RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : OCTOBER 2014

MAM202 (GROUPS and GEOMETRY)

AVAILABLE MARKS : 55
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. TRUE or FALSE ?

(a) If an isometry fixes three distinct points, then it must be the identity.

(b) An isometry has a unique fixed point if and only if it is a non-identity rotation.

(c) The product of any two rotations is a rotation.

(d) If $\rho_{\alpha(C),r} = \rho_{C,r}$ for isometry α, then α fixes the point C.

[2,2,2,2]

Question 2.

(a) Define the terms *odd isometry*, *involutory isometry*, and *glide reflection*.

(b) Prove the following statements :

- Given two isometries α and β, the product $\alpha\,\beta\,\alpha^{-1}$ is an involution if and only if β is an involution.
- Translation τ commutes with reflection $\sigma_{\mathcal{C}}$ if and only if τ fixes line $\mathcal{C}$.

[3,12]

Question 3. PROVE or DISPROVE :

(a) The set of all glide reflections forms a group.

(b) Every translation is a product of two reflections in parallel lines.

[8,8]

Question 4. Consider the lines

$$(\mathcal{L}) \quad 2x + y - 3 = 0 \quad \text{and} \quad (\mathcal{M}) \quad 2x + y - 8 = 0.$$

(a) Write the equations for each of the following transformations :

i. the reflection $\sigma_{\mathcal{L}}$;

ii. the reflection $\sigma_{\mathcal{M}}$;

iii. the product of reflections $\alpha = \sigma_{\mathcal{M}}\sigma_{\mathcal{L}}$.

(b) Find the *image* and the *preimage* of the line $\mathcal{L}$ under the transformation α.

[6,4]

Question 5. Consider the isometry with equations

$$\begin{cases} x' = \dfrac{1}{5}(-3x + 4y) + 4 \\ y' = \dfrac{1}{5}(-4x - 3y) + 2. \end{cases}$$

Determine all fixed points (if any) of this transformation. Hence deduce the nature of this isometry.

[6]