RHODES UNIVERSITY
DEPARTMENT of MATHEMATICS (Pure & Applied)
CLASS TEST No. 2 : MAY 2010

AM3.4 (DIFFERENTIAL GEOMETRY)

AVAILABLE MARKS : 54
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1.

(a) Define the terms *principal curvature* and *principal vector* (of a smooth patch) and *normal curvature* (of a smooth curve). Hence, calculate the principal curvatures of the circular cylinder

$$\sigma(u,v) = (\cos u,\, \sin u,\, v)\,.$$

(b) Let γ be a curve on a surface patch σ, and let κ_1 and κ_2 be the principal curvatures of σ.

i. Give a formula for the normal curvature of γ in terms of κ_1 and κ_2.

ii. Hence, use this formula to prove that the principal curvatures (at a point of a surface) are the maximum and the minimum values of the normal curvature of all curves on the surface that pass through that point.

(c) Compute the *normal curvature* of the circle

$$\gamma(t) = (\cos t,\, \sin t,\, 1)$$

on the elliptic paraboloid

$$\sigma(u,v) = \left(u,\, v,\, u^2 + v^2\right).$$

[8,9,10]

Question 2.

(a) Define the terms *gaussian curvature* and *mean curvature* (of a surface patch). Hence, calculate the gaussian and mean curvatures of the surface

$$\sigma(u, v) = (u + v,\, u - v,\, uv)\,.$$

(b) Let κ_1 and κ_2 be the principal curvatures of a surface patch σ, and suppose that $|\kappa_1|, |\kappa_2| \leq C$. Let σ^λ be the corresponding *parallel surface* of σ, where $|\lambda| < \frac{1}{C}\cdot$ Prove that the principal curvatures of σ^λ are

$$\frac{\kappa_1}{1 - \lambda\,\kappa_1} \quad \text{and} \quad \frac{\kappa_2}{1 - \lambda\,\kappa_2}\cdot$$

Hence, derive the gaussian and mean curvatures of σ^λ.

(c) Show that if σ is a surface patch with constant non-zero mean curvature H, then for $\lambda = \frac{1}{2H}$, the parallel surface σ^λ has constant gaussian curvature $4H^2$.

[12,10,5]