RHODES UNIVERSITY
DEPARTMENT OF MATHEMATICS (Pure & Applied)

EXAMINATION : JUNE 2006

MATHEMATICS II/APPLIED MATHEMATICS II

PAPER 3

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AVAILABLE MARKS : 117
FULL MARKS : 100
DURATION : 2 HOURS

M2.1 - TRANSFORMATION GEOMETRY

NB : All questions may be attempted. All steps must be clearly motivated. Marks will not be awarded if this is not done.

Question 1. [**12** marks]

TRUE or FALSE ?

(a) A product of two involutions is an involution or the identity transformation.

(b) $\rho_{C,r}^{-1} = \rho_{C,-r} = \sigma_C$ for any point C.

(c) An odd isometry is a product of three reflections.

(d) Some letters of the (Latin) alphabet (written in most symmetric from) have two points of symmetry.

(e) A dilatation is a similarity.

(f) A collineation is a product of strains and similarities.

[2,2,2,2,2,2]

Question 2. [**30** marks]

(a) Define the terms : *collineation, similarity, isometry,* and *glide reflection.*

(b) Name two kinds of collineations which are *not* similarities, two kinds of similarities which are *not* isometries, and two kinds of isometries which are *not* glide reflections. In each case, give equations for your selected transformations.

(c) Prove the following two statements :

i. Any similarity is a collineation.

ii. The product of two reflections in intersecting lines is a rotation.

(d) State clearly the *Classification Theorem for Isometries.* Identify the *even isometries* and the *involutory isometries.*

(e) Define the terms *shear* and *strain.* Is the product of two shears always a shear ? Make a clear statement and then prove it.

[4,9,8,3,6]

Question 3. [**32** marks]

PROVE or DISPROVE :

(a) For any two points P and Q,

$$\sigma_Q \sigma_P = \tau^2_{Q,P}.$$

(b) The transformations σ_P and $\sigma_{\mathcal{L}}$ *commute* if and only if P lies on $\mathcal{L}$.

(c) The set of all dilations forms a *group.*

(d) If α is an isometry, then

$$\alpha\, \tau_{A,B}\, \alpha^{-1} = \tau_{\alpha(A),\alpha(B)}.$$

[8,8,8,8]

Question 4. [**16** marks]

Prove ONLY TWO of the following statements :

- An isometry that fixes two distinct points fixes that line pointwise, hence it is either a reflection or the identity.
- A rotation of r° followed by a rotation of s° is a rotation of $(r+s)^\circ$ unless $(r+s)^\circ = 0^\circ$, in which case the product is a translation.
- A similarity (of ratio r) has equations

$$\begin{aligned} x' &= ax - by + h \\ y' &= \pm(bx + ay) + k \end{aligned}$$

  with $a^2 + b^2 = r^2$ and, conversely, such equations are those of a similarity (of ratio r).
- A collineation is an affine linear transformation.

[8,8]

Question 5. [**27** marks]

Consider the points

$$A = (1, -3), \quad B = (3, 5) \quad \text{and} \quad C = (6, 0)$$

and the line

$$\mathcal{L} : \quad x + 4y - 6 = 0.$$

(a) Let $\mathcal{M}$ be the line through A and B. Write the equation of $\mathcal{M}$. Find the coordinates of the midpoint M of A and B. Verify that the point M lies on $\mathcal{M}$.

(b) Find the equation of the line $\mathcal{N}$ through the point C and parallel to $\mathcal{M}$. Verify that $\mathcal{N}$ is perpendicular to $\mathcal{L}$.

(c) Write the equations for the following transformations :

i. the halfturn σ_M;
ii. the reflection $\sigma_{\mathcal{L}}$;
iii. the translation $\tau_{A,B}$;
iv. the rotation $\rho_{C,45}$;
v. the isometry $\sigma_{\mathcal{L}}\sigma_{\mathcal{M}}$;
vi. the dilation $\delta_{C,-2}$.

(d) Find the image of the origin under each of the transformations in (c).

(e) Find the equations of the unique *direct similarity* that sends O (the origin) to B and A to C. (HINT : Use the general equations for a similarity.)

(f) Determine

i. the ratio

ii. the fixed point

of the similarity in (e). Hence identify this transformation.

[2,2,11,3,5,4]

END OF THE EXAMINATION PAPER