RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 1 : MARCH 2006

# M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 58

FULL MARKS : 50

DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1.   TRUE or FALSE ?

(a) The *identity transformation* $\iota$ is in every group of transformations $\mathfrak{G}$.

(b) The image of any line $\mathcal{L}$ under a given *collineation* is a line *parallel* to $\mathcal{L}$.

(c) The product of *three* halfturns is a halfturn.

(d) Reflection $\rho_{\mathcal{L}}$ *fixes* line $\mathcal{M}$ if and only if $\mathcal{L} \perp \mathcal{M}$.

[2,2,2,2]

Question 2.

(a) Define the terms *involution*, *collineation*, and *group of transformations*.

(b) Give *with justification* an example of an involution and an example of a collineation which is *not* an involution.

(c) Prove ONLY ONE of the following statements :

- The set of all collineations forms a group.
- A reflection is an isometry.

[4,6,4]

Question 3.   PROVE or DISPROVE :

(a) Any translation is a product of two halfturns *and* any halfturn is a product of two translations.

(b) A rotation preserves midpoints and segments.

[8,8]

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Question 4.

(a) Given the points $A = (1, 3)$ and $B = (4, 4)$, write the equations for each of the following transformations :

   i. $\sigma_A$;

   ii. $\sigma_B$;

   iii. $\sigma_B \sigma_A$.

(b) Is transformation $\sigma_B \sigma_A$ a *collineation* ? Find the *image* of the line $\mathcal{N}$ with equation $x - 3y + 1 = 0$ under $\sigma_B \sigma_A$.

(c) Write the equations for the *reflection* $\sigma_{\mathcal{L}}$, where $\mathcal{L}$ is the line with equation $x - y + 1 = 0$.

(d) Find the image of the point $P = (1, 1)$ under $\sigma_{\mathcal{L}}$.

(e) What happens with the line $\mathcal{M}$ with equation $x + y = 0$ under $\sigma_{\mathcal{L}}$ ? Justify your answer.

[4,6,4,1,5]