

RHODES UNIVERSITY  
DEPARTMENT of MATHEMATICS (Pure & Applied)  
CLASS TEST No. 2 : OCTOBER 2014

MAT314 (DIFFERENTIAL GEOMETRY)

AVAILABLE MARKS : 54  
FULL MARKS : 50  
DURATION : 1 HOUR

NB : All questions may be attempted.
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Question 1.

- (a) Define the following terms: *surface patch*, *atlas* and *smooth surface*.
- (b) Let  $U$  and  $\tilde{U}$  be open subsets of  $\mathbb{R}^2$  and let  $\sigma : U \rightarrow \mathbb{R}^3$  be a regular surface patch. Let  $\Phi : \tilde{U} \rightarrow U$  be a bijective smooth map with smooth inverse map. Show that

$$\tilde{\sigma} = \sigma \circ \Phi : \tilde{U} \rightarrow \mathbb{R}^3$$

is a *regular surface patch*.

- (c) Show in detail that the unit sphere

$$\mathbb{S}^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$$

is a *smooth surface*.

[5,8,12]

Question 2. Show that, if the surface patch  $\sigma$  is the generalized cylinder

$$\sigma(u, v) = \gamma(u) + v \mathbf{a},$$

then

- (a) the curve  $\tilde{\gamma}(u) = \gamma(u) - (\gamma(u) \bullet \mathbf{a}) \mathbf{a}$  is contained in a plane perpendicular to  $\mathbf{a}$ .
- (b)  $\sigma(u, v) = \tilde{\gamma}(u) + \tilde{v} \mathbf{a}$ , where  $\tilde{v} = v + \gamma(u) \bullet \mathbf{a}$ .
- (c)  $\tilde{\sigma}(u, \tilde{v}) = \tilde{\gamma}(u) + \tilde{v} \mathbf{a}$  is a reparametrization of  $\sigma$ .

[4,4,3]

Question 3.

- (a) Explain what is meant by saying that a diffeomorphism (between surfaces)  $f : \mathcal{S}_1 \rightarrow \mathcal{S}_2$  is *conformal*.
- (b) Find all smooth functions  $f$  for which the surface patch

$$\sigma(u, v) = (u \cos v, u \sin v, f(u))$$

is conformal.

[3,15]

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