RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : MAY 2010

M2.1 (TRANSFORMATION GEOMETRY)

AVAILABLE MARKS : 55
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) An odd isometry is a product of *three* reflections.

(b) If $\rho_{\alpha(C),r} = \rho_{C,r}$ for isometry α, then α fixes C.

(c) An isometry that does not fix a point is a glide reflection.

(d) If $\mathcal{M}$ is any line, then every odd isometry is the product of $\sigma_{\mathcal{M}}$ followed by an even isometry.

[2,2,2,2]

Question 2.

(a) Define the terms *even isometry*, *dilatation*, and *glide reflection*.

(b) Prove the following statements :

- If α is an isometry, then

$$\alpha\,\sigma_P\,\alpha^{-1} = \sigma_{\alpha(P)}.$$

- The rotation $\rho_{O,r}$ (about the origin) has equations

$$\begin{aligned} x' &= (\cos r)x - (\sin r)y \\ y' &= (\sin r)x + (\cos r)y. \end{aligned}$$

[3,8,8]

Question 3. PROVE or DISPROVE :

(a) The set of *all* rotations forms a group.

(b) A translation that fixes line $\mathcal{C}$ *commutes* with the glide reflection with axis $\mathcal{C}$.

[8,8]

Question 4.

(a) Show that the transformation

$$\begin{aligned} x' &= x + h \\ y' &= -y + k \end{aligned}$$

is an *odd isometry*.

(b) For what values (if any) of the parameters h and k is the transformation in (a)

i. a glide reflection ?

ii. a reflection ?

In each case, find the (equation of) the line *fixed* by the isometry.

[2,10]