RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST : APRIL 2013

MATHEMATICS & APPLIED MATHEMATICS II

MAM 201 (GEOMETRY)

AVAILABLE MARKS : 52
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) The mapping $(x, y) \mapsto (x, x^2 + y)$ is a transformation.

(b) If τ is a translation and $\mathcal{L}$ is a line, then $\tau(\mathcal{L}) \parallel \mathcal{L}$.

(c) A halfturn σ_P fixes a line $\mathcal{L} \iff P \in \mathcal{L}$.

(d) The set of all halfturns is a group.

[2,2,2,2]

Question 2.

(a) Define the terms *parallel*, *collineation*, *dilatation* and *isometry*.

(b) Prove ONLY ONE of the following statements :

- The set $\mathfrak{D}$ of dilatations forms a group.
- Every isometry preserves midpoints.

[4,8]

Question 3. PROVE or DISPROVE :

(a) A product of three halfturns is a halfturn. In particular, if points P, Q, R are not collinear, then $\sigma_R\sigma_Q\sigma_P = \sigma_S$, where $\square PQRS$ is a parallelogram.

(b) The set $\mathfrak{T}$ of translations forms a commutative group.

[8,8]

Question 4. Consider the points

$$A = (1,1), \quad B = (2,0), \quad C = (2,2)$$

and the lines

$$\mathcal{L} \,:\, y = x, \quad \mathcal{M} \,:\, x + y - 2 = 0.$$

(a) Write the equations for each of the following transformations :

i. the translations $\tau^2_{A,B} = \tau_{A,B}\tau_{A,B}$ and $\tau_{A,C}$.

ii. the product of halfturns $\sigma_B\,\sigma_A$.

iii. the reflection $\sigma_{\mathcal{L}}$.

iv. the reflection $\sigma_{\mathcal{M}}$.

v. the product of reflections $\sigma_{\mathcal{M}}\sigma_{\mathcal{L}}$.

(b) Find

i. the preimage of the point C under the reflection $\sigma_{\mathcal{M}}$.

ii. the image of the point A under the product of reflections $\sigma_{\mathcal{M}}\sigma_{\mathcal{L}}$.

iii. the image of the line $\mathcal{M}$ under the translation $\tau_{A,C}$.

[10,6]