is work in progress and joint work with C. Budde (Wuppertal, Germany)

Geometrical constants for Morrey spaces

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SAMS Classification: 09

Special Session: Functional Analysis and Operator Theory

There are several geometrical constants defined on Banach spaces, such as James constant, which measures the nonsquareness of the unit ball; and Von Neumann-Jordan constant, which measures how close the space is to being a Hilbert space.

In this talk, I will discuss these constants and their importance in describing the geometrical properties of Banach spaces. In particular, I will discuss these constants for Morrey spaces.

Cauchy-Riemann equations for non-commutative functions

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SAMS Classification: 09

Special Session: Functional Analysis and Operator Theory

Let $\mathcal{V}$ be a finite dimensional vector space over $\mathbb{K}$ ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$). We then define the non-commutative (nc) space over $\mathcal{V}$ as

$$\mathcal{V}_{\text{nc}} = \bigsqcup_{n=1}^{\infty} \mathcal{V}^{n \times n},$$

the disjoint union over all $n \in \mathbb{N}$ of $n \times n$ matrices over $\mathcal{V}$. A nc set is then a subset of $\mathcal{V}_{\text{nc}}$ which is closed under the taking of direct sums. A mapping from a nc set to some nc space is called a nc function, if it maps $n \times n$ matrices to $n \times n$ matrices, and respects direct sums and similarities. If a nc function acts on a nc set with an additional property, referred to as right admissible, then this nc function is analytic. This leads us to the question whether there exists some kind of Cauchy-Riemann equations for these functions. In this talk we will give a brief overview of some of the facts regarding nc functions mentioned here and then introduce appropriate Cauchy-Riemann equations. This talk is based on joint work with Sanne ter Horst.

Why we need a non-commutative extension of the Feller convolution?

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SAMS Classification: 09 OPERATOR ALGEBRA AND FUNCTIONAL ANALYSIS

The Feller convolution fails to completely describe the setting of two-space homogeneous Markov transition functions intertwined by the extended Chapman-Kolmogorov equation. The failure is due to the commutative nature of the Feller convolution in the framework of distributions.

We construct a suitable extension of the Feller convolution in the framework of admissible homomorphisms introduced by W. S. Lee and N. Sauer. The two-space distributions are represented by $\mathbb{C} \times \mathbb{C}$ -valued admissible homomorphisms, and their product expresses the uni-directional extended Chapman-Kolmogorov equation as an empathy relation. Thus the Riesz representation of a two-space distribution is a $\mathbb{C} \times \mathbb{C}$ -valued admissible homomorphism.

Furthermore, the Fokker-Planck equations for intertwined two-space homogeneous processes are described in terms of admissible homomorphisms as an implicit evolution equation. It is not surprising that a pair of generators are involved as opposed to a single generator in classical Fokker-Planck equations.

On entropy for general quantum systems

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SAMS Classification: 09, 13

Special Session: Functional Analysis and Operator Theory

We revisit the notion of relative entropy for both classical and quantum systems, and provide some new descriptions of this notion respectively based on the theories of the Connes cocycle

derivative, and noncommutative L^p -spaces. We then introduce the notion of entropy for a single state of a general quantum system, and show that this notion agrees with von Neumann entropy in the case of semifinite von Neumann algebras. In closing we investigate the relationship between this notion of entropy and relative entropy, and identify an Orlicz space which forms the home for all states with “good” entropy.

Right inverses of Banach space quotients

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SAMS Classification: 09

Special Session: Functional analysis and Operator Theory

The existence of continuous/uniform continuous/Lipschitz right inverses of quotients of Banach spaces has been employed by a number of authors (including luminaries like Johnson, Lindenstrauss and Kalton) to study the non-linear geometry of Banach spaces.

We will discuss how this concept is used in the negative solution to the Lipschitz isomorphism problem for Banach spaces, and also its relevance to the Lipschitz decomposition problem for ordered Banach spaces.

A Projected Subgradient-Proximal Method for Split Equality Equilibrium Problems of Pseudomonotone Bifunctions in Banach Spaces

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SAMS Classification: XX 09 Operator algebra and functional analysis

Special Session: Operator algebra and functional analysis

In this paper, we propose a simultaneous projected subgradient-proximal type iterative algorithm to solve split equality equilibrium problem for pseudomonotone bifunctions in 2-uniformly convex Banach spaces which are uniformly smooth. We obtain strong convergence result using the proposed algorithm under some mild conditions on the equilibrium bifunctions. Furthermore, we give some application of our result to domain decomposition for PDEs. The results of this paper extend and complement existing results on split equality equilibrium problem and its special cases in the literature.

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Fixed points results for a general class of nonexpansive mappings in Banach spaces

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SAMS Classification: xx 09 Operator algebra and functional analysis

Special Session: Functional Analysis and Operator Theory (FAOT)

We present some existence and convergence results for an incipient class of nonexpansive mappings in Banach spaces. Some illustrative examples show the usefulness of our results. We also study the convergence behaviour of various iteration processes using numerical computations. Finally, we provide an application of our results to nonlinear integral equations.

Radius preserving (semi)regularities and Fredholm theory in Banach algebras

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SAMS Classification: 09

Special Session: FAOT

We investigate the effect of radius preserving (semi)regularities in Fredholm Theory of Banach algebras.

Weighted Non-commutative Banach Function Spaces

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Special Session: Functional Analysis and Operator Theory

The theory of non-commutative Banach function spaces is by now a mature and highly refined field. Recently Labuschagne and Majewski, motivated by applications to Quantum Statistical Mechanics, introduced the concept of weighted non-commutative Banach function spaces.

We briefly illustrate the physical motivation for defining weighted non-commutative Banach function spaces and then give an overview of two competing methods of describing these spaces. In describing these spaces we use a weighted analogue of a trace τ_x . Letting τ_x play the role of a trace, we can construct τ_x -measurable operators, a type of topology of convergence in measure, and a weighted analogue to the singular value function and can relate each concept back to their tracial versions.

Finally, we investigate the interpolation theory of weighted non-commutative Banach function spaces, and in particular describe the exact monotone interpolation spaces of the Banach couple $(L_x^1(\widetilde{\mathcal{M}}), L^\infty(\widetilde{\mathcal{M}}))$.

The discrete twofold Ellis-Gohberg inverse problem

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SAMS Classification: 09

Special Session: Functional Analysis and Operator Theory

In a series of papers Ellis and Gohberg considered various one-fold and two-fold operator equations associated with scalar- and matrix-valued orthogonal functions in the Wiener class, culminating in the book [1]. In this talk we discuss the inverse problem for one such problem for orthogonal matrix functions in the Wiener class over the unit circle. Under reasonable conditions, the problem is reduced to an invertibility condition on an operator that is defined using the Hankel and Toeplitz operators associated to the Wiener class functions that comprise the data set of the inverse problem. The talk is based on [2].

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Elements of the Proof of the Furstenberg Multiple Recurrence Theorem

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SAMS Classification: 09

Special Session: Functional Analysis and Operator Theory

Szemerédi's Theorem, proven in 1975, is a well known theorem in Ramsey Theory that can be stated as:

A subset of the integers A with positive upper density contains arithmetic progressions of the form,

$$a, a + d, a + 2d, a + 3d, \dots, a + (k - 1)d$$

for arbitrarily large $k \in \mathbb{N}$.

The Furstenberg Multiple Recurrence Theorem, a result in Ergodic Theory, was proven in 1976 and shown to be equivalent to Szemerédi's Theorem.

In this talk we will lay out some of the central ideas and techniques behind the proof of the Furstenberg Multiple Recurrence Theorem. This will include the definitions of weak mixing and compact systems, examples of these and a dichotomy result that characterizes all measure preserving systems in terms of weak mixing and compactness. Having introduced the necessary tools, we shall discuss how these results are brought together to prove the Furstenberg Multiple Recurrence Theorem.

n th roots of H -selfadjoint matrices

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SAMS Classification: 02

Special Session: FAOT

We call a matrix A H -selfadjoint for some invertible hermitian matrix H , if $HA = A^*H$. Given an H -selfadjoint matrix B , we seek to find necessary and sufficient conditions for the existence of an H -selfadjoint matrix A such that $B = A^n$, and if possible, describe the H -selfadjoint matrices A with this property. The matrix A is called an n th root of B . The case for $n = 2$ was done by Bolshakov et al. in [1] and we present a generalization of their

result to arbitrary n .

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Unbounded order convergence in $C(X)$

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SAMS Classification: 09

Special Session: Functional Analysis and Operator Theory

Unbounded order convergence of sequences in a Riesz space is an abstraction of convergence almost everywhere of sequences of measurable functions. In this talk, we study this notion of convergence in the space $C(X)$ of continuous functions on a Tychonoff space X . It is shown that, analogously to the measure-theoretic case, unbounded order convergence in $C(X)$ can be characterised as pointwise convergence everywhere except on a negligible set. Remarkably, the measure-theoretic case can be seen as a special case of our result for $C(X)$.

Applications of non-normed topological $*$ -algebras to relativistic quantum field theory

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SAMS Classification: 09,13

Special Session: Functional Analysis and Operator Theory (FAOT)

Relativistic quantum field theory is one of the biggest triumphs in physics, and is an attempt to combine quantum mechanics with the special theory of relativity. It does, however, not yet have a firm rigorous mathematical foundation. In 1956, the American physicist Arthur Wightman formulated, what was in his view, the basic axioms that any relativistic quantum field theory should have (see (1) below). A few years later, H. J. Borchers and A. Uhlmann realized that Wightman's axioms can be reformulated in terms of the language of non-normed topological $*$ -algebras and their representation theory (see (2) below). This development helped to motivate the general study of topological $*$ -algebras and their representation theory, although the general theory of Banach $*$ -algebras (including C^* -algebras) was fairly well developed at around the early sixties. The purpose of this talk is to give an introduction to the work of Borchers and Uhlmann and how the study of topological $*$ -algebras impacted on relativistic quantum field theory. If time allowing, a few results of the author with applications to quantum mechanics will also be given.

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AKS-chemotaxis model generalized well-posedness, blow-up and controllability dynamics

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SAMS Classification: xx

Special Session:

In this talk, we discuss the generalized well-posedness in Sobolev scale spaces $E_q^\alpha := H^{2\alpha,q}(\Omega)$, $-1 \leq \alpha \leq 1$, $1 < q < \infty$ to the classical Keller-Segel model, and deduce in $\Omega \subset \mathbb{R}^N$, $N = 2, 3$ using a density argument that solutions to the model with initial data in $L^2(\Omega)$ are in $L^\infty[0, T; L^2(\Omega)) \cap L^2(0, T; H^1(\Omega))$ with time derivatives in $L^2(0, T; L^2(\Omega))$. More concretely, we show that the model defines a perturbed analytic semigroup in the scale spaces, and solutions decay at time infinity to the cross product of spatial averages and null solutions. Blow-up dynamics are obtained at the borderline scale spaces

$E_q^\alpha, \alpha = \frac{N}{2q}$ independent of time $t > 0$ if the chemo-attraction coefficient bypasses the Moser-Trudinger threshold value. In particular, working with solutions to the model in $L^\infty[0, T; L^2(\Omega)) \cap L^2(0, T; H^1(\Omega))$ we prove the existence of a finite time blow-up lifespan with an upper finite time blow-up bound given by [1] Levine's concavity method, and lower finite time blow-up bound yielded by [2] Payne & Song differential inequality technique. In controllability dynamics, we prove null and approximate controllability at any finite time $T > 0$ of the system with initial data in $L^2(\Omega)$ using Carleman's inequality, and appropriate energy functionals.

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Lexicographic cones

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SAMS Classification: 09

Special Session: Functional Analysis and Operator Theory

We introduce lexicographic cones, a method of assigning an ordered vector space $\text{Lex}(S)$ to a poset S , generalising the standard lexicographic cone. These lexicographic cones are then used to prove that the projective tensor cone of two arbitrary cones is a cone, and to find a new characterisation of finite-dimensional vector lattices.

Classes of Dunford-Pettis-type operators

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SAMS Classification: 09

Special Session: Functional Analysis

In this talk we will introduce the notions of operators that are sequentially p -limited, p -convergent, weak p -convergent and weak* p -convergent on Banach spaces. They will all be either weaker or stronger variants of the so-called Dunford-Pettis operators. Corresponding to these classes of operators are therefore weaker or stronger variants of the Dunford-Pettis property on Banach spaces. The significance of these properties to the weakly compact operators defined on the Banach spaces is also investigated.

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Asymptotics of the eigenvalues of a class of fourth order Birkhoff regular problems with eigenvalue parameter dependent boundary conditions

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Special Session: Functional Analysis and Operator Theory

A class of fourth order Birkhoff regular problems with eigenvalue dependent boundary conditions is considered. These problems have quadratic operator representations with non self-adjoint operators. The first four terms of the asymptotics of the eigenvalues of the problems are evaluated explicitly.

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Special Session: Graph Theory and Combinatorics

Oddness to resistance ratios in cubic graphs

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SAMS Classification: 14

Special Session: Graph Theory and Combinatorics.

Let G be a bridgeless cubic graph. Oddness (weak oddness) is defined as the minimum number of odd components in a 2-factor (an even factor) of G , denoted as $\omega(G)$ [4] ($\omega'(G)$ [2]). Oddness and weak oddness have been referred to as measurements of uncolourability [1,2,3,4], due to the fact that $\omega(G) = 0$ and $\omega'(G) = 0$ if and only if G is 3-edge-colourable. Another so-called measurement of uncolourability is resistance, defined as the minimum number of edges that can be removed from G such that the resulting graph is 3-edge-colourable, denoted as $r(G)$ [4]. It is easily shown that $\omega(G) \geq \omega'(G) \geq r(G)$. While it has been shown that the difference between any two of these measures can be arbitrarily large, it has been conjectured that $\omega'(G) \leq 2r(G)$, and that if G is a snark then $\omega(G) \leq 2r(G)$ [1]. In this talk, we disprove the latter by showing that the ratio of oddness to weak oddness can be arbitrarily large. We also offer some insights into the former conjecture by defining what we call resistance reducibility, and conjecturing that $r(G) > 2$ implies that G is such resistance reducible. A conjecture which if true, implies that indeed $\omega'(G) \leq 2r(G)$.

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Extremal unicyclic graphs with short segments

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Special Session: Graph Theory and Combinatorics

A segment in a graph G is a maximal path such that all its internal vertices have degree 2 in G . The lengths of all the segments of G form its segment sequence, in analogy to the degree sequence. We study the structure of extremal unicyclic graphs with fixed segment sequence relative to various well-known invariants such as the number of subtrees, the number of independent subsets and the Wiener index. The talk will focus on the case where no segment is long enough to form one cycle (all of length at most 2). For completeness, results on the case where segments can be longer than 2 will also be pointed out.

Edge-connectivity of C_4 -free Graphs

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SAMS Classification: 14

Special Session: Graph Theory and Combinatorics

Let G be a connected graph. The edge-connectivity $\lambda(G)$ is the minimum number of edges whose removal renders G disconnected. Since removing the edges incident with a vertex of smallest degree always yields a disconnected graph, we have $\lambda(G) \leq \delta(G)$, where $\delta(G)$

denotes the minimum degree of G . A graph G for which $\lambda(G) = \delta(G)$ is said to be maximally edge-connected.

A classical result due to Chartrand states that every graph of order n satisfying

$$\delta(G) \geq \frac{n-1}{2} \quad (*)$$

is maximally edge-connected. There is a considerable number of generalisations of this result in the graph theory literature. In this talk we show that for graphs that do not contain a 4-cycle as a subgraph, the sufficient condition $(*)$ can be replaced by the much weaker condition

$$\delta(G) \geq \sqrt{\frac{n}{2}} + 1$$

to guarantee that the graph is maximally edge-connected. We also present similar results for graphs that contain neither a 4-cycle nor a 5-cycle, and for graphs whose girth is at least 6.

Local Properties of Graphs and the Hamilton Cycle Problem

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Special Session: Graph Theory

We say a graph G is locally $\mathcal{P}$, abbreviated LP , if for every vertex v in $V(G)$ the open neighbourhood $N(v)$ of v is non-empty and induces a graph with property $\mathcal{P}$. For example, a graph G without isolated vertices is locally connected (LC) if $N(v)$ induces a connected graph for each $v \in V(G)$. Oberly and Sumner [1] and Saito [2] made conjectures regarding how strong a local condition has to be to guarantee that a connected graph is hamiltonian. We investigate and compare graphs that are locally connected, locally traceable, locally hamiltonian, nested locally hamiltonian, locally Hamilton-connected, locally Chvátal-Erdős and locally Ore to the conditions of their conjectures. Parameters that are investigated include

the minimum order that a nonhamiltonian graph can have, and the smallest value of the maximum degree for which the HCP becomes NP-complete. We conclude that a high degree of local connectedness is insufficient to guarantee that a connected graph is hamiltonian. Rather, it is the relation between local connectedness and the local independence number that is important in this regard.

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Matula extremal trees among topological trees

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SAMS Classification: 14, 03

Special Session:

Half a century ago, David Matula gave an explicit bijection between the set of all rooted trees and the set of all positive integers. Let T be a rooted tree with branches $T_1, T_2, \dots, T_r$ and p_m the m -th prime number ($p_1 = 2, p_2 = 3, p_3 = 5, \dots$). The Matula number $M(T)$ of T is $p_{M(T_1)} \cdot p_{M(T_2)} \cdot \dots \cdot p_{M(T_r)}$, starting with $M(\bullet) = 1$. In 1996, Gutman and Ivić found the Matula extremal trees among rooted trees with given number of vertices. In the present talk, we determine sharp upper and lower bounds for $M(T)$ which hold for all topological trees (rooted trees without vertices of outdegree 1) T with given number of leaves, and also characterise the cases of equality.

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Total domination in maximal outerplanar graphs

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SAMS Classification: 14

Special Session: Graph Theory and Combinatorics

The total domination number of a graph is the minimum size of a set S such that every vertex has a neighbor in S . In this talk, we survey recent results on the total domination number of maximal outerplanar graphs, and briefly discuss a relation between total domination in maximal outerplanar graphs and the concept of watched guards in simple polygons.

(1,1,2,3)-Colourings of Subcubic Graphs

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Special Session: Graph Theory and Combinatorics.

For $1 \leq i \leq k$, let $S = (x_1, x_2, \dots, x_k)$ be a non-decreasing sequence of integers such that $x_i \in \mathbb{Z}^+$. An S -packing colouring of a graph G is a function $\rho : V(G) \mapsto \{x_1, x_2, \dots, x_k\}$ such that for any two vertices $u, v \in V(G)$, $\rho(u) = \rho(v) = x_i$ if and only if the distance between u and v is greater than x_i . [1] asked whether it is true that any subcubic graph except the Petersen graph is $(1,1,2,3)$ -colourable. In this paper we show using initially a similar approach as in [2] that every subcubic graph containing no Petersen graph as its component, is $(1,1,2,3)$ -colourable. Since a $(1,1,2,2)$ -colouring of a graph G does not necessarily imply that G is also $(1,1,2,3)$ -colourable (note that the converse is true), we remark that by proving every subcubic graph is $(1,1,2,3)$ -colourable (with an exception of the Petersen graph), we have a stronger result of Theorem 3.2 (If a graph G is a generalized prism of a cycle, then G is $(1, 1, 2, 2)$ -colourable if and only if G is not the Petersen graph.) in [3].

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The k -Ramsey number for some odd cycles

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SAMS Classification: 06, 14

Given any two graphs F and H , the Ramsey number $R(F, H)$ is defined as the smallest positive integer n such that for every red-blue coloring of the edges of the complete graph of order n (K_n) there will either be a subgraph in K_n isomorphic to F whose edges are all colored red (a red F) or a subgraph in K_n isomorphic to H whose edges are all colored blue (blue H). In their article Stars and their k -Ramsey Numbers [1], Andrews, Chartrand, Lumduanhom and Zhang defined the k -Ramsey number $R_k(F, H)$ to be the Ramsey number of a red F and a blue H to be in the k -partite complete graph such that each partite set is either of order $\lfloor \frac{n}{k} \rfloor$ or $\lceil \frac{n}{k} \rceil$ for $2 \leq k \leq n$. In their article they found the k -Ramsey number for when $F = H = C_4$, the cycle of order 4, and for $F = K_{(1,t)}$ and $H = K_{(1,s)}$, the complete bipartite graphs for $s, t \geq 3$ called stars. We continue their work by investigating the case when F and H are any odd cycle.

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On Diameters in Graph and Digraphs

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SAMS Classification: Graph Theory

Special Session: Graph Theory and Combinatorics

In a connected, finite graph or a strong, finite digraph G of order n , the distance $d_G(u, v)$ between two vertices u and v is the length of a shortest $u - v$ path in G . The diameter $diam(G)$ of G is the largest of the distances between all pairs. The (vertex)-connectivity $\kappa(G)$ and edge-connectivity $\lambda(G)$ of G are the minimum number of vertices and edges, respectively, whose removal results in a graph that is not connected or a digraph that is not strong.

Bounds on diameter in terms of order, size and vertex-connectivity were given by Ore in 1968 for graphs and the extension to strong digraphs by Dankelmann. In the late 80's

Caccetta and Smyth strengthened these bounds for edge-connectivity $\lambda \geq 8$ instead of vertex-connectivity. Sharp bound on the diameter for the remaining values of λ , i.e, for $2 \leq \lambda \leq 7$ were given by Dankelmann who also extended these results to Eulerian digraphs.

In this talk, we present these existing results and the extension to the results of Caccetta and Smyth, and Dankelmann to new results for bipartite graphs with close consideration of results presented by Mukwembi on order, size, diameter and minimum degree in 2013. Finally we will discuss also the extension of these new results for bipartite graphs to Eulerian bipartite digraphs.

Keywords: Graph, digraph, bipartite, vertex-connectivity, edge-connectivity, diameter.
Category: Discrete Mathematics.

Values of k for which the k -defect polynomial of a graph is the zero polynomial.

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SAMS Classification: 14

Special Session: Graph Theory and Combinatorics

In this talk we begin by giving a brief introduction to the k -defect polynomials. Analogous to the chromatic polynomial (the 0-defect polynomial), the k -defect polynomials count the number of ways it is possible to colour a graph with λ colours when allowing k bad edges. As a result, the k -defect polynomials form the coefficients of the Bad Colouring polynomial which is equivalent to the Tutte polynomial. We identify some values of k for which the k -defect polynomial of a graph is the zero polynomial. We then give exact values of such k for certain classes of graphs. Finally we pose a few questions which can be explored further.

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Conditioned Galton-Watson Trees

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SAMS Classification: 14

Special Session: Graph Theory and Combinatorics

Given a nonnegative integer-valued random variable ξ , a Galton-Watson tree is constructed in the following way: starting with the root, give each node a number of children generated by a copy of ξ , where the number of children of different nodes are mutually independent. It is well known that if $\mathbb{E}\xi \leq 1$, then the tree is finite almost surely. In this talk, we consider the case where $\mathbb{E}\xi = 1$, also known as the critical case, and the tree is conditioned to have exactly n nodes. This is what we call the conditioned Galton-Watson tree model. Classical examples include the random rooted labelled tree (corresponding to a Poisson distribution for ξ), the random plane tree (corresponding to a geometric distribution for ξ) and the random binary tree (with a distribution whose support is $\{0, 2\}$). In this talk, I will discuss typical properties of conditioned Galton-Watson trees when the number of nodes n is large. These include graph theoretic properties such as the number of independent sets, the number of matchings, and the number of dominating sets. This talk is based on a joint work with Matas Šileikis (The Czech Academy of Sciences, Institute of Computer Science) and Stephan Wagner (Stellenbosch University).

On partition functions with restrictions on odd and even parts

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SAMS Classification: 13

Special Session: Graph Theory and Combinatorics

It turns out that the number of partitions of a positive integer n into distinct odd parts is equal to the number of self-conjugate partitions of n . We generalise this result and derive some parity and recurrence formulas of related partition functions. This is a joint work with Darlison Nyirenda.

Irredundance graphs

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Special Session: Graph Theory and Combinatorics

An *irredundant set* D of a graph $G = (V, E)$ is a set of vertices such that each vertex is either isolated in the subgraph induced by D or adjacent to a vertex in $V - D$ that is nonadjacent to all other vertices in D . The *upper irredundance number* $\text{IR}(G)$ is the largest cardinality of an irredundant set of G and an $\text{IR}(G)$ -set is an irredundant set of cardinality $\text{IR}(G)$.

The *IR-graph* of G has the $\text{IR}(G)$ -sets as vertex set, and sets D and D' are adjacent if and only if D' is obtained from D by exchanging a single vertex of D for an adjacent vertex in D' . In this talk we discuss the realizability of graphs as IR -graphs and show that all disconnected graphs are IR -graphs, but some connected graphs are not.

Ternary trees and equinumerous combinatorial objects

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Special Session: Graph Theory and Combinatorics

The class of ternary trees has been shown to be bijective to several other classes of combinatorial objects. Furthermore, these objects are enumerated by the generalised Catalan number $\frac{1}{2n+1} \binom{3n}{n}$. In this talk I will briefly discuss some of the objects known to be bijective to ternary trees, as well as introduce two new equinumerous objects.

Coding Theory
The Singleton and Hamming bounds.

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SAMS Classification: XX 14 Combinatorics

Special Session:

A block code, denoted by (n, M, d) , is defined by its word length n , the number of codewords M , and its minimum distance d . A good (n, M, d) – code has small n (for fast transmission of messages), large M (to enable transmission of a wide variety of messages) and large d (to detect and correct many errors). The Singleton bound was developed from an ordinary binary (n, M, d) – code C , by creating a new list of codewords L , each of length $n - d + 1$. The Singleton bound is given by $M \leq 2^{n-d+1}$. An alternative bound called the Hamming bound, is given by $M \leq 2^n \left[\sum_{i=0}^n \binom{n}{i} \right]^{-1}$ [1]. In this talk we compare the two bounds and prove that the Hamming bound, in general, is more powerful than the Singleton bound.

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Eigenvalue distribution in random trees

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Special Session: Graph Theory and Combinatorics

In this talk, several results on the distribution of eigenvalues of large random trees will be presented. Combining methods from spectral graph theory, enumerative and probabilistic combinatorics, we find that the multiplicity of eigenvalues satisfies a central limit theorem under different models of randomness, for both the adjacency matrix and the Laplacian matrix.

In some special cases, a simple explicit expression can be determined for the limiting proportion of a fixed eigenvalue. In general, however, we have to numerically evaluate certain infinite sums, or solve infinite systems of equations.

Special Session: In honour of Jim McKenzie

Appreciation of Jim McKenzie

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- David Mason: Outline of Jim McKenzie's CV
 - Michele Dougherty: Appreciation
 - Sunil Maharaj: Appreciation
 - Thama Duba: Appreciation
 - Gary Zank: Video
 - Loyiso Nongxa: Video
 - Invitation to anyone present to say a few words
-

**Stochastic Navier-Stokes type equations: Well posedness and
Deviations Principles**

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SAMS Classification: 10, 11, 17

Special Session: Session in honour of Prof Jim McKenzie

In this paper we investigate a stochastic 3D Navier-Stokes α model driven by multiplicative Gaussian noise. We rely on weak convergence methods to LDP in order to prove a large deviations principle of the strong unique solution of the stochastic 3D Navier stokes equations as the viscosity $\nu \rightarrow 0$.

Black hole as particle accelerator

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SAMS Classification: 13

Special Session: Session in honour of Prof Jim Mckenzie

Bañados, Silk and West (BSW) demonstrated that the extremal Kerr black hole can act as a particle accelerator with arbitrarily high center-of-mass energy (E_{CM}) when the collision of particles takes place in the vicinity of the event horizon. We consider the rotating Hayward black hole for our study to discuss this phenomenon in more detail. As we know the rotating Hayward black hole, apart from mass (M) and angular momentum (a), has a charge g that provides a deviation from the Kerr black hole. We demonstrate that for each value of g , there exists extremal angular momentum a_E and extremal horizon r_H^E , which corresponds to a regular extremal black hole with degenerate horizons. The condition $a < a_E$ describes a regular non-extremal black hole with outer and inner horizons. We discuss the structure of horizons and ergospheres for the rotating Hayward black holes. Furthermore, we apply the BSW process on the rotating Hayward black hole and demonstrate numerically that the E_{CM} diverges in the vicinity of the horizon for the extremal cases. It turns out that a rotating Hayward black hole can act as a particle accelerator and thus in turn provide a suitable framework for Planck-scale physics. Moreover, it turns out for a non-extremal case that there always exist a finite upper bound for the E_{CM} , which increases with the deviation parameter g .

Radiation collapse in five-dimensional Einstein-Gauss-Bonnet gravity

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SAMS Classification: 13

Special Session: Session in honour of Prof. Jim McKenzie

We investigate the continual gravitational contraction of a spherically symmetric radiation shell in five-dimensional Einstein-Gauss-Bonnet gravity. We show that the final fate of such a collapse is an extended and weak curvature naked conical singularity at the centre, which then subsequently becomes covered by an apparent and event horizon. This process is completely different from the five-dimensional general relativity counterpart, where a strong curvature singularity develops at the centre. Since the singularity in the case of Einstein-Gauss-Bonnet gravity is sufficiently weak, we argue that the spacetime can be extended through it, which gives us an elegant way of constructing regular black holes in higher dimensions without violating any energy conditions. We also extend our study to spacetimes with null and string fluids, which are the counterpart of generalised Vaidya spacetimes in general relativity. We show that similar end states are also possible in those cases. Furthermore, we show that in dimensions higher than five, a strong curvature singularity forms upon the cessation of the collapse.

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Understanding planetary interiors and plasma processes from spacecraft missions to the outer planets.

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SAMS Classification: 13

Special Session:

The international NASA/ESA Cassini-Huygens spacecraft mission to Saturn and its moons spent over 13 years in orbit around the Saturn system. The mission ended with the spacecraft burning up in Saturn's atmosphere on 15th September 2017, after a critical end of mission phase with 22 orbits closer to the planet than any spacecraft had gone before. This end of mission phase has confirmed an extremely small tilt between the dipole and rotation axis of Saturn, a real surprise, since planetary dynamo theory suggests that internal planetary magnetic fields cannot be generated without such a tilt being present. The implications of this discovery will be described as well as some of the highlights from analysis of the magnetic field data from the rest of the mission. We will also look forward to the European Space Agency JUICE mission to Jupiter and its moon, due to be launched in 2022.

In pursuit of atmospheric and ocean waves

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SAMS Classification: 13

Special Session: In honour of Prof Jim McKenzie (1938-2015)

In this paper we give an overview of ocean and atmospheric waves, how they are generated and their characteristics. We further discuss their restoring forces together with their propagation properties and give geometrical representations of their evolution patterns.

The Jeans instability in a first order theory of viscosity

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SAMS Classification: 13

Special Session: In honour of Prof Jim McKenzie

We examine the stability of a self-gravitating fluid with bulk viscous pressure. The bulk viscosity is governed by a recently proposed first order theory due to Disconzi. We determine the critical length and mass scales for the system and establish the damping of density perturbations. Our results are compared to those obtained in the classical Jeans analysis.

Thermal plumes in Lake Kivu

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SAMS Classification: 13

Special Session: In honour of Prof Jim McKenzie

Lake Kivu is situated on the border between Rwanda and the Democratic Republic of the Congo. It contains large quantities of dissolved carbon dioxide and methane gas due to the surrounding volcanic activity. Volcanic activity could produce sufficient thermal energy to form a rising thermal plume which could carry water with dissolved gases to a level where the water is over-saturated and gas bubbles could form triggering a gas eruption. Two-dimensional and axisymmetric thermal plumes in Lake Kivu are studied. The Boussinesq approximation is made for the buoyancy term. The Navier-Stokes equation is approximated for large values of the Grashof number. A conserved quantity for each plume is derived. The general form for the similarity solution is obtained and a system of two coupled ordinary differential equations is derived for the two-dimensional and axisymmetric plumes. A shooting method with the conserved quantity as target is used to derive numerical solutions.

The Karmakar condition in 5 dimensions

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SAMS Classification: 13

Special Session: In honour of Prof Jim McKenzie (1938 - 2015)

The Karmakar condition [2] in 4 dimensions is the class 1 embedding condition for a spherical symmetric metric to the flat 5 dimensional metric. To find a similar condition for 5 dimensions one should consider the spherical symmetric metric

$$ds^2 = B^2(r, t)dr^2 + Y^2(r, t) (d\theta^2 + \sin^2 \theta d\phi^2 + \sin^2 \theta \sin^2 \phi d\psi^2) - A^2(r, t)dt^2 \quad (1)$$

and simplify the class 1 embedding conditions [1]

$$\text{Gauss: } R^{\mu\nu}_{\gamma\delta} = \pm (b^\mu_\gamma b^\nu_\delta - b^\mu_\delta b^\nu_\gamma) \quad (2)$$

$$\text{Codazzi: } b^\mu_{\nu;\gamma} = b^\mu_{\gamma;\nu} \quad (3)$$

where $R^{\mu\nu}_{\gamma\delta}$ is the Riemann tensor. Substituting (1) into (2) and solving the resulting system we obtain

$$b^1_1 = \frac{\epsilon R^{12}_{12}}{\sqrt{\epsilon R^{23}_{23}}} \quad (4)$$

$$b^2_2 = b^3_3 = b^4_4 = \sqrt{\epsilon R^{23}_{23}} \quad (5)$$

$$b^5_5 = \frac{\epsilon R^{25}_{25}}{\sqrt{\epsilon R^{23}_{23}}} \quad (6)$$

$$b^1_5 = -\frac{\epsilon R^{12}_{25}}{\sqrt{\epsilon R^{23}_{23}}} \quad (7)$$

where $\epsilon = \pm 1$. The solution (4)-(7) is valid if the following condition is satisfied

$$R^{15}_{15} = \frac{R^{12}_{12} R^{25}_{25}}{R^{23}_{23}} - \frac{R^{12}_{25} R^{25}_{12}}{R^{23}_{23}} \quad (8)$$

Substituting (4)-(7) into (3) and using the Bianchi equations we again obtain (8). Thus (8) is the Karmakar condition for 5 dimensions.

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Thermal instability in double diffusive natural convection in an inclined open square cavity

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SAMS Classification: 13

Special Session: In honour of Prof Jim McKenzie (1938 - 2015)

In this talk we discuss the thermal instability in fluid layer in an inclined open square cavity with an inclined magnetic field. A Galerkin-type method is used to solve the equations in the case of linear stability, and in the nonlinear case a truncated Fourier series is used to obtain a system of five general Lorenz type equations. A multi-domain spectral collocation method is used to solve the differential equations that describe the evolution of the disturbances in the nonlinear regime. The influence of the important physical parameters on the thermal instability is investigated. The results are presented in terms of streamlines, isotherms, iso-concentrations, the Nusselt and the Sherwood numbers. A trapping region for amplitude trajectories is obtained. A limited phase space analysis is presented.

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On thermal stability in a reactive Powell-Eyring fluid in a pipe with two-step exothermic kinetics

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SAMS Classification: 17

Special Session: In honour of Prof Jim McKenzie (1938 - 2015)

This study investigates the effects of viscous heating, radiative heat loss and convective cooling on the flow and thermal stability of a reactive Powell-Eyring fluid with two-step exothermic kinetics in a circular pipe. The reactive non-Newtonian fluid is stimulated by sensitized chemical kinetics and propelled by a constant pressure gradient. The analytical solution to the momentum equation is obtained using a modified Adomian decomposition method while the energy equation is solved numerically using the spectral quasi-linearization method due to its nonlinearity. The effect of some embedded thermo-physical parameters on the thermal critical condition, fluid flow rate and energy balance are examined and discussed.

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Nonlinear radiation in convective Casson nanofluid flow

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SAMS Classification: 13

Special Session: Session in honor of Prof Jim McKenzie

We investigate the impact of nonlinear thermal radiation and variable transport properties on the two-dimensional flow past a moving wedge of an electrically conducting Casson nanofluid filled with gyrotactic microorganisms. It has always been assumed that fluid viscosity and thermal conductivity are temperature dependent in some previous studies. However, this study assumes viscosity, thermal conductivity and the nanofluid properties, are dependent on the solute concentration. The spectral local linearization method is used to solve the conservation equations. The values computed by our method are compared with those in literature, and we discuss the convergence and accuracy of the method. The impact of some parameters on the skin friction, heat and microorganisms mass transport are discussed.

Numerical discretization of input-to-state-stabilization of linear hyperbolic systems of
balance laws with boundary disturbances

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SAMS Classification: Not Member

Special Session: In honour of Prof Jim McKenzie (1938 - 2015)

In this talk, a linear hyperbolic system of balance laws with boundary disturbances in one dimension is considered. An operator splitting technique is required for numerical boundary feedback stabilization of such a system. Then, the finite volume method is applied to

discretize the split system. An explicit candidate Input-to-State Stability (ISS)-Lyapunov function in L^2 -norm is considered and discretized to investigate conditions for ISS of the discretized split system. Finally, results are experimented on a test example and applied to the isothermal Euler equations with boundary disturbances.

Special Session: Lie Symmetries and ODEs

A study of Kadomtsev-Petviashvili-Boussinesq (KPB) equation

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SAMS Classification: 07 LIE GROUP AND REPRESENTATION

In this talk we study the Kadomtsev-Petviashvili-Boussinesq non-linear evolution partial differential equation. We present exact solutions of this equation. Moreover, we obtain its conservation laws by employing Ibragimov's conservation theorem.

**Double Symmetry Reductions of Solute Transport Model with
Space Dependent Water Velocity**

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SAMS Classification: 07 07 Lie groups and representations
Special Session:

We study the non-linear convection-dispersion equation arising in contaminant or solute transport. Here the model contains the non-constant water velocity which depends on spatial variable. The Lie method of algebra is employed to reduce the convection-dispersion equation (CDE) into ordinary differential equations which can be solved by various standard methods of solutions. The double reduction aided the transformation of partial differential equation (PDE) into ordinary differential equation (ODE). The transient and steady state of oil dispersion is considered. Exact solutions are constructed and some given in terms of special functions.

**Transient response of longitudinal parabolic fins to step change in base heat flow
conditions and base temperature**

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SAMS Classification: 07

In this paper, solutions for model describing heat transfer in longitudinal parabolic fins are constructed. The heat coefficients and thermal conductivity are assumed to be power law temperature dependent. The boundary conditions are the step change in base flow conditions and in base temperature. Both local and nonlocal symmetry techniques are employed to analyze the problem at hand. In one case the reduced equation transforms to the Ermakov-Pinney equation. Nonlocal symmetries are admitted when some arbitrary constants appearing in the governing equations are specified. Since the obtained general exact analytical solution satisfy only the zero initial temperature we sort numerical solutions. The effects of the thermo-geometric fin parameter and the power law exponent on temperature distribution are studied.

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On the stochastic Lagrangian Euler Equations: wellposedness and regularity

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SAMS Classification: xx

In this talk we establish the existence of weak-martingale solution to a stochastic Lagrangian Averaged Euler equations on Lipschitz domain through time discretization. Using mollification we establish the continuity in time of the solution of weak solution.

Extensions of sub-Riemannian structures on Lie groups

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SAMS Classification: 04

We define the extension of a left-invariant sub-Riemannian structure in terms of an extension of the underlying Lie group and compatibility of the respective distributions and metrics. We show that geodesics of a structure can be lifted to geodesics of any extension of the structure. In the case of central extensions, we show that the normal geodesics of the minimal extension are the projection (in a sense) of the normal geodesics of any other compatible extension. Several illustrative examples are discussed.

Applications of Maths to the Sciences and Mathematical Physics

Valuation of price sensitivities using Malliavin calculus

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SAMS Classification: xx 17 Applications of Math. to the sciences
Special Session:

We present a probabilistic method for evaluating price sensitivities (Greeks) in finance [1,2]. Our approach rely on the integration by parts formula as developed in the Malliavin calculus. We focus on discontinuous path-dependent payoff functionals.

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Entropy Analysis of Variable Viscosity Cu-H₂O Nanofluid Couette Flow in a Microchannel with Nonlinear Radiative Heat Transfer

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SAMS Classification: 17

One of the classical problems in fluid dynamics with widespread applications in industries and engineering systems is the Couette flow where the fluid motion is induced by movement of the bounding surface. Such applications can be found in shear flow of coolant fluids used in the industrial processes as well as tribology and lubrication technology which deals with the design, friction, wear, and lubrication of interacting surfaces in relative motion. Recently, the introduction of a new class of nanotechnology-based heat transfer fluids known as nanofluid provides a better ultrahigh-performance coolants and lubricants for many industrial technologies. This paper theoretically investigates the effects of thermal radiation, variable viscosity, nanoparticles shape and volume fraction on the thermal performance of a water-Copper nanofluid under a microchannel Couette flow scenario. The nonlinear governing equations are obtained and tackled numerically using shooting method with Runge-Kutta-Fehlberg integration scheme. Pertinent results are presented graphically and discussed with respect to the nanofluid velocity and temperature profiles, skin friction, Nusselt number, entropy generation rate and Bejan number.

Grey Optimization

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SAMS Classification: 17

MSC classification: 90-xx; 91-xx

Decision making is a process of selecting an action from the set of feasible actions in order to optimize a given measure of performance, called the objective. When these kind of problems are formulated mathematically, they become a mathematical optimization problem. Mathematical optimization problems can broadly be classified as deterministic and non-deterministic problems. If all the information are known certainly then the problem is called deterministic, otherwise non-deterministic. Optimization problems modeled from real problems usually involve uncertain parameters. If the level of available information used during problem formulation is not complete but sufficiently enough to construct probability density functions for the uncertain parameters or values, then stochastic optimization approach can be used to solve the formulated problem. Similarly, if the information is enough to construct the fuzzy membership function then fuzzy optimization approach can be used. However in some cases, the available information is minimal where either using

stochastic or fuzzy optimization is not possible. Mostly in these cases, a given parameter will be represented by an interval without any likelihood of the number in the interval to represent the parameter. Hence, the problem will involve an interval number or grey number. This paper aims to discuss some of the concepts and solution approaches regarding grey optimization problem, problems involving grey numbers.

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A Deterministic and Stochastic Dynamics of Rhino Poaching with Optimal Control and Cost Effectiveness Analysis

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SAMS Classification: 17

Rhinoceros populations across Africa have faced considerable reduction in numbers due to human-induced causes such as illegal trade in wildlife trophies. This makes long-term population persistence of this iconic species very uncertain. Poaching has been attributed to be the major cause of drastic decline in Rhino population. However, there have been increased anti-poaching efforts in recent years, leading into population recovery. In this paper, both deterministic and stochastic Rhino poaching dynamics are investigated with optimal control and cost effectiveness strategies. The compartmental deterministic model is analysed under constant and time dependent control strategies with incremental cost effective ratio. A threshold number that guarantee poachers free equilibrium is obtained together with its local and global sensitivity indices. The stochastic model is based on Rhino fertility and survival rates life table. The projection matrix is obtained and analysed quantitatively in order to determine the effect of poaching on the Rhino future stable population distribution

and the corresponding growth rate. It is found that with poaching, the population of this iconic species may soon go into extinction. Numerical simulation is performed in order to validate the obtained quantitative results.

Game-Theoretic Integration and Differential Equations

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SAMS Classification: 17

In the recent years, the revival of the game-theoretic probability free has gain interest both in practise and academic. In particular, an outer-measure of a set of prices has been defined in continuous-time which correspond to a minimal superhedging price. This lead to several properties of price paths associated with randomness deduced. In this talk, we present integration by parts formula for càdlàg price path. In addition, we present the existence and uniqueness solution of a multidimensional differential equations in continuous-time.

**On SDEs with Lipschitz coefficients, driven by continuous,
model-free price paths**

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SAMS Classification: xx 17 Applications of Math. to the sciences
Special Session:

Using similar assumptions as in Revuz and Yor's book [1] we prove the existence and uniqueness of the solutions of SDEs with Lipschitz coefficients, driven by continuous, model-free price paths. The main tool in our reasonings is a model-free version of the Burkholder-Davis-Gundy inequality for integrals driven by model-free, continuous price paths.

References

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Sensitivity of path-dependent derivatives, Functional Itô Calculus

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SAMS Classification: XX 17 Applications of Math. to the sciences
Special Session:

We look at Dupire's Functional Itô Calculus [1] and the computation of sensitivities (Greeks). We compute Greeks of path-dependent derivatives prices. We deal with weakly path-dependent functionals, these are functionals with zero Lie bracket.

References

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P-V critically of a black hole BTZ Black Hole

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SAMS Classification: MATHEMATICAL PHYSICS

In this paper, we investigate the BTZ black hole in P-V critical point of view. As we know, the BTZ black hole does not have P-V critical behavior. So in this case, we introduce some anstaz that give us the modified BTZ black hole. Finally, we show that this modified BTZ black hole is satisfied by the equation of state of Liquid-gas phase transition.

Unsteady Magnetohydrodynamic Mixed Convective Flow of a Reactive Casson Fluid through a Porous Medium

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SAMS Classification: 13

This article analyses the thermal decomposition in an unsteady MHD mixed convection flow of a reactive, electrically conducting Casson fluid within a vertical channel filled with a saturated porous medium and the influence of the temperature dependent properties on the flow. The fluid viscosity is considered to vary exponentially with temperature. The flow is subjected to an externally applied uniform magnetic field. The exothermic chemical kinetics inherent in the flow system give rise to heat dissipation. A technique based on a semi-discretization finite difference scheme and the shooting method is applied to solve the dimensionless governing equations. The effects of the temperature dependent viscosity,

the magnetic field and other important parameters on the velocity and temperature profiles, the wall shear stress and the wall heat transfer rate are presented graphically and discussed quantitatively.

A generalised Bondi system on null infinity

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SAMS Classification: 13

The formulae for computing the Bondi components (i.e. energy and momenta) of an asymptotically flat space-times have been known for many years. These formulae are explicitly derived (for example in the Spinor and Space-time books by Penrose and Rindler) by making assumptions about the conformal factor and gauge on $\mathcal{I}^+$ to simplify the calculations. There are only a few numerical frameworks able to compute asymptotically flat space-times where $\mathcal{I}^+$ is included in the computation domain, of which one is the initial boundary value formulation of the conformal field equations given in 2017 by Stevens et. al. This formulation fixes a conformal factor for the whole space-time based on the initial data set, allowing one to know where the conformal boundary is before a numerical evolution is done. In general this conformal factor cannot be chosen to satisfy the simplifying assumptions of Penrose and Rindler and thus their expressions for the Bondi-mass and the accompanying mass-loss formula are not valid. This talk summarises recent results by the author that generalises these formulae by shedding the current literature's simplifying assumptions, in order for compatibility with the aforementioned numerical framework.

Computational Methods

Chebyshev polynomials based spectral collocation method for fractional partial differential equations

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SAMS Classification: 16

We focus on developing accurate numerical scheme for fractional partial differential equations (FPDEs). The numerical method is based on the shifted Chebyshev polynomial of the first kind and interpolation using the Gauss–Lobatto quadrature. The independent variable is approximated such that it satisfies the orthogonality condition at the shifted Chebyshev–Gauss–Lobatto collocation points. We obtain the corresponding arbitrary differentiation matrix, and solve linear and non-linear FPDEs. To explore the performance of the developed method, we solve the space fractional diffusion equation, the space fractional advection dispersion equation, and the time fractional non-linear reaction sub-diffusion equation. The results obtained confirm the accuracy and convergence of the method.

The mixed finite element method for a vibrating Timoshenko beam

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SAMS Classification: 16

We consider a model for the vibration of a Timoshenko beam. A new proof for convergence is derived using a symmetric formulation and the real reason for the impressive numerical results using the mixed finite element method is determined.

Finite time singularities of the complex Burgers and KdV equations

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SAMS Classification: 16

The inviscid Burgers equation, $u_t + uu_x = 0$, is known to develop multivalued solutions (shocks) in finite time. When viscosity is added, $u_t + uu_x = \delta u_{xx}$, shock formation is prevented and the solution remains single valued for all time. The same is true if dispersion instead of viscosity is added, $u_t + uu_x = \delta u_{xxx}$ (KdV). These statements are true under appropriate initial and boundary conditions, and assuming u to be a real-valued function of x and t . When considering complex-valued solutions, however, the situation changes. Both the complex Burgers and KdV equations can develop singularities in finite time on the x -axis. Using spectral methods and Padé approximation, we examine the dynamics of these singularities in the complex plane until blow-up. (Joint work with Marco Fasoldini and Nick Hale.)

Finite element solution of the reaction-diffusion equation

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SAMS Classification: XX 16 Computational methods

In this study, a Galerkin finite element method (FEM) is used to solve the diffusion-reaction equation. The existence and uniqueness of a weak solution to the corresponding variational formulation will be proved using the lax-Milgram lemma. The FEM seeks an approximation to the weak solution in the space of continuous piecewise linear functions. Computer implementation of the FEM will make use of MATLAB, which is computer software for performing numerical computations. In conclusion, we derive suitable error estimates for proving convergence of the FEM..

PDEs and ODEs

Solution of nonlinear PDEs via the Finite Volume Method with Applications to Fluid Dynamics

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SAMS Classification: 11,16

This talk explores intuitive and robust numerical methods for the solution of the nonlinear systems of coupled partial differential equations governing the flow of complex fluids in complex geometries. In particular, we present the finite volume method as the most intuitive such numerical method, for a broad range of computational fluid dynamics applications, as compared to the finite difference and finite element methods.

We begin by explaining the fundamentals of the finite volume method using generic convection-diffusion type partial differential equations. Such equations are prototypical of generic transport equations in fluid flow. We illustrate and explain the several numerical issues that are symptomatic of the numerical methods for fluid dynamical problems. Such issues include:

- (a) mesh skewness and related flux correction methods,
- (b) numerical discretization of convective fluxes along mesh boundaries using either:
 - (i) basic methods such as central differencing or upwind discretization,
 - (ii) or more advance schemes such as quadratic upwind interpolation of convective kinematics or total variation diminishing schemes,
- (c) consistent methods for pressure-velocity coupling,
- (d) numerical stability and implementation of relevant remedies such as the DEVSS technique, LCR technique, etc.

We finally proceed to provide illustrative fluid dynamical examples in more complex geometries in order to demonstrate the versatility of the finite volume method.

Operational matrices: A tools for solving variable order differential equations

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SAMS Classification: 12,16

A numerical technique for solving a class of nonlinear variable order differential equations (VODEs) is introduced by using Bernstein Polynomials (BPs). We solve the following type of VODEs:

$${}_0D_t^{\kappa(t)}y(t) + \lambda_1 y'(t) + \lambda_2 y(t) + \lambda_3 y''(t) y(t) = g(t),$$

$$y(0) = y_0,$$

where $g(t), y(t) \in L^2[0, 1]$ are known and unknown functions, $\lambda_1, \lambda_2, \lambda_3$ and y_0 are all constants. ${}_0D_t^{\kappa(t)}$ is variable order Caputo derivative.

Here we apply the operational matrix of Bps. With this matrix, the main equation is transformed into a system of algebraic equations by expanding the solution as Bernstein polynomials with unknown coefficients. Then, by solving algebraic equations, the numerical solutions are obtained. The results of numerical examples indicate that the proposed method is computationally efficient.

Nonlinear Timoshenko model for small vibrations of a beam

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SAMS Classification: 11

Three different models for small vibrations of a beam will be considered, resulting from various assumptions. In all 3 models the constitutive equations for the classical Timoshenko model are used. Numerical approximations are used to investigate the influence of longitudinal vibrations on transverse vibrations and vice versa.

Existence and uniqueness results for implicit differential equations with ψ -Hilfer fractional derivative

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SAMS Classification: Fractional Calculus; ψ -Hilfer fractional derivatives; Implicit differential equation; Existence; fixed point.

Motivated by the works of Vanterler et al [1] and Vivik et al [2]. Using Banach and Krasnoselkii fixed point theorem we study the existence of solutions implicit fractional differential system with ψ -Hilfer fraction derivatives.

1. J. Vanterler da, C. Sousa, E. C. de Oliveira, On the ψ - Hilfer fractional derivative. department of applied maths. Imecc-Unicamp, 13083-859, Campinas, SP, Brazil.
2. R. Vivek, K. Kanagarajan, S. Harikrishnan, Existence and uniqueness results for implicit differential equations with generalized fractional derivatives, Journal Nonlinear Anal. Appl., 1 (2018) 27-33.

Modelling the effects of Silicosis in a Mining Community

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SAMS Classification: 12

A mathematical model for the transmission dynamics of silicosis in a mining environment is designed and its qualitative analysis is given. The model takes into account the severity of silica dust exposure in a mining environment. It is shown that the disease free and endemic equilibria are globally asymptotically stable in the absence as well as in the presence of silica dust particles in the air, respectively. The epidemiological implications of these results are discussed. Numerical simulations are presented to support the theoretical analysis.

Second order hyperbolic type problems with applications

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SAMS Classification: 11

We refer to a generalization of the multi-dimensional wave equation in variational form as the general linear second order hyperbolic problem. To the various special cases we refer as second order hyperbolic type problems. The theory can be applied to the vibration of elastic bodies and systems of elastic bodies.

In the presentation we focus on the importance of the theory for numerical approximations and real world applications. New existence results are given and some recent applications are discussed.

Probability Theory

Remark on constants in maximal inequalities for Bessel processes

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Let $X = (X_t)_{t \geq 0}$ be a Bessel process with dimension $\alpha > 0$ starting at $x \geq 0$, and let $p > \max\{0, 2 - \alpha\}$ and $0 < q < \infty$. It is shown that there exists a positive constant $H_x(p, q, \alpha)$ such that

$$\mathbf{E}_x \left(\max_{0 \leq t \leq \tau} X_t \right)^p \leq H_x(p, q, \alpha) \left(\mathbf{E}_x(\tau) \right)^{\frac{p}{p+q}} + x^p$$

for any stopping time τ of X .

The present maximal inequality includes a result of Dubins and Schwarz (1988) as a special interesting case, and also settles a question raised by Dubins, Shepp and Shiryaev (1993).

SAMS Classification: 10

Approximation of distributional risk under perturbation by Wasserstein distance

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SAMS Classification: 10

We consider an idealized problem related to the risk that the distribution that one is assuming is misspecified within a certain margin of error. The problem is $\inf E_\mu(H(X))$, where the infimum is taken over all distributions μ that are within a certain Wasserstein distance of a baseline distribution ν . With certain (arguably) mild assumptions we prove existence of a minimum, and existence of a minimizing sequence of discrete distributions.

Finite Groups and Topology

Balanced ideals in cozero parts of frames

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ABSTRACT: Balanced filters in point-free topology were introduced and studied by Dube in [1]. In this talk we dualise the results presented in the article, amongst other things, we show that a maximal ideal is regular if and only if it is open-generated.

References

[1] T. Dube, *Balanced and closed-generated filters in frames*, Quaest. Math., **26** (2003), 73-81.

The conjugacy class ranks of M_{23}

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SAMS Classification: 02

Special Session: Algebra

Abstract

Let G be a finite group and X be a conjugacy class of G . The *rank* of X in G , denoted by $\text{rank}(G : X)$, is defined to be the minimum number of elements of X generating G . We investigate the ranks of the sporadic simple group M_{23} . We use the structure constants method to determine the ranks of all the non-trivial classes of M_{23} .

**On the projective character tables of an inertia factor group
 $2^6:(6 \times 2)$ of a maximal subgroup in $G_2(4)$**

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SAMS Classification: 02

Special Session: Categories, Algebra and Topology

The Chevalley-Dickson simple group $G_2(4)$ of Lie type G_2 over the Galois field $GF(4)$ and of order $251596800 = 2^{12} \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 13$ has a class of maximal subgroups [1] of the form $2^{4+6}:(A_5 \times 3)$, where 2^{4+6} is a special 2-group with center $Z(2^{4+6}) = 2^4$. Since 2^4 is normal in $2^{4+6}:(A_5 \times 3)$, the group $2^{4+6}:(A_5 \times 3)$ can be constructed as a non-split extension group of the form $\overline{G} = 2^4 \cdot (2^6:(A_5 \times 3))$. Two inertia factor groups, $H_1 = 2^6:(A_5 \times 3)$ and $H_2 = 2^6:(6 \times 2)$, are obtained if $\overline{G}$ acts on 2^4 . In this paper, the author presents a method (based on the work done in [3] and [4]) to compute all projective character tables of H_2 . These tables become very useful if one wants to construct the ordinary character table of $\overline{G}$ by means of Fischer-Clifford theory [2].

References

- [1] J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker, and R.A. Wilson, *Atlas of Finite Groups*, Oxford University Press, Oxford, 1985.
- [2] B. Fischer, *Clifford-matrices*, Progr. Math. **95**, Michler G.O. and Ringel C.(eds), Birkhauser, Basel (1991), 1 - 16.
- [3] R.J. Haggarty and J.F. Humphreys, *Projective characters of finite groups*, Proc. London Math. Soc. (3)**36** (1975), 176 - 192.
- [4] A.L. Prins, The character table of an involution centralizer in the Dempholff group $2^5 \cdot GL_5(2)$, *Quaestiones Mathematicae* **39** (2016), 561-576.

On Hausdorff quasi-pseudometric modular

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SAMS Classification: 05

Special Session: Categories, Algebra and Topology

Chistyakov introduced and developed a concept of modular metric for an arbitrary set in order to generalise the classical notion of modular on a linear space. In this talk, we discuss modular metric which does not satisfy the symmetric axiom of a modular metric and we called it modular quasi-pseudometric.

It is well-known that for a nonsymmetric distance function, there are several completeness concepts. In this talk, we discuss Cauchy sequence and left(right) Cauchy sequences in the setting of modular quasi-pseudometric spaces. Furthermore, we examine the Hausdorff pseudo-modular on the power set of a non-empty set equipped with a modular quasi-pseudometric modular.
