RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : MAY 2006

M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 58
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) An isometry that fixes a point is an involution.

(b) A product of five reflections is an involutory isometry.

(c) The square of a glide reflection is a glide reflection.

(d) If $\mathcal{L}$ is any line, then every odd isometry is the product of $\sigma_{\mathcal{L}}$ followed by an even isometry.

[2,2,2,2]

Question 2.

(a) Define the terms *odd isometry*, *involutory isometry*, and *glide reflection*. Prove that a glide reflection is an odd isometry. Is it an involution ? Justify your answer.

(b) Prove ONLY ONE of the following statements :

- If lines $\mathcal{L}$ and $\mathcal{M}$ are parallel, then the transformation $\sigma_{\mathcal{M}}\sigma_{\mathcal{L}}$ is a translation.
- Rotation $\rho_{O,r}$ (about the origin) has equations

$$\begin{aligned} x' &= (\cos r)x - (\sin r)y \\ y' &= (\sin r)x + (\cos r)y. \end{aligned}$$

[8,6]

Question 3. PROVE or DISPROVE :

(a) Every isometry is either a product of five reflections or a product of six reflections.

(b) Any two rotations do commute.

[8,8]

Question 4.

(a) If $\sigma_{\mathcal{M}}\sigma_{\mathcal{L}}((x,y)) = (x+3, y-6)$, find equations for lines $\mathcal{L}$ and $\mathcal{M}$. Is the solution unique ? Justify your answer.

(b) Show that the transformation

$$(x,y) \mapsto (-x+h, y+k), \quad h,k \in \mathbb{R}$$

represent an *odd isometry*.

(c) For what values (if any) of the parameters h and k is the isometry in (b)

i. a glide reflection ?

ii. a reflection ?

In each case, find the (equation of) the line *fixed* by the isometry.

[8,2,10]