RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : OCTOBER 2008

M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 58
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) Every isometry is the product of three reflections.

(b) Only the identity is a translation and a rotation.

(c) An isometry that does not fix a point is a glide reflection.

(d) $x' = \pm ax - by + h$ and $y' = \pm bx + ay + k$ are equations for an isometry if $a^2 + b^2 = 1$.

[2,2,2,2]

Question 2.

(a) Define the terms *odd isometry*, *conjugate of an isometry* (by another isometry), and *glide reflection*.

(b) What is the conjugate of a halfturn by any given isometry ? Make a clear statement and then prove it.

(c) Prove ONLY ONE of the following statements :

- Nonidentity rotations $\rho_{C,r}$ and $\rho_{D,s}$ do not commute unless $C = D$.
- The rotation $\rho_{O,r}$ about the origin has equations

$$\begin{aligned} x' &= (\cos r)x - (\sin r)y \\ y' &= (\sin r)x + (\cos r)y. \end{aligned}$$

[4,8,8]

Question 3. PROVE or DISPROVE :

(a) Given a point P and a line $\mathcal{L}$, $\sigma_P \sigma_{\mathcal{L}} = \sigma_{\mathcal{L}} \sigma_P$ if and only if $P \in \mathcal{L}$.

(b) Involutory isometries form a group.

[8,8]

Question 4. Let $a, b \in \mathbb{R}$ such that $a^2 + b^2 = 1$. Consider the point $P = (a, -b)$, the line $\mathcal{L}$ with equation $bx + ay = 0$, and the isometry α with equations

$$\begin{aligned} x' &= ax - by - 1 \\ y' &= bx + ay. \end{aligned}$$

(a) Verify that if $a = -1$, then α is a *halfturn* σ_C. Hence find C.

(b) Verify that if $a = 1$, then α is a *translation*.

(c) Show that if $a \neq \pm 1$, then α is a *noninvolutory rotation* $\rho_{D,r}$. Hence find the centre D and the directed angle r.

(d) Compute $\alpha((0,0))$ and $\alpha(P)$.

(e) Find the image of the line $\mathcal{L}$ under α in TWO DIFFERENT WAYS.

[2,1,5,1,5]