RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 1 : AUGUST 2009

M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 54
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) For any transformations α and β, $(\alpha\beta)^{-1} = \alpha^{-1}\beta^{-1}$.

(b) For any points A and B, $\sigma_B\sigma_A = \tau^2_{A,B}$.

(c) The image of any line under a given dilatation is a line.

(d) Every involution is a halfturn.

[2,2,2,2]

Question 2.

(a) Define the terms *transformation*, *group of transformations*, *isometry*, and *collineation*.

(b) Give *with justification* an example of a transformation which is *not* a collineation, and an example of a collineation which is *not* an isometry.

(c) Prove ONLY ONE of the following statements :

- Every isometry is a collineation.
- Every rotation is an isometry.

[4,6,8]

Question 3. PROVE or DISPROVE :

(a) The set of all halfturns forms a group.

(b) $\tau_{B,D}\sigma_A\tau_{D,C} = \sigma_D$, where A is the midpoint of B and C, and $D = \sigma_B(A)$.

[8,8]

Question 4. Consider the points

$$A = (2, -1), \quad B = (2, 3)$$

and the line

$$(\mathcal{L}) \quad x + y - 3 = 0.$$

(a) Write the equations for each of the following transformations :

i. the translation $\tau_{A,B}$;

ii. the halfturns σ_A and σ_B;

iii. the reflection $\sigma_{\mathcal{L}}$.

(b) Is transformation $\alpha = \sigma_B \sigma_A$ a collineation ? Find the image of $\mathcal{L}$ under α.

(c) What happens with line $\mathcal{M}$ with equation $x - y = 0$ under the reflection $\sigma_{\mathcal{L}}$? Justify your answer.

[4,4,4]