RHODES UNIVERSITY

DEPARTMENT of MATHEMATICS (Pure & Applied)

CLASS TEST No. 1 : MARCH 2010

AM3.4 (DIFFERENTIAL GEOMETRY)

AVAILABLE MARKS : 52
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1.

(a) Define the following terms : *level curve, parametrised curve, regular curve, unit-speed curve, arc-length, reparametrisation.*

(b) Calculate the *arc-length* of the logarithmic spiral

$$\gamma(t) = \left(\mathrm{e}^t \cos t, \mathrm{e}^t \sin t\right)$$

from $\gamma(0)$ to $\gamma(t)$.

(c) Which curves have unit-speed reparametrisation ? Give a unit-speed reparametrisation of the circle

$$\gamma(t) = (a + R\cos t, b + R\sin t)$$

about (a, b) with radius R.

[6,7,7]

Question 2.

(a) Let $\kappa : (\alpha, \beta) \to \mathbb{R}$ be any smooth function. Show that there is a unit-speed curve $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ with signed curvature κ.

(b) Use the *Fundamental Theorem* (for plane curves) to show that any plane curve γ with constant curvature κ can be obtained from a circle of suitable radius by a rigid motion.

[7,7]

Question 3.

(a) Given the unit-speed curve γ, assume the *Serret-Frenet formulas*

$$\dot{\mathbf{t}} = \kappa\,\mathbf{n} \quad \text{and} \quad \dot{\mathbf{b}} = -\tau\,\mathbf{n}.$$

Hence, derive the equality

$$\dot{\mathbf{n}} = -\kappa\,\mathbf{t} + \tau\,\mathbf{b}.$$

(b) Compute the *curvature* κ, the *torsion* τ and the vectors of the Serret-Frenet frame $\mathbf{t}, \mathbf{n}$ and $\mathbf{b}$ for the (unit-speed) curve

$$\gamma(t) = \left(\frac{4}{5}\cos t, 1 - \sin t, -\frac{3}{5}\cos t\right) \cdot$$

[6,12]