RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 1 : AUGUST 2007

M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 50
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) There exist points P, Q, R such that $PR \geq PQ + QR$.

(b) If $\tau_{A,B}(C) = D$ then $\tau_{A,B} = \tau_{D,C}$.

(c) The product of *five* halfturns is a halfturn.

(d) Every rotation is an isometry.

[2,2,2,2]

Question 2.

(a) Define the terms *collineation*, *isometry*, and *group of transformations*.

(b) Prove ONLY ONE of the following statements :

- The set of all collineations forms a group.
- Every isometry is a collineation.

[4,4]

Question 3. PROVE or DISPROVE :

(a) The product of two reflections is a reflection.

(b) $\sigma_P \tau_{A,B} \sigma_P = \tau_{C,D}$ where $C = \sigma_P(A)$ and $D = \sigma_P(B)$.

[8,8]

Question 4. Consider the points

$$A = (-1, -1), \quad B = (0, 1), \quad \text{and} \quad C = (1, 1).$$

(a) Write the equations for each of the following transformations :

i. σ_A;
ii. σ_B;
iii. σ_C;
iv. $\tau_{B,A}$.

(b) Find (the point) X such that

$$\sigma_A \sigma_B \sigma_C = \sigma_X.$$

(c) Find (the point) Y such that

$$\sigma_C \tau_{A,B} = \sigma_Y.$$

[4,2,2]

Question 5. Consider the point $P = (h, k)$ and the line $\mathcal{L}$ with equation $y = 0$.

(a) Write the equations of the halfturn σ_P.

(b) Write the equations of the reflection $\sigma_{\mathcal{L}}$.

(c) Is the product $\sigma_P \sigma_{\mathcal{L}}$ a *collineation* ? Justify *carefully* your answer.

(d) Determine (necessary and sufficient) conditions under which the following identity holds :

$$\sigma_P \sigma_{\mathcal{L}} = \sigma_{\mathcal{L}} \sigma_P.$$

Express your result in geometric terms.

[1,1,4,4]