

RHODES UNIVERSITY
DEPARTMENT of MATHEMATICS (Pure & Applied)
CLASS TEST No. 1 : AUGUST 2008

M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 50
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

- (a) There *exist* points P, Q, R such that $PR \geq PQ + QR$.
- (b) The *preimage* of any line under a given transformation is a line.
- (c) If transformations α and β are in the group \mathfrak{G} , then $\alpha\beta = \beta\alpha$.
- (d) Every involution is a reflection.

[2,2,2,2]

Question 2.

- (a) Define the terms *transformation*, *collineation*, and *dilatation*.
- (b) Give *with justification* an example of a transformation which is *not* a collineation, and an example of a collineation which is *not* a dilatation.
- (c) Prove **ONLY ONE** of the following statements :
 - If P, Q, R are distinct points and $Q = (1 - t)P + tR$ for some $0 < t < 1$, then $PQ + QR = PR$.
 - If A, B, C are noncollinear points, then $\tau_{A,B} = \tau_{C,D}$ if and only if $\square CABD$ is a parallelogram.

[3,6,5]

Question 3. PROVE or DISPROVE :

- (a) If Q is the midpoint of P and R , then $\sigma_P\sigma_Q = \tau_{P,R}$.
- (b) Any isometry *preserves* segments and lines.

[8,8]

Question 4. Consider the points

$$A = (1, 1), \quad B = (-3, -3)$$

and the line

$$(\mathcal{L}) \quad x + y - 1 = 0.$$

- (a) Write the equations for each of the following transformations :
- the translation $\tau_{A,B}$;
 - the product of halfturns $\sigma_M\sigma_B$, where M is the midpoint of A and B ;
 - the reflection $\sigma_{\mathcal{L}}$;
 - the reflection $\sigma_{\mathcal{M}}$, where \mathcal{M} is the line through A and M .
 - the product of reflections $\sigma_{\mathcal{M}}\sigma_{\mathcal{L}}$.
- (b) Find the fixed points (if any) and the fixed lines (if any) of the transformation $\sigma_B\sigma_M$.

[9,3]
