RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : SEPTEMBER 2007

M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 50
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) If an isometry fixes three points, then the isometry must be the identity.

(b) An isometry that fixes exactly one point is a nonidentity rotation.

(c) $\rho_{B,r}\rho_{A,-r}$ is the translation that takes A to B.

(d) Every odd isometry is the product of an even isometry followed by a reflection.

[2,2,2,2]

Question 2.

(a) Define the terms *even isometry*, *involution*, *dilatation*, and *glide reflection*.

(b) Prove ONLY ONE of the following statements :

- Translation τ commutes with the reflection $\sigma_{\mathcal{L}}$ if and only if τ fixes $\mathcal{L}$.
- The rotation $\rho_{O,r}$ about the origin has equations

$$\begin{aligned} x' &= (\cos r)x - (\sin r)y \\ y' &= (\sin r)x + (\cos r)y. \end{aligned}$$

[4,8]

Question 3. PROVE or DISPROVE :

(a) If α and β are isometries such that $\alpha^2 = \beta^2$, then $\alpha = \beta$ or $\alpha = \beta^{-1}$.

(b) Every isometry is either a product of five reflections or a product of six reflections.

[8,8]

Question 4. Consider the points

$$A = (1, 3) \quad \text{and} \quad B = (5, 5)$$

and the line $\mathcal{C}$ with equation

$$x - 2y = 0.$$

(a) Write the equations for the *reflection* σ in line $\mathcal{C}$.

(b) Find the image A' of the point A under the reflection σ.

(c) Write the equations for the *translation* τ such that $\tau(A') = B$.

(d) Write the equations for the (counter-clockwise) *rotation* ρ about the point A through an angle of $45°$.

(e) Find the (equations of the) *glide reflection* γ with axis $\mathcal{C}$ such that $\gamma(A) = (7, 1)$.

[2,1,1,2,4]

Question 5. Find the (coordinates of the) *centre* of the rotation with equations

$$\begin{aligned} x' &= (\cos r)x - (\sin r)y + 2 \\ y' &= (\sin r)x + (\cos r)y, \quad r° \neq 0°. \end{aligned}$$

[4]