# RHODES UNIVERSITY DEPARTMENT of MATHEMATICS (Pure & Applied) CLASS TEST No. 1 : MARCH 2010

## M2.1 (TRANSFORMATION GEOMETRY)

### AVAILABLE MARKS : 56 FULL MARKS : 50 DURATION : 1 HOUR

NB : All questions may be attempted.

#### Question 1. TRUE or FALSE ?

- (a) The mapping  $(x, y) \mapsto (x, \cos y)$  is a transformation.
- (b) Any collineation has an inverse.
- (c) The product of 2010 halfturns is a translation.
- (d) Every involution is a halfturn.

[2,2,2,2]

#### Question 2.

- (a) Define the terms transformation, group of transformations, involution, and collineation.
- (b) Prove the following statements :
  - $\sigma_M \sigma_A = \tau_{A,B}$  where M is the midpoint of A and B.
  - Every isometry preserves betweenness and segments.

[4,8,8]

#### Question 3. PROVE or DISPROVE :

- (a) The set of all involutions forms a group.
- (b) Translations and halfturns are isometries.

[8,8]

Question 4. Consider the points

$$A = (1, 3), \quad B = (4, 4).$$

- (a) Write the equations for each of the following transformations :
  - i. the translation  $\tau_{A,B}$ ;
  - ii. the halfturns  $\sigma_A$  and  $\sigma_B$ ;
  - iii. the reflection  $\sigma_{\mathcal{L}}$ , where  $\mathcal{L}$  is the line with equation x y + 1 = 0.
- (b) Is transformation  $\alpha = \sigma_B \sigma_A$  a collineation? Justify your answer.
- (c) Find the image of the line  $\mathcal{M}$  with equation x 3y + 1 = 0under  $\alpha$ .
- (d) Find
  - the image of the point B under the translation  $\tau_{A,B}$ .
  - the preimage of the point B under the transformation  $\alpha$ .

[4,2,4,2]