RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : OCTOBER 2009

M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 55
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) The product of four reflections is a product of two reflections.

(b) The product of reflections in *four* intersecting lines is a reflection.

(c) The square of a halfturn is a *nonidentity* translation.

(d) $x' = (\cos r)x + (\sin r)y$ and $y' = (\sin r)x - (\cos r)y$ are equations for a *rotation*.

[2,2,2,2]

Question 2.

(a) Define the terms *involutary isometry*, *conjugate of an isometry*, and *dilatation*.

(b) What is the *conjugate* of a translation (by a given isometry) ? Make a clear statement and then prove it.

[4,9]

Question 3. PROVE or DISPROVE :

(a) The square of a glide reflection is a translation.

(b) Involutory rotations form a group.

[8,8]

Question 4. Consider the points

$$A = (1, 2) \quad \text{and} \quad B = (2, -1)$$

and the line $\mathcal{L}$ with equation

$$x - 3y = 0.$$

(a) Write the equations for the *reflection* $\sigma_{\mathcal{L}}$.

(b) Find the image of the point A under the reflection $\sigma_{\mathcal{L}}$.

(c) Write the equations for the *translation* $\tau_{A,B}^{-1}$.

(d) Determine the equations for the isometry $\rho_{B,-90}\,\rho_{A,90}$.

(e) Find the equations of all *four* isometries sending the segment $\overline{OA}$ onto the segment $\overline{OB}$, where $O = (0, 0)$.
(HINT : Use the general equations for an isometry.)

[2,1,1,6,8]