RHODES UNIVERSITY
DEPARTMENT of MATHEMATICS (Pure & Applied)
CLASS TEST No. 1 : MARCH 2013

MAT314 (DIFFERENTIAL GEOMETRY)

AVAILABLE MARKS : 54
FULL MARKS : 50
DURATION : 1 HOUR

NB : All questions may be attempted.

Question 1.

(a) Define the terms *regular curve, reparametrization* and *unit-speed curve.* Is the parametrized curve

$$\gamma(t) = \left(\frac{4}{5}\cos t,\ 1-\sin t,\ -\frac{3}{5}\cos t\right)$$

unit-speed ? Justify your answer.

(b) Prove that a parametrized curve has a unit-speed reparametrization if and only if it is regular.

(c) Show that

$$\gamma(t) = \left(\cos^2 t - \frac{1}{2},\ \sin t\cos t,\ \sin t\right)$$

is a parametrization of the (curve of) intersection of the circular cylinder of radius $\frac{1}{2}$ and axis the z-axis with the sphere of radius 1 and centre $\left(-\frac{1}{2}, 0, 0\right)$.

[6,12,12]

Question 2.

(a) Define the terms *curvature* and *torsion* for a regular (not necessarily unit-speed) space curve. Hence state (but DO NOT PROVE) the *Fundamental Theorem for Space Curves.*

(b) Let γ be a regular curve in $\mathbb{R}^3$ with nowhere vanishing curvature. Prove that the image of γ is contained in a plane if and only if (the torsion) τ is zero at every point of the curve.

[8,16]