Integration within and across mathematics

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Introduction

What does "integration" in the curriculum mean, in principle, and what does it mean for the learning and teaching of mathematics in school? In this paper, we will grapple with these questions. We do so from the perspective of our collective knowledge and experience, working across research and development projects in mathematics education.[1]

In recent years the principle of knowledge integration has been central in much of the general education documentation, as well as more specific curriculum policy documentation that has been produced and disseminated in South Africa. Integration in and for education first emerged in this country in the early 1990s although the Peoples' Education Movement had subscribed to such principles long before this (Julie,1991). At this time new policy initiatives investigated the separation of education and training in South African education and began to argue for greater alignment between workplace competence and informal knowledge on the one hand, and the more formal knowledge acquired in school on the other. The motivation for closer links between education and training was related to a very large work force, who had terminated their schooling after a few years because of apartheid education. Many had become enskilled through employment but were blocked from promotion possibilities because they had not successfully completed the required level of formal schooling. A system was thus needed that would enable articulation between vocational and formal education, and that would recognise knowledge and skills acquired across different contexts (NEPI, 1993; ANC, 1994).

As policy investigation continued, motivations and explorations diversified and deepened. Contradictions emerged, possibly the greatest being the separation of Training from Education by placing it within the Ministry of Labour and not Education. It is beyond the scope of this paper to delve deeply into all the policy developments that are part of the context of this paper. Jansen (1999) provides a historical overview of the development of curriculum for post-apartheid South Africa. We will focus, rather, on current interpretations of "integration" as these have materialised in policy and curriculum documentation surrounding Curriculum 2005 and insofar as they relate to mathematics.

Despite the fact that some of the design features of Curriculum 2005 such as phase and programme organisers are likely to be dropped, integration remains one of the driving principles of South Africa's outcomes-based curriculum. This is a response to traditional curriculum practices where knowledge that is transmitted and acquired in schools tends to be fragmented, abstract and inert. The boundaries around subject areas in the current curriculum are to be either blurred or fully collapsed in the new curriculum through the establishing of Learning Areas, and integration is to take place across these learning areas. The collapsing and blurring of boundaries is further supported by conceptualising knowledge, skills and values as outcomes rather than content-laden inputs.

Although it might appear that mathematics has not been affected by the learning area model, this is not the case. The extended name of the "maths" Learning Area: Mathematical Literacy, Mathematics and the Mathematical Sciences (MLMMS) indicates that the boundaries have been collapsed between pure and applied mathematics, and statistics. The name also suggests that the boundary around formalised mathematics has been blurred and mathematical literacy is advocated as a desired democratic competence. MLMMS defines mathematics as:

> ... the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a *human activity that deals with patterns, problem-solving, logical thinking*, etc., in an attempt to *understand the world and make use of that understanding*. This *understanding is expressed, developed and contested through language*, symbols and

[1] This paper has its origins in a workshop for all mathematics education staff at Wits, those in the departments of mathematics and education, and those in RADMASTE. It has since been presented at a Gauteng Institute for Curriculum Development (GICD) seminar, and discussed as part of a Grade 7 Curriculum 2005 implementation workshop in the Gauteng Department of Education.

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social interaction. (Department of Education, 1997: 2 (MLMMS); our emphasis)

MLMMS thus aims to induct learners into a broader field of mathematics than the current mathematics curriculum. This will enable connections across various components of mathematics, and between mathematics and real world problem solving. MLMMS shifts from a narrow focus on mathematical abstraction, to an increased emphasis on mathematical meaning—learning in school is to be a meaningful affair, where learners can make sense of and use the knowledge, skills and values that have been encoded into the curriculum. In this sense, then, school learning is to become relational, shifting from its current production of inert knowledge, to knowledge that is meaningful, transferable and flexible. This means that integration must take place at several levels: integration of the various components of mathematics; between mathematics and everyday real world knowledge; and, where appropriate, across learning areas.

One interpretation of the principle of integration is in relation to the notion of "relevance", and to connecting mathematics with the day-to-day experiences of learners and with the mathematical meanings they bring to class—and hence to the notion of learner-centredness. In the past, the fragmented curriculum has been tied to a pedagogy where the teacher selects knowledge and demonstrates (usually by working through examples with the class), and learners then copy, practice and apply it to further examples. The emphasis here is on the reproduction and application of given procedures. Little emphasis is placed on connecting with learners' meanings or with day-to-day practices, both of which are essential elements of a learner-centred pedagogy.

This is all known—at least at the level of policy and advocacy. Our purpose in this paper is to open up for debate the issues involved when integration within and across mathematics takes shape in two domains of curriculum implementation: the day-to-day practices of school mathematics classrooms and mathematics teacher education activity.

We begin the paper with an introduction to who we are and where are we coming from. The substance of the paper follows in the form of three examples of what we call *integration interpretation*. As we describe and then analyse each example, we pose the questions of desirability on the one hand, and feasibility on the other. Is integration, as teachers interpret it, desirable? Is it feasible? We then briefly point to key theoretical issues around integration that inform our experience and hence our position. Thereafter we conclude the paper with an interpretation of what integration within and across mathematics means for teaching and learning mathematics in school, and the demands it makes on teachers. We hope that the questions and issues raised in our paper provide debate firstly within the covers of *Pythagoras* and thus within AMESA, and then beyond.

Who we are

In April 1999, a number of the staff and graduate students involved in Mathematics Education at the University of the Witwatersrand came together for a full day "indaba". One of our foci was the notion of "integration" and what we thought it meant, both in terms of policy and practice in mathematics education. We were particularly interested in the issue of integration since mathematics in the school curriculum, while now including mathematical literacy and the mathematical sciences, remains a separate learning area. The issue of integration might then be understood as simply (sic) working across learning areas through themes. Yet, our collective experience was also suggesting that there were significant integration issues within mathematics as a learning area itself. In particular, most of us had experienced various difficulties in "pulling mathematics out" when working within a theme (e.g., AIDS and farming). Many of us had also experienced difficulties and observed teachers with similar difficulties in attempting to integrate across school mathematics and mathematics in everyday life. Hence the title: Integration within and across mathematics. We emerged from a lively and mutually informative two hour discussion, resolved to produce and develop the points we had discussed. This paper is the outcome of that process, taken forward by the three authors, but, as with most knowledge, owes a debt to all participants at the indaba.

The mathematics education staff and graduate students at Wits bring together a diversity of activity, experience and expertise. Some staff lecture on the formal programmes, degrees and courses in mathematics education in the University and supervise graduate students' research. Included in these programmes are both pre- and in-service mathematics education courses for teachers. Through the formal teaching programme, and related research (both directly and indirectly linked to this teaching), staff have engaged with and are building an understanding of the issue of integration theoretically and practically. Practical understanding has grown through working with teachers on the one hand, at both pre- and in-service levels, and through research in mathematics classrooms.

Other staff work mainly in the RADMASTE Centre, with more non-formal in-service professional development on the one hand and school mathematics curriculum development on the other. Their experience and growing expertise derives from many hours spent in school-based teacher development support, in running workshops for mathematics teachers across the country, and in developing and testing learning support materials.

Our first story is drawn from our experience with pre-service mathematics teachers with a focus on materials development. It was this experience which motivated us to look further into the issue of integration and thus to investigate the issues with practising teachers. The second story provides insight into a Grade 7 teacher's attempts at integration *across the curriculum* as seen within a research project related to the formal in-service Further Diplomas in Education programme. The third story derives from the RADMASTE context, specifically a non-formal in-service professional development programme run through the centre. It describes one lesson during which a Grade 7 teacher works towards integration *within* mathematics.[2]

Preservice mathematics teacher education

In this story we relate some of our experiences with a group of final-year pre-service teacher education students enrolled in the postgraduate Higher Diploma in Education (HDipEd). All the students in this group were graduates with several students having honours degrees in mathematics or mathematically-related disciplines.

Students were given the task of developing an integrated theme-based learning programme covering 12-15 hours of classwork. The learning programme had to integrate maths with at least two other learning areas, had to be adaptable to various school contexts in South Africa and therefore had to work within a restricted budget. Assessment strategies had to focus on critical and specific outcomes, including assessment criteria and performance indicators. Students worked in groups of three and chose a variety of themes which reflected their combination of teaching subjects. The themes chosen were: *Food and nutrition; Bushmen; Developing, marketing and selling a product;* and *The world we live in.* Although the project was to be of an integrated nature, it was initiated within a mathematics course and therefore students were expected to foreground mathematics more than might have been necessary had this task been initiated within an education theory and practice course.

The groups struggled to integrate maths into their themes with some groups achieving greater success than others. The maths topics covered in the various themes included ratio, integers, percentage, profit and mark-up, scientific notation, conversion between SI units, area, volume, plotting of co-ordinates in the Cartesian Plane and statistics. In some places, the integration was superficial and was restricted to textbook-style word problems relating to the theme. In other places the mathematics was contextualised appropriately: for example, ratio in recipes, application of area and volume in packaging products, and the use of scientific notation in representing distances between planets.

Students experienced difficulty in trying to teach new mathematical concepts within the theme. For the most part, the mathematics content was restricted to concepts that learners should have already learnt and could now apply to the given context. However, there were some instances where students attempted to introduce new mathematics concepts. One of the more successful attempts came from the "food" theme where simultaneous equations were introduced through a shopping context. These mathematics activities resembled those that have been developed by the Realistic Mathematics Education (RME) movement (see for example, de Lange, 1996) which was impressive since the students were not familiar with the work of RME. However, in contrast to RME, they used the contexts to develop and teach specific strategies for solving simultaneous equations as opposed to allowing learners to develop their own initial strategies.

Another group designed an activity in which they superimposed the Cartesian Plane on a world map to provide a different frame of reference. The point at which the Greenwich Meridian and the equator intersect was used as the origin and so the northern hemisphere formed quadrants I and II and the southern hemisphere quadrants III and IV. Learners were then required to use Cartesian co-ordinates instead of lines of latitude and longitude to do various map-related activities.

[2] It should be noted that each of our stories took place when there was much emphasis on specific outcomes, assessment criteria, range statements, and phase and programme organisers in the development of learning programmes.

Students generally had difficulty in addressing social, political, cultural and economic aspects of their themes. For example, the food and nutrition theme did not address the issue of culture in relation to food. The students in this group were all B.Sc. graduates which might account for their particular approach to the theme. Students with a background in the humanities would no doubt have approached the theme in a very different way. However, it is an issue of concern that the group did not address an aspect that is so obviously central to the theme that they had chosen. Vithal (1997) reports that her pre-service students experienced similar difficulties in addressing these types of issues.

Of the four themes that were chosen, some clearly provided greater possibilities for developing mathematical concepts than others. The way in which the "Bushmen" theme developed did not lend itself easily to learning much meaningful maths. As a result, students tended to impose a "mathematical gaze" (Dowling, 1995) on the culture and lifestyle of the bushmen in order to include mathematics in the theme. Although this group had shown cultural sensitivity in most aspects of their project, their attempts to incorporate maths in a culturally sensitive way were not always successful.

It is possible that several of the difficulties that students experienced in developing their integrated learning programmes can be traced back to their own histories of learning mathematics. The proliferation of textbook-type word problems in their learning programmes, albeit original and creative ones, reflects a history where applications did not extend beyond "dressed up" word problems with a real-world veneer. This type of mathematical experience is possibly similar to the experiences of other mathematics teachers in this country.

While the possibilities of creating exciting and relevant integrated learning programmes suggest that integration is desirable, the difficulties which the students faced raise questions about both the desirability and the feasibility of integrating maths across the curriculum. This was a highly motivated group of students who were committed to producing materials of an exceptionally high standard. They had a wealth of resources (in terms of both materials and knowledge) at their disposal, not least of which was a sound understanding of the mathematical concepts which they were attempting to integrate into their themes. In addition, the students were continually aware of the need to extract mathematics from their themes and sub-themes. Despite all this, they experienced enormous difficulties in producing conceptually sound, integrated materials. The maths tended to "get lost" in several instances. Ironically, the group that integrated maths most successfully, set up the learning programme in such a way that subject domains were clearly identifiable in each lesson. Thus, while the learning programme formed an integrated whole, individual lessons focussed on specific issues that were located in specific subject domains. The groups that attempted more integration within each lesson tended to be less clear about the goals of the lesson. This suggests that integration might only be feasible, from the point of view of mathematics, if one takes mathematics as the starting point and then proceeds towards themes and integrated programmes. Failure to do this results in losing the mathematics within the theme and hence mathematical goals might not be achieved.

The FDE Research Project

Mrs Shongwe (pseudonym) is an experienced Grade 7 mathematics teacher. She passed Grade 10 mathematics at school, and completed a Foundation Mathematics course during her initial teacher training. She attended a number of mathematics in-service workshops prior to joining the FDE programme. She graduated with her FDE (Mathematics Teaching) in December 1997 and enjoys learning more mathematics.

Mrs Shongwe teaches in a primary school in the Northern Province in South Africa, in a relatively poor, small township. Her school is a "can do" school with a good reputation. It has basic physical infrastructure, a qualified, stable and collegial staff, and reasonable teacher : learner ratios (1:35). There are, however, no extras, for example, no staff room, no photocopier, no science laboratory. The main language of most of the teachers and learners is TshiVenda. English is an additional language, but also the language of instruction.

In the third year of the research project, 1998, at a time when Mrs Shongwe had completed her FDE studies, she was observed in her Grade 7 classes over four consecutive days. In broad terms she was working with "statistical graphs" during that time. The first lesson was centred on the collection, categorisation (sorting) and counting of garbage in homes/school. Learners were asked to collect garbage in their homes or the school grounds. In the lessons that followed learners worked in groups to record the information in a bar chart, to discuss in their groups why they thought there was more fruit waste than other garbage, to evaluate their contributions as individuals and as a group, and then to evaluate the graphs of other groups. Mrs Shongwe's goals were that the sorted garbage would be a stimulus not simply for drawing bar charts, but for integrating mathematics with environmental issues, thereby integrating across subjects (science and English language), and for integrating self, group and peer assessment into the task at hand.

Mrs Shongwe asked learners to "classify" the garbage that had been collected, but as she continued with the instructions of what to do, she ended up providing the classifications (fruit waste, tins, paper, plastics) herself. One particular incident is important to point out here. Amongst the waste was a polystyrene plate. There was confusion as to how to classify this: was it paper or plastic? A quick vote was taken and it was placed in the plastic pile—a pragmatic strategy, perhaps, to getting on with the task and producing the bar chart but there is another crucial issue here regarding the status of knowledge. Although the vote happened to produce the correct solution, there was no guarantee of this. As the teacher, Mrs Shongwe did not indicate whether she agreed with the outcome of the vote or not: perhaps she was unsure herself, perhaps it did not matter to her since this was not the focus of the lesson. What is of concern, however, is what learners might take from the voting approach—"it's OK to vote when you don't know the answer and you can rely on a vote to produce the correct answer". It is these kinds of difficulties that emerge when we bring the everyday world into the mathematics classroom. Suddenly the teacher is expected to possess a broad general knowledge of matters unrelated to mathematics and possibly also to be an expert in other subject areas. This is clearly seldom possible and might leave the teacher feeling powerless to cope with the new demands. Thus teachers might resort to voting-type strategies as Mrs Shongwe did in the interests of time and other such pressures.

During the lesson described above, individual learners who were called up to carry out the classification had no difficulties producing four discrete piles. Each group then counted and recorded the number of pieces in each pile which were then to be represented in a bar chart. However, moving from the classification to its graphical representation was not easy:

T: We are going to make a graph, you understand?
Ls: (In chorus) Yes.
T: You are going to see in the graph which one is more ... tell me (pointing to the amount of each class of waste had recorded on the board) which one is more
Ls: (In chorus) Fruit waste.
T: Fruit waste?
Ls: (In chorus) Yes.
T: What does this tell us? For the family that has all th things there in their dustbin. If there is lots of fruit wa what does this tell us?
Ls: (some) The family eats more [i.e., lots of] fruit.
T: We are going to see it in a graph. Take out your penci your rubber, ruler ...

She then demonstrated how to set up th vertical axis and scale. She did not focus on th reasons for doing so.

T: The distance of the points must be equal. So if you decid you want the space to be this size (holding up a ruler an pointing to a 2 cm length) then you must have 2 centimetres equal to one centimetre [i.e., 1 unit] ... If yo want the distance to be more you can have 3 centimetre equals 1 ... (pause) 1 centimetre [i.e., 1 unit]. Yo understand?
Ls: (In chorus) Yes!
T: The distance you want is allowed ... any distance. So what distance do you want to use? They must be equal.

With that, learners were instructed to draw their graphs in their groups.

It is interesting to note a few issues that are hidden in the task, and remained unexplored. There was no discussion as to what constituted a "unit" of waste. It was assumed that any "piece" of peel, or paper or plastic was a unit in itself. Does this matter? It is only on re-examination of the video-tape amongst the authors here that this emerged as an interesting point of reflection. The issue is that in graphing quantities, the units being counted are critical. While the task does not depend on this here, it in fact opens the way for this important discussion in mathematics and graphing of quantities. The point here is that bringing in everyday materials like waste enables and provokes important mathematical issues for discussion and hence an argument for the inclusion of the real world in the mathematics curriculum. But it requires much of the teacher to first "see" and then be able to "exploit" such opportunities.

As learners started on the task of constructing a graph in their groups, Mrs Shongwe moved around the class to see what learners were doing and to assist them. She quickly realised that many learners did not understand what they were meant to be doing and so she returned to the board to repeat instructions about setting up the axes and the vertical scale, and moved further to demonstrate how the graph was to be drawn. She attempted to involve the class in her demonstration through rather typical question and answer strategy where learner answers were restricted to mainly "yes" or "no". For example: "We can show the amount of fruit waste here, ne?" She ended up drawing all four bars on the board when she could have just done one or two and got the class to try the rest on their own.

Eventually, most groups were able to work from her demonstration to produce a bar chart on poster size paper. However, the task was reduced to copying from the board. Moreover, the wider goals of "seeing" relationships in the graph did not extend beyond "seeing" that "fruit waste was more".

What we can see as the week's activities unfolded is that Mrs Shongwe faced a number of new challenges. Learners did not know how to construct an appropriate scale for the graph. Thus (aside from the issue of the "unit" mentioned above) while the categorisation and counting of the various types of garbage was rather easy at Grade 7 level, learners struggled to represent this information on a bar chart. Mrs Shongwe introduced the class to the idea of horizontal and vertical axes and what needed to be measured along these lines. However, learners still had difficulties resulting in her finally drawing the scales and bars on the chalkboard for them to copy. The issue we wish to point to here is that in the whole week of activity around this innovative task, the mathematics focus on representation as a bar graph, was ultimately reduced to copying from the board.

Focussing on mathematics in integrated learning tasks is not straight forward. Mrs Shongwe had not anticipated the mediation she would need to do to convey the concepts of scale so that learners could understand. This led to difficulties with sequencing and task demands. For example, there was no sequence of tasks that could enable learners to come to terms with what different scales could do and what they meant, and this was what they were to construct for themselves. In the end the demands made on them were not mathematical—the mathematical task was reduced to one of copying from the chalkboard.

In Mrs Shongwe's attempts at integrating mathematics, science, language and everyday life, the purposes of integration appear to have taken precedence over cognitive demands, resulting in low-level demands across all three learning areas, particularly mathematics. Mrs Shongwe's activities were creative and relevant to her learners. Her programme showed evidence of addressing several critical outcomes as well as specific outcomes from other learning areas such as Natural Sciences (NS) and Language, Literacy and Communication (LLC). However, it was the outcomes of MLMMS that she had greatest difficulty addressing. Had this lesson formed part of a geography or biology lesson, it is likely that we would not be as concerned with the manner in which the learners copied the graphs from the board—after all, the mathematics teachers should be teaching learners about axes and scale! But this is precisely the problem, in dealing with real life issues, the focus has shifted from a sequence of mathematically-based tasks for developing learners' notions of units of measure, scale and data handling, to one of applying this knowledge. The learners were unable to apply the knowledge because they did not have it. It would appear that such integration is only feasible if thematic work is done in co-operation with other teachers and other subjects. This would alleviate the expectation that all teachers need to be experts in all areas and allow them to focus on developing learners' competences within each discipline. However, it does raise the question as to whether it is both feasible and desirable for all teachers to be working in an integrated way all the time.

RADMASTE Mathematics Inservice Project

This lesson took place in a Grade 7 mathematics class in Eldorado Park in October 1999 (the final school term). While the school can be classified as "traditionally disadvantaged", it is well resourced with a dedicated and enthusiastic teaching staff. The teacher, Mr Fons (pseudonym), had participated in a RADMASTE inservice course since January 1999. He participated enthusiastically in the course and showed a willingness to take up the challenges of the new curriculum on both a practical and theoretical level.

The lesson focussed on decimal fractions and Mr Fons began by explaining to the learners the structure of the lesson - that there would be four 15 minute sessions which would focus on different parts of the topic. He encouraged the class to help him keep to the time restrictions.

Session 1

Mr Fons had pre-prepared two different calendars on a flipchart—the Roman calendar with 10 months and the Julian with 12 months. He asked the class the differences between the calendars and then continued to discuss the history of both calendars. He focussed on helping the learners understand where the names of the months came from such as the prefix "oct" in October and asked the meaning of "oct". He asked them to name an animal in the sea with eight legs. In a similar way he explained the relationship between December and the number 10. It was clear from the flipchart that October was the eighth month of the year in the Roman calendar but the tenth month of the Julian calendar. Similarly December was originally the tenth month and is now the twelfth. He discussed historical background of these anomalies, referring to the influences of Julius and Augustus Caesar. This provided the introduction for the next session in the lesson.

Session 2

Mr Fons gave the class a photocopied worksheet which they worked on in groups. The worksheet had different grids of blocks. Learners were required to shade in $\frac{1}{10};\frac{1}{100};\frac{1}{1000}$ on each of the different grids. They then had to cut these out. He moved around checking that everyone was working, giving assistance where necessary and reminding learners about time management. He told them to cut out only one set per group to save time. They then used these cuttings to help answer questions about which fractions are bigger, smaller, the same etc. He did several of these exercises together with the class on the board and asked questions to guide them. He also asked questions which challenged learners to explain why they said that, for example, $\frac{1}{10}=\frac{10}{100}$ or $\frac{1}{10}=\frac{100}{1000}$. He played devil's advocate with them around the meaning of "equal". He explored the difference between mathematical equivalence (equal quantities) and everyday equivalence (things that look the same). He gave other examples such as two R1 coins are equal to one R2 coin but they are not the same.

After the discussion he left them to complete the activity on the worksheet. Each learner was to complete his/her own worksheet although they could work together in groups or pairs. He walked around all the time checking, encouraging and challenging.

Session 3

Mr Fons introduced a pre-prepared conversion table of decimals such as $\frac{1}{10}=0{,}1;\frac{10}{100}=0{,}10;\frac{100}{1000}=0{,}100$. He had colour-coded the tens and the hundreds digits on each side of the equal sign and used pattern recognition to help learners see the connection between decimals and fractions. He use a second way of seeing the connection between fractions and decimals by getting learners to use long division to see that $\frac{27}{10}=2{,}7$ and $\frac{36}{10}=3{,}6$, etc. He then extracted the pattern and rules for what decimals mean and how they can be read, i.e., tenths, hundredths, etc.

Session 4

He moved on to powers. He explained that "10 to the power of 10" means 10 x 10 x 10 x ...(ten times). He then challenged them by asking what 10^6 is. Some said ten million, he made them check. They worked in groups and in time learners said it was one million. He then wrote the pattern on the board and asked them to generalise about the number of "x" signs for each exponent.

$10^1 = 10$
$10^2 = 10 \times 10$
$10^4 =$
$10^{12} = ?$ How many "x" signs in this? Why do you say so. Can you explain a rule?

The lesson ended with a recap of what had been done, and getting learners to finish the worksheet on powers.

The lesson flowed very well and was well focussed. Learners participated actively and eagerly responded to questions. A healthy inquisitive mathematics stance was evident in the class and rich mathematics was being learnt. The integration was primarily within mathematics and not across learning areas. The integration came from the maths; that is, the explicit outcomes were mathematical and other critical outcomes supported these. The connections that were made within mathematics made for a mathematically rich lesson. The lesson was supported by a great deal of resources including supportive learning materials, good prior knowledge of learners' abilities, photocopied worksheets and time—a double mathematics period.

In reflecting on integration within this lesson, it is clear that Mr Fons integrated several specific mathematics outcomes in the lesson: understanding of number; manipulating patterns; history of maths; working with shape; using mathematical language; and justifying conjectures. While the focus of the lesson is on mathematical outcomes, these are supported by attention to several critical outcomes: identifying and solving problems;

working in groups; effective management of time; communicating in a variety of modes; and considering the "world as a set of related systems".

Desirable and feasible integration: issues from the field of practice

Whereas Mrs Shongwe and the HDipEd students took an integrated theme as their starting points, Mr Fons started with mathematics. He related and connected different aspects of decimals and he sequenced and graded a range of mathematical activities that lead to the development of concepts and the use of decimal fractions. This was located in a mathematical context but aspects of history and language were brought in where these were appropriate and meaningful to learners.

Our three stories highlight the fact that integration both within and across mathematics is no simple task. When starting with a theme as Mrs Shongwe and the HDipEd students did, there is a danger of the mathematics being overshadowed by the theme and while tasks might be challenging and meaningful to learners, they might not support conceptual development of mathematics. When one begins with mathematics and then explores links between the specific mathematics content and other learning areas, it is easier to maintain a mathematics focus but this occurs at the expense of extended integrated and thematic work. Either way there is a need to be able to zoom in on the mathematics and out again repeatedly, and it is essential to maintain a balance between the two.

In addition, we need to consider the diverse contexts of teaching and learning which we encounter in South Africa. Mr Fons was able to make connections within mathematics because his learners already "possessed" the mathematical knowledge and skills that he was linking to the new work. This made it possible for him to integrate successfully within mathematics. In Mrs Shongwe's class this was not the case. The gaps in learners' mathematical knowledge undermined her attempts at integration. Instead of being able to move on and focus on interpreting the data from the bar charts, she was forced to teach (in fact "demonstrate") how to set up the required scale and this detracted from the flow of her learning programme. There are many teachers in this country who face similar difficulties to Mrs Shongwe on a daily basis, having to cope with learners who have vast gaps in their mathematical knowledge.

In terms of what is desirable and what is feasible *in practice*, integration *within* mathematics is clearly desirable. But what of integration across the curriculum? Two of our stories suggest that it is not a simple task and perhaps it is not feasible on a practical level. Moreover, if it is accompanied by a persistent neglect of conceptual mathematical development and tasks with low mathematical demands, then it is also undesirable.

To this point, we have focussed on practice, on issues of interpretation of curriculum integration by various agents in mathematics education. We need, however, to extend our deliberations. Curriculum integration, learner-centredness and relevance are not new ideas in education. There is a vast body of educational theory and research in general, and mathematics education research in particular that continues to grapple with the issues of integration and relevance. For the purposes of this paper, we shall draw on the work of just three authors to elaborate at a theoretical level some of the concerns about integration that we have raised. In so doing we hope to bring theory, research and practice to bear on each other.

Discussion

Sociologists of education have enabled us to see that the means by which a society selects, transmits and evaluates the knowledge it considers worthwhile (so constituting the curriculum) is inevitably a reflection of relationships of power and social control in the society (Bernstein, 1971). As a sociologist of education, Bernstein's concerns are with how knowledge is distributed in and across society, who has access to education, who succeeds at school, and where and how issues of race, class and gender inequalities are produced. Those who do succeed are ultimately privileged in access to societal goods. The critical point here is that when curriculum transformation is on the agenda, as it is in South Africa right now, this is not simply a matter of selecting different knowledge and new forms of pedagogy, but more critically a matter of how these combine into new power relations and social control in the society. What is up for change are issues of access, success and who comes to benefit. With such power dynamics at stake, it is no wonder that curriculum change is such a highly contested matter and Curriculum 2005 is no exception.

In the 1970s, as Bernstein developed his sociology of education, there was a curriculum reform movement in the United Kingdom, and particularly a move towards integrated curricula and learner-centred practice. Bernstein developed (and has continued to refine) several concepts which are useful for an analysis of curriculum. Bernstein (1971, 1996) distinguishes between *integrated codes* and *collection codes* with respect to curriculum. A collection code is constructed by means of strong boundaries between subjects (what he calls "classification") and strong teacher control over aspects such as pacing and sequencing of content (what he calls "framing"). The "old" South African curriculum is, in many ways, an example of a collection code. Subjects are clearly demarcated in the curriculum. Strong boundaries around subjects are evidenced in the time-table and in subject departments within schools. In addition, the delivery of the curriculum, the selection and pacing of lessons is strongly controlled by the teacher (in fact in many ways by the pre-scribed textbook and matriculation examination).

By contrast, an integration code is characterised by weak classification and framing with boundaries between subjects being blurred or weakened (e.g., by the formation of learning areas where subjects are combined), and less rigid teacher control over pacing and sequencing (e.g., through a more democratic learner-centred pedagogy). The first two stories provide examples of interpretations of an integrated code as does Curriculum 2005.

What Bernstein helps us to see is that when we attempt to shift from a collection to an integrated code there are all kinds of consequences, both intended and unintended. For example, as mathematics teachers we have all been trained (particularly at the secondary level) into subject specialisation. Our identities in the school are closely tied to the subjects that we teach. We come to view ourselves and our competence as tied to our subject expertise. An integrated curriculum requires a new set of competences for us. As mathematics teachers, other issues emerge besides retraining. Mathematics currently has high status - we are often privileged in the school, in the location of mathematics in the time-table, in the rewards we get for good pass rates and so on. Perhaps some of our resistance to integrated curricula is that we will lose this status. Furthermore, in the mathematics classroom we are the experts, in control of how the curriculum is delivered, the concepts taught and the meanings attached. As we move to more learner-centred practices, we will be expected to enable and encourage learner meanings and new meanings. Are we comfortable doing this? When Mr Fons encouraged learners to make links between word prefixes and their meanings, he could quite easily have been "caught out" by a learner asking why October is the tenth month of the year but its prefix means "eight". Fortunately he had anticipated this issue and done the necessary research. Mrs Shongwe had not anticipated the problem of the polystyrene plate and had to make a decision on the spur of the moment. In both cases it is not the teacher's mathematical knowledge that is at stake but his/her wider knowledge. Either way, the situation might arise where "the teacher doesn't know the answer"—is no longer the expert. Repeated experiences such as these may lead to teachers losing confidence in themselves and learners losing confidence in their teachers. Will teachers simply retreat to their "comfort zones" and teach mathematics as they did previously because it is easier and less threatening?

It could be argued that the teacher should encourage learners to explore answers to their own questions and Curriculum 2005 promotes this approach. Consider for example the maths lesson on scientific notation from the HDipEd projects. In the middle of the lesson, a learner might ask the teacher about the size of the planets, or how and when they were created, or whether there are other galaxies besides the Milky Way. Assuming that most mathematics teachers would not be able to provide answers to some or all these questions on the spot, a natural response might be to tell learners to investigate for themselves. For learners who have access to libraries, encyclopaedias and/or the Internet, this is an appropriate response but what about those learners who do not have access to such resources? Is it the teacher's responsibility to do the research? Should the teacher ignore the questions, and with what consequences? And what about the mathematics?

Curriculum 2005 advocates a blurring of subject boundaries between learning areas and calls for a similar blurring of boundaries between school and everyday knowledge. As mathematics teachers we need to be aware of how these changes are affecting us, where our support and resistance is coming from and how this affects us.

We move now from Bernstein per se to a related but separate area of research. Research into links between mathematics in and outside of school is not a new area of interest, and has been diversely motivated. Pedagogical motivations revolve around attempts to render mathematics learned in school more meaningful—the assumption here is that if mathematical concepts are connected with learner experiences then they will be able to make more sense of the mathematics they are learning. Epistemological arguments are more around mathematics itself, and where and how it is and can be used both to solve real world problems, and to reflect critically on the world in which we live.

Several researchers have investigated links between school mathematics and everyday mathematics (see for example, Lave, Murtaugh and de la Rocha, 1984; Carraher, Carraher and Schliemann, 1985; Scribner, 1986; Lave, 1988; Saxe, 1990; Harris, 1991). This body of research has revealed that mathematical goals in school differ from the goals of everyday or work-related practice. A well rehearsed example is "best buys". When we are asked to decide on what will be a best buy (most cost-efficient) in the classroom, we are asked to consider the arithmetic of the problem and what kind of purchase is cheapest or best value. When we are buying in the store a whole host of other issues enter our decisions: the brands we like, what will fit into our storage space at home and so on. As a result, the mathematics that is learned in school is not readily used in out-of-school contexts. Thus research warns us that the notion of *transfer* of knowledge across contexts of use—which is at the heart of integration in the South African curriculum—needs to be explored in greater depth and not simply taken as a given.

Lave & Wenger's (1991) work helps us to think deeply about the issue of transfer. They argue that knowledge and skills cannot be neatly lifted out of one setting and imported ready-to-use into a new setting. Rather, in a new setting, the individual will call upon existing knowledge and skills if s/he deems them to be appropriate and then adapt these in order to achieve her/his goals within the new setting. In other words, we will only draw on our mathematical knowledge learned in school if such knowledge is appropriate to another setting and adaptable to its use there. Lave & Wenger argue that the development of knowledge and the formation of identity are closely linked to each other and both are tied to the setting in which they are acquired. This has important implications for the new South African curriculum. According to Lave & Wenger, a learner is unlikely to use school knowledge easily to solve everyday problems because everyday life and school life constitute two very different settings and the learner will have developed different identities in each of these settings. What Lave & Wenger's work pushes us to think about, is the nature of knowledge developed in school and its appropriateness and adaptability across settings. This is a tough call in a subject like mathematics because much of what constitutes the discipline is not applicable in any direct sense in everyday life. The question for us then is should our selection of mathematics into the school curriculum be restricted to those elements of mathematics that are applicable?

The question implies a context—people, place and purpose—since we cannot speak of application apart from context. Thus the response to the question must include "to whom?" and "for what purpose?" The mathematics that an engineer needs is very different to the mathematics required by a taxi-driver or the mathematics necessary to calculate bond repayments. Besides, learning mathematics is as much about particular content as it is about mathematical processes and thinking. It is not feasible to restrict the mathematics curriculum to content that is applicable. Take Mrs Shongwe's lesson as an example. The difficulties that she experienced were due largely to the fact that her learners did not have the mathematical knowledge necessary to apply to the integrated task. If at some previous stage her learners had learned about scale, axes and representation of data *as an end in themselves*, the difficulties in the lesson might have been avoided. To the learners such a lesson might have appeared "meaningless" from an applications point of view. However, once they had the knowledge, they would potentially be able to use it for the type of task that she had set up. While this might seem a trivial example, the need to lay foundations for more advanced mathematics at all levels appears to be essential for integrated work. The difficulties that the HDipEd students experienced in developing integrated activities to teach new mathematical content is further support for this claim.

The issue of mathematical applications is not just one of relevance and motivation, it is also about issues of access and equity. Mrs Shongwe cannot teach her Grade 7 learners only the mathematics that she thinks they will need in a rural area in the Northern Province. If she does this, they may be confined to that situation and with restricted access to further mathematics and even to further study in general since it is well known that mathematics acts as a filter to higher education—hence Bernstein's concerns about access and success.

At this point we need to clarify that our concerns about integration do not lead us to advocate that mathematics lessons should still consist of endless exercises of procedural tasks that learners must master before they begin to apply the principles that they have learned. We are merely cautioning against a pendulum swing that emphasises relevance and integration at the expense of conceptual mathematical knowledge and algorithmic and computational efficiency. Our third author, Boaler (1997), provides some insight into possibilities for increasing transfer of mathematical knowledge and skills from the classroom to everyday life. Unlike the previous two authors, her focus in specifically on mathematics.

Boaler (1997) argues that transfer is more likely to take place if learners learn mathematics in a more integrated way through a problem-solving approach. Boaler compared the mathematical experiences and achievements of learners in two working class schools in the UK, schools she named Amber Hill and Phoenix Park. The schools adopted very different approaches to teaching and learning maths. At Amber Hill, mathematics lessons consisted of rule-based, procedural activities with much drill and practice. In contrast, mathematics lessons at Phoenix Park were far more open-ended with learners spending most of their time working on problem solving tasks. As a result of these different approaches, learners gained vastly different experiences of mathematics and developed different forms of mathematical knowledge. The majority of learners at Amber Hill were unable to apply their knowledge to new problems and situations since the knowledge they had developed was rigid, consisting essentially of remembering and applying rules. Learners at Phoenix Park were able to cope with new problems far more easily since they had developed more flexible forms of knowledge. For them, mathematics did not consist of following rules—it required careful thought and the ability to apply what they already knew to the problem at hand.

Boaler's research suggests that transfer is more likely to take place when mathematics has been learnt in a setting such as Phoenix Park. There are several reasons for this. Firstly, in Bernstein's terms, the boundaries between school mathematics and everyday practices had been weakened. Secondly, learners had developed their own strategies to solve problems and had made connections between various parts of mathematics—an ideal example of integration within mathematics.

Our three stories do not provide an example of a Phoenix Park environment. However, Mr Fons' lesson provides a glimpse of how various aspects of mathematics can be connected to develop a network of related concepts. He helps learners to place decimal fractions within the broader decimal system, links to base ten calculations, to common and improper fractions and provides a visual representation of the fractions. We acknowledge that the learners might not yet have made the connections for themselves but the lesson is an example of how one teacher approached integration within mathematics.

Conclusion

We have described three stories to illustrate our concerns about integration within and across mathematics. Whereas both forms of integration are promoted in the new curriculum, it would seem that integration of mathematics across the curriculum is not feasible from either a practical or a theoretical point of view. Our first two stories provided evidence for the practical difficulties of attempting to integrate mathematics within themes without losing focus on the development of conceptual mathematics knowledge. At the same time, the work of Bernstein suggests that the changes that the new curriculum will bring with it, will impact far more than the content and pedagogy of the mathematics classroom: they will effect a shift in power relations between teachers and learners. Lave & Wenger argue against the fundamental assumption that integration and relevance will promote transfer of learning beyond the mathematics classroom. This all suggests that the promises of the new curriculum might be unfounded and that what might be desirable at the level of policy and advocacy might not be feasible at either a practical or a theoretical level.

Boaler provides evidence that integration within mathematics is not simple to achieve and needs to be explicitly developed through suitable classroom practices. The extent to which teachers and learners are able to cope with such practices will determine the extent to which learners will develop mathematical knowledge that is sufficiently flexible and robust to be applied to a variety of different problem situations within mathematics. There can be no doubt that integration within mathematics is desirable but this approach will place new demands on teachers. In order to succeed it will require more realistic time frames, assessment practices that incorporate integrated mathematical tasks, and with extensive teacher support.

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