

RHODES UNIVERSITY  
DEPARTMENT of MATHEMATICS (Pure & Applied)  
CLASS TEST : SEPTEMBER 2011

MATHEMATICS & APPLIED MATHEMATICS II  
MAM 202 (GEOMETRY)

AVAILABLE MARKS : 52  
FULL MARKS : 50  
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

- (a) A transformation is necessarily a collineation.
- (b)  $\sigma_A \sigma_B \sigma_C = \sigma_B \sigma_C \sigma_A$  for points  $A, B, C$ .
- (c)  $\sigma_{\mathcal{L}} = \sigma_P^{-1}$  if point  $P$  is on line  $\mathcal{L}$ .
- (d) An isometry that fixes a point is an involution.

[2,2,2,2]

Question 2.

- (a) Define the terms *betweenness*, *dilatation*, *isometry*, and *involution*.
- (b) Prove ONLY ONE of the following statements :
  - Any translation is a dilatation.
  - Every isometry preserves betweenness and segments.

[4,8]

Question 3. PROVE or DISPROVE :

- (a)  $\sigma_A \sigma_M = \sigma_M \sigma_B$  if and only if  $M$  is the midpoint of  $A$  and  $B$ .
- (b) The set of all involutions forms a group.

[8,8]

Question 4. Consider the points

$$A = (0, -1), \quad B = (4, 1)$$

and the line

$$(\mathcal{L}) \quad x + 2y - 1 = 0.$$

- (a) Write the equations for each of the following transformations :
- the translation  $\tau_{A,B}$ ;
  - the product of halfturns  $\sigma_A \sigma_M$ , where  $M$  is the midpoint of  $A$  and  $B$ .
  - the reflection  $\sigma_{\mathcal{L}}$ .
  - the rotation  $\rho_{B,90}$ .
- (b) Find
- the image of the point  $B$  under the translation  $\tau_{A,B}$ .
  - the preimage of the point  $A$  under the rotation  $\rho_{B,90}$ .
- (c) What happens with the line  $\mathcal{M}$  with equation  $2x - y + 2 = 0$  under the transformation  $\sigma_{\mathcal{L}}$ ? Justify your answer.
- (d) Determine a line  $\mathcal{B}$  such that  $\rho_{B,90} = \sigma_{\mathcal{B}} \sigma_{\mathcal{A}}$ , where  $\mathcal{A}$  is the line with equation  $x - y - 3 = 0$ . Is the line  $\mathcal{B}$  unique? If yes, find its equation.

[6,2,4,4]

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