RHODES UNIVERSITY
DEPARTMENT OF MATHEMATICS (Pure & Applied)

EXAMINATION : NOVEMBER 2008

MATHEMATICS II/APPLIED MATHEMATICS II

PAPER 2

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AVAILABLE MARKS : 116
FULL MARKS : 100
DURATION : 2 HOURS

M2.1 - TRANSFORMATION GEOMETRY

NB : All questions may be attempted. All steps must be clearly motivated. Marks will not be awarded if this is not done.

Question 1. [**12** marks]

TRUE or FALSE ?

(a) The image of any line under a given collineation is a line.
(b) Every reflection is an involution.
(c) An odd isometry that does not fix a point is a glide reflection.
(d) The symmetry group of a rectangle has four elements .
(e) Any dilatation is a dilation.
(f) The point $\delta_{A,r}(B)$ is on (the ray) $AB^{\rightarrow}$ if $A \neq B$.

[2,2,2,2,2,2]

Question 2. [**30** marks]

(a) Define carefully the terms : *collineation, involution, isometry*, and *similarity*.

(b) Use equations to give two *concrete* examples of collineations which are *not* similarities. Justify your claim.

(c) Name two kinds of similarities which are *not* isometries and two kinds of isometries which are *not* involutions. In each case, give *general* equations for your selected transformations.

(d) Prove the following two statements :

- The product of two halfturns about different points is a non-identity translation.
- If α is an involution, then $\alpha\,\beta\,\alpha^{-1}$ is an involution for any transformation β.

(e) State clearly the CLASSIFICATION THEOREM FOR SIMILARITIES. Define carefully *all* terms involved in your statement.

[4,6,8,8,4]

Question 3. [**32** marks]

PROVE or DISPROVE :

(a) Nonidentity rotations $\rho_{A,r}$ and $\rho_{B,s}$ *commute* if and only if $A = B$.

(b) The set of *all* dilations form a *group*.

(c) A figure can have exactly two *points of symmetry*.

(d) If α is a similarity, then $\alpha\,\sigma_P\,\alpha^{-1} = \sigma_{\alpha(P)}$.

[8,8,8,8]

Question 4. [**24** marks]

Prove ONLY THREE of the following statements :

- If $\mathcal{L}$ and $\mathcal{M}$ are *parallel* lines, then the product $\sigma_{\mathcal{M}}\,\sigma_{\mathcal{L}}$ is a *translation*.
- If α is an *isometry*, then $\alpha\,\tau_{A,B}\,\alpha^{-1} = \tau_{\alpha(A),\alpha(B)}$.
- A finite symmetry group is *either* a cyclic group *or* a dihedral group.
- Dilation $\delta_{P,r}$ about $P = (h,k)$ has equations

$$\begin{aligned} x' &= rx + (1-r)h \\ y' &= ry + (1-r)k. \end{aligned}$$

[8,8,8]

Question 5. [**18** marks]

Consider the points

$$A = (0,1), \quad B = (1,2), \quad C = (-1,6) \quad \text{and} \quad D = (-15,4).$$

(a) Write the equations for the following transformations :

i. the translation $\tau_{A,B}$.

ii. the halfturn σ_C.

iii. the reflection $\sigma_{\mathcal{L}}$, where $\mathcal{L} = \overleftrightarrow{AC}$.

iv. the dilation $\delta_{D,-2}$.

(b) If $\sigma_{\mathcal{N}}\sigma_{\mathcal{M}}((x,y)) = (x+6, y-3)$, find equations for $\mathcal{M}$ and $\mathcal{N}$.

(c) Let α denote the similarity such that

$$\alpha(A) = B, \quad \alpha(B) = C \quad \text{and} \quad \alpha(C) = D.$$

i. Find its ratio s.

ii. Determine the equations for α. (Hint : Use the general equations for a similarity.)

[8,4,6]

END OF THE EXAMINATION PAPER