RHODES UNIVERSITY

DEPARTMENT OF MATHEMATICS (Pure & Applied)

EXAMINATION : NOVEMBER 2009

MATHEMATICS II

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AVAILABLE MARKS : 110
FULL MARKS : 100
DURATION : 2 HOURS

M2.1 - TRANSFORMATION GEOMETRY

NB : All questions may be attempted. All steps must be clearly motivated. Marks will not be awarded if this is not done.

Question 1. [**12** marks]

TRUE or FALSE ?

(a) Two parallel lines are mapped by a given isometry into two parallel lines.

(b) Every reflection is a product of two rotations.

(c) An even isometry that does not fix a point is a translation.

(d) The symmetry group of a square has exactly four elements.

(e) A dilatation that is not a translation must have a fixed point.

(f) Any similarity is a collineation.

[2,2,2,2,2,2]

Question 2. [**22** marks]

(a) Define carefully the terms : *collineation, halfturn, stretch rotation*, and *dilatation*.

(b) Prove the following two statements :

- The product of two reflections in parallel lines is a translation.
- A rotation about the origin has equations

$$\begin{aligned} x' &= (\cos r)x - (\sin r)y \\ y' &= (\sin r)x + (\cos r)y. \end{aligned}$$

(c) State clearly the CLASSIFICATION THEOREM FOR SIMILARITIES.

[4,8,8,2]

Question 3. [**32** marks]

PROVE or DISPROVE :

(a) $\sigma_A \sigma_B = \sigma_B \sigma_C$ <u>if and only if</u> B is the *midpoint* of A and C.

(b) The set of all reflections and glide reflections forms a *group*.

(c) If α is an *even* isometry, then

$$\alpha\, \rho_{C,r}\, \alpha^{-1} = \rho_{\alpha(C),r}.$$

(d) Halfturns σ_A and σ_B *commute* <u>if and only if</u> $A = B$.

[8,8,8,8]

Question 4. [**16** marks]

Prove ONLY TWO of the following statements :

- A product of four reflections is a product of two reflections.
- A rotation of r° followed by a rotation of s° is a rotation of $(r+s)$ unless $(r+s)^\circ = 0^\circ$, in which case the product is a translation.
- A similarity preserves betweenness, midpoints and segments.
- A dilatation is a translation or a dilation.

[8,8]

Question 5. [**28** marks]

Consider the points

$$A = (2,1), \quad B = (2,-2), \quad C = (-2,3), \quad D = (4,3)$$

and the line $\mathcal{L}$ with equation

$$x + 3y = 0.$$

(a) Write the equations for the following transformations :

i. the translation $\tau_{A,B}^{-1}$;

ii. the rotation $\rho_{B,-90}$;

iii. the reflection $\sigma_{\mathcal{L}}$;

iv. the dilation $\delta_{A,-3}$.

(b) How many similarities are there sending the segment $\overline{AB}$ onto the segment $\overline{CD}$? Justify your claim.

(c) Find the equations of the unique direct similarity such that $A \mapsto C$ and $B \mapsto D$.

(d) Find the equations of the unique opposite similarity such that $A \mapsto D$ and $B \mapsto C$.

[8,4,8,8]

END OF THE EXAMINATION PAPER