RHODES UNIVERSITY
DEPARTMENT OF MATHEMATICS (Pure & Applied)

EXAMINATION : NOVEMBER 2007

# MATHEMATICS II/APPLIED MATHEMATICS II

## PAPER 3

Examiners : Dr C.C. Remsing
Dr G. Lubczonok

AVAILABLE MARKS : 105
FULL MARKS : 100
DURATION : 2 HOURS

---

M2.1 - TRANSFORMATION GEOMETRY

NB : All questions may be attempted. All steps must be clearly motivated. Marks will not be awarded if this is not done.

Question 1. [**12** marks]

TRUE or FALSE ?

(a) Any translation is a product of two rotations.
(b) The set of all halfturns forms a group.
(c) A nonidentity isometry that fixes a point is a rotation.
(d) The symmetry group of an isosceles triangle that is not equilateral has three elements .
(e) A dilation is a product of two strains.
(f) An equiaffine transformation is either an isometry or a shear.

[2,2,2,2,2,2]

Question 2. [**20** marks]

(a) Define the terms : *odd isometry, dilation, shear,* and *affine transformation.*

(b) Give general equations for each transformation as in (a).

(c) Prove ONLY ONE of the following statements :

- Any dilation is a similarity, but not every similarity is a dilation.
- Any shear is an equiaffine transformation.

(d) Define the term *glide reflection* and then prove that the square of a glide reflection is a translation.

[4,4,6,6]

Question 3. [**32** marks]

PROVE or DISPROVE :

(a) The set of *all* rotations forms a *group.*

(b) Reflections $\sigma_{\mathcal{A}}$ and $\sigma_{\mathcal{B}}$ *commute* <u>if and only if</u> $\mathcal{A} = \mathcal{B}$.

(c) For any *similarity* $\alpha$, the *conjugate* of the translation $\tau_{P,Q}$ by $\alpha$ is the translation $\tau_{P',Q'}$, where $P' = \alpha(P)$ and $Q' = \alpha(Q)$.

(d) An equiaffine similarity is an isometry.

[8,8,8,8]

Question 4. [**16** marks]

Prove ONLY TWO of the following statements :

- An even isometry has equations

$$\begin{aligned} x' &= ax - by + h \\ y' &= bx + ay + k \end{aligned}$$

  with $a^2 + b^2 = 1$ and, conversely, such equations are those of an even isometry.
- A finite symmetry group is either a cyclic group or a dihedral group.
- A dilatation is a translation or a dilation.
- The affine transformation

$$\begin{aligned} x' &= ax + by + h \\ y' &= cx + dy + k \end{aligned}$$

  (with $ad - bc \neq 0$ ) is a similarity (of ratio $r$ ) <u>if and only if</u>

$$a^2 + c^2 = b^2 + d^2 = r^2 \quad \text{and} \quad ab + cd = 0.$$

[8,8]

Question 5. [**25** marks]

Consider the points

$$O = (0,0), \quad A = (1,0), \quad B = (2,2) \quad \text{and} \quad C = (-1,6)$$

and the transformations $\rho$ with equations

$$\begin{aligned} x' &= \frac{1}{\sqrt{5}}(x - 2y) + \frac{1}{\sqrt{5}} \\ y' &= \frac{1}{\sqrt{5}}(2x + y) + \frac{\sqrt{5}-1}{2\sqrt{5}} \end{aligned}$$

and $\delta$ with equations

$$\begin{aligned} x' &= \sqrt{5}x \\ y' &= \sqrt{5}y + \frac{1-\sqrt{5}}{2}. \end{aligned}$$

(a) Show that $\rho$ is a rotation. Find its centre $C$ and its angle $r$.

(b) Show that $\delta$ is a stretch about some point $P$. Find the coordinates of $P$ and the ratio $s$.

(c) Write the transformations $\delta$ and $\rho$ in matrix form. Hence compute the product $\rho\,\delta$.

(d) Let $\alpha$ denote the similarity such that

$$\alpha(O) = A, \quad \alpha(A) = B \quad \text{and} \quad \alpha(B) = C.$$

  i. Find its ratio $t$.
  ii. Determine the equations for $\alpha$. (HINT : Use the general equations for a similarity.)

(e) Compare the transformations $\rho\,\delta$ and $\alpha$ as in (c) and (d), respectively. Hence identify the transformation $\alpha$.

[8,4,4,7,2]

END OF THE EXAMINATION PAPER