RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST : APRIL 2012

MATHEMATICS & APPLIED MATHEMATICS II
MAM 201 (GEOMETRY)

AVAILABLE MARKS : 52

FULL MARKS : 50

DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1.   TRUE or FALSE ?

(a) The mapping $(x, y) \mapsto (x^2, y)$ is a transformation.

(b) Every halfturn is an involution.

(c) If $\tau_{A,B}(C) = D$, then $\tau_{A,B} = \tau_{C,D}$.

(d) A translation has no fixed points.

[2,2,2,2]

Question 2.

(a) Define the terms *transformation, halfturn, dilatation*, and *involution*.

(b) Prove ONLY ONE of the following statements :

- Any halfturn is a dilatation.
- Every isometry preserves segments.

[4,8]

Question 3.   PROVE or DISPROVE :

(a) If M is the midpoint of A and B, then $\sigma_A \sigma_M = \tau_{A,B}$.

(b) The set of all rotations about the same point P forms a group.

[8,8]

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Question 4. Consider the points

$$A = (1, 2), \quad B = (2, 1)$$

and the lines

$$(\mathcal{L}) \quad x - y = 0, \quad (\mathcal{M}) \quad y - 1 = 0.$$

(a) Write the equations for each of the following transformations :

    i. the translation $\tau_{A,B}$.

    ii. the product of halfturns $\sigma_A \sigma_M$, where M is the midpoint of A and B.

    iii. the reflection $\sigma_{\mathcal{L}}$.

    iv. the reflection $\sigma_{\mathcal{M}}$.

    v. the product of reflections $\sigma_{\mathcal{M}} \sigma_{\mathcal{L}}$.

(b) Find

    i. the preimage of the point M under the translation $\tau_{A,B}$.

    ii. the image of the point A under the reflection $\sigma_{\mathcal{L}}$.

    iii. the image of the point B under the reflection $\sigma_{\mathcal{M}}$.

    iv. the image of the line $\mathcal{L}$ under the reflection $\sigma_{\mathcal{M}}$.

(c) What happens with the point M under the action of the transformation $\sigma_{\mathcal{L}}$. Justify your answer.

[9,5,2]