RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 2 : OCTOBER 2009

AM3.4 (DIFFERENTIAL GEOMETRY)

AVAILABLE MARKS : 55
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1. Let $\mathcal{S}$ be a subset of $\mathbb{R}^3$ and let γ be a unit-speed curve in $\mathbb{R}^3$.

(a) Explain carefully what is meant by saying that $\mathcal{S}$ is a *smooth surface*. Hence, show that the *unit sphere*

$$\mathbb{S}^2 = \left\{(x, y, z) \in \mathbb{R}^3 \,|\, x^2 + y^2 + z^2 = 1\right\}$$

is a smooth surface.

(b) Define the *tangent developable* $\mathcal{T}$ associated with the curve γ. Give conditions under which $\mathcal{T}$ is a smooth surface.

(c) Prove that any tangent developable is isometric to (part of) a plane.

[7,3,10]

Question 2. Consider the elliptic cylinder

$$\mathcal{S} = \left\{ (x, y, z) \in \mathbb{R}^3 \,|\, \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right\}.$$

(a) Show that $\mathcal{S}$ is a smooth surface.

(b) Compute the *first fundamental form* of the surface patch

$$\sigma(u, v) = (a \cos u,\, b \sin u,\, v).$$

(c) Define the term *ruled surface*. Is $\mathcal{S}$ a ruled surface? Justify your answer.

(d) Define the term *surface of revolution*. Is $\mathcal{S}$ a surface of revolution? Justify your answer.

[3,4,5,5]

Question 3.

(a) Explain what is meant by saying that a diffeomorphism $f : \mathcal{S}_1 \to \mathcal{S}_2$ (between two smooth surfaces) is *conformal*.

(b) Find *all* smooth functions f for which the surface patch

$$\sigma(u, v) = (u \cos v,\, u \sin v,\, f(u))$$

is conformal.

[2,16]