RHODES UNIVERSITY

DEPARTMENT OF MATHEMATICS (Pure & Applied)

EXAMINATION : MAY 2010

MATHEMATICS II

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AVAILABLE MARKS : 110
FULL MARKS : 100
DURATION : 2 HOURS

M2.1 - TRANSFORMATION GEOMETRY

NB : All questions may be attempted. All steps must be clearly motivated. Marks will not be awarded if this is not done.

Question 1. [**12** marks]

TRUE or FALSE ?

(a) For *any* three points P, Q and R, $PR = PQ + QR$.

(b) The image of any line under a given collineation is a line.

(c) Every isometry is a product of *three* reflections.

(d) The product of *any* two rotations is a rotation.

(e) The symmetry group of a square has *exactly* four elements.

(f) A dilatation that is not a translation *must* have a fixed point.

[2,2,2,2,2,2]

Question 2. [**22** marks]

(a) Define carefully the terms : *involutive transformation, halfturn, stretch reflection*, and *dilatation*.

(b) Prove the following two statements :

- The product of two reflections in *intersecting* lines is a rotation.
- An *odd isometry* has equations

$$\begin{aligned} x' &= ax + by + h \\ y' &= bx - ay + k \end{aligned}$$

where $a^2 + b^2 = 1$.

(c) State clearly the CLASSIFICATION THEOREM FOR ISOMETRIES.

[4,8,8,2]

Question 3. [**32** marks]

PROVE or DISPROVE :

(a) $\sigma_A \sigma_B = \sigma_B \sigma_C$ <u>if and only if</u> B is the *midpoint* of A and C.

(b) The set of all dilations forms a *group*.

(c) For any $r \neq 0$ (and some point C)

$$\delta_{B,r}\, \delta_{A,\frac{1}{r}} = \tau_{A,C}.$$

(d) Reflections $\sigma_{\mathcal{A}}$ and $\sigma_{\mathcal{B}}$ *commute* <u>if and only if</u> $\mathcal{A} = \mathcal{B}$.

[8,8,8,8]

Question 4. [**16** marks]

Prove ONLY TWO of the following statements :

- If α is an even isometry, then $\alpha\,\rho_{C,r}\,\alpha^{-1} = \rho_{\alpha(C),r}$.
- The square of a glide reflection is a translation.
- A finite symmetry group is *either* a cyclic group *or* a dihedral group.
- Any dilation is a dilatation *but* not every dilatation is a dilation.

[8,8]

Question 5. [**28** marks]

Consider the points

$$A = (-2,-1), \quad B = (0,3), \quad C = (2,1)$$

and the line $\mathcal{L}$ with equation

$$2x - y + 3 = 0.$$

(a) Write the equations for the following transformations :
   i. the translation $\tau^2_{A,B}$;
   ii. the halfturn σ_B;
   iii. the rotation $\rho_{C,r}$;
   iv. the reflection $\sigma_{\mathcal{L}}$;
   v. the glide-reflection $\sigma_{\mathcal{L}}\sigma_C$;
   vi. the dilation $\delta_{A,-3}$.

(b) Find the axis of the glide-reflection $\gamma = \sigma_{\mathcal{L}}\sigma_C$.

(c) How many similarities are there sending the segment $\overline{AB}$ onto the segment $\overline{BC}$? Justify your claim.

(d) Find the equations of the unique direct similarity (as in (c)) such that $A \mapsto B$ and $B \mapsto C$.

(e) Find *all* dilatations taking the circle with equation $x^2 + y^2 = 1$ to the circle with equation $x^2 + (y-2)^2 = 4$.

[8,4,4,8,4]

END OF THE EXAMINATION PAPER