RHODES UNIVERSITY

DEPARTMENT OF MATHEMATICS (Pure & Applied)

EXAMINATION : NOVEMBER 2014

MATHEMATICS & APPLIED MATHEMATICS II

Paper 2

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AVAILABLE MARKS : 110
FULL MARKS : 100
DURATION : 3 HOURS

MAM 202 - GROUPS AND GEOMETRY

NB : All questions may be attempted. All steps must be clearly motivated. Marks will not be awarded if this is not done.

Question 1. [**16** marks]

TRUE or FALSE ?

(a) For any three points P, Q and R, $PR \geq PQ + QR$.
(b) Every transformation is a collineation.
(c) Every reflection is an involution.
(d) If α and β are isometries and $\alpha^2 = \beta^2$, then $\alpha = \beta$ or $\alpha = \beta^{-1}$.
(e) The symmetry group of a square has *exactly* four elements.
(f) Any dilation is a dilatation.
(g) A similarity with two fixed points is an isometry.
(h) The opposite similarities form a group.

[2,2,2,2,2,2,2,2]

Question 2. [**28** marks]

(a) Define the terms : *involutive transformation, odd isometry, stretch reflection*, and *dilatation*.

(b) Prove ONLY THREE of the following statements :

- A halfturn is an involutory dilatation.
- If lines $\mathcal{L}$ and $\mathcal{M}$ are parallel, then the product $\sigma_{\mathcal{M}}\sigma_{\mathcal{L}}$ is the translation through twice the directed distance from $\mathcal{L}$ to $\mathcal{M}$.
- A rotation of r° followed by a rotation of s° is a rotation of $(r+s)^\circ$ unless $(r+s)^\circ = 0^\circ$, in which case the product is a translation.
- Let $\mathfrak{G}$ be a finite group of isometries. Then there is a point C in the plane which is left fixed by every element of $\mathfrak{G}$.

[4,8,8,8]

Question 3. [**32** marks]

PROVE or DISPROVE :

(a) $\sigma_P\, \tau_{A,B}\, \sigma_P = \tau_{C,D}$, where $C = \sigma_P(A)$ and $D = \sigma_P(B)$.

(b) The square of a glide reflection is a translation.

(c) The set of all dilations forms a *group*.

(d) Dilatations $\delta_{P,r}$ and $\tau_{P,Q}$ *never* commute if $P \neq Q$.

[8,8,8,8]

Question 4. [**6** marks]

Find a glide reflection γ such that $\gamma^2 = \tau_{A,B}$, where $A = (1,1)$ and $B = (2,2)$. Is the solution unique ? Justify your answer.

[6]

Question 5. [**28** marks]

Consider the points

$$A = (2, 1), \quad B = (2, -2), \quad C = (-2, 3), \quad D = (4, 3)$$

and the line $\mathcal{L}$ with equation

$$x - y + 2 = 0.$$

(a) Write the equations for the following transformations :

  i. the translation $\tau^2_{A,B}$;

  ii. the halfturn σ_B;

  iii. the rotation $\rho_{C,r}$;

  iv. the reflection $\sigma_{\mathcal{L}}$;

  v. the glide-reflection $\sigma_{\mathcal{L}}\sigma_C$;

  vi. the dilation $\delta_{A,-2}$.

(b) How many similarities are there sending the segment $\overline{AB}$ onto the segment $\overline{BC}$? Justify your claim.

(c) Find the equations of the unique *direct similarity* (as in (b)) such that $A \mapsto C$ and $B \mapsto D$.

(d) Find the equations of the unique *opposite similarity* (as in (b)) such that $A \mapsto D$ and $B \mapsto C$.

[8,4,8,8]

END OF THE EXAMINATION PAPER