LEARNING FROM PRACTICE: WHAT MATHEMATICS KNOWLEDGE IS NEEDED FOR DEVELOPING NUMBER SENSE?

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There is concern about poor learner mathematics performance in South Africa which, it is argued, begins from the Foundation Phase (FP). Research has identified contributing sources of the crisis to be a lack of both content and pedagogical skills of South African mathematics teachers. There is little research that examines the mathematical content and pedagogical knowledge of competent teachers in South Africa. These teachers possess particular knowledge that is not common among the majority of the South African mathematics teachers. In this respect, I considered it important to investigate, learn and understand from such teachers more about the knowledge they employ, and how they employ it in their teaching, that results in effective teaching and learning. This research attempts to understand the content and pedagogical knowledge that one 'expert' Grade 2 teacher employs, and the ways in which she employs them in her teaching, in order to highlight the mathematics knowledge for teaching that FP teachers need. The study found that FP teaching requires the knowledge and tactful employment of all six knowledge domains as described by Ball, Thames and Phelps (2008). In this paper, I illuminate these six knowledge domains in the teaching of counting to Grade 2 learners by one expert teacher.

Key words: counting principle; Horizon Content Knowledge; Knowledge of Content and Curriculum; decuple;

INTRODUCTION

Poor learner performance in mathematics has a long-standing record in South Africa. More than two decades after attainment of democracy South Africa is still seeking ways of addressing this crisis. Research around poor mathematics performance points to a number of factors, the dominant being that South African teachers lack both mathematics content and pedagogical knowledge to teach it effectively. Ball, Thames and Phelps (2008) refer to the knowledge to teach mathematics effectively as Mathematics Knowledge for Teaching (MKfT). MKfT combines the subject matter knowledge and the pedagogical content knowledge. Mathematics teachers in South Africa are said to lack the MKfT to teach mathematics in ways that enhance conceptual understanding and the effect of this deficiency is felt as far back in the education system as Foundation Phase. Research suggests Foundation Phase teachers do not develop the learners' number sense well enough to equip them with essential mathematical strategies and proficiency that would help them learn mathematics as they move up the grades. This deficit then expands as learners move up the grades.

As part of my Masters research, I conducted a qualitative case study research to investigate MKfT enacted in the teaching of an expert Foundation Phase teacher. The research focused on how she developed her Grade 2 learners' number sense particularly in relation to counting. A key aim was learning from the practice of an expert teacher the key aspects of MKfT required in developing number sense so as to inform fellow Foundation Phase teachers and Foundation Phase teacher educators, both in-service and pre-service. The broader study found that Foundation Phase teaching requires employment of all the domains of the MKfT to develop number sense to Grade 2 learners. These domains are complexly interconnected and interdependent and the research shows that while one needs the full set to be able to teach effectively, the expertise becomes visible in the seamless and somewhat automated interweaving of these domains.

In this paper, I share an excerpt from a case study of an expert teacher, which is analysed and unpacked to illuminate (1) the various domains of MKfT required in the teaching of counting and (2) their interconnectedness. Due to space limitations I only provide one such transcript of the teacher. I have chosen this specific excerpt from the transcript of one lesson for its richness and potential in illuminating the MKfT required by the teacher for the effective teaching of counting.

BACKGROUND TO THE STUDY

The problem of poor learner performance in South Africa is well documented (Fleisch, 2008; Spaull 2013; Taylor & Taylor, 2013; Graven, 2016). Research in South African mathematics education has identified a number of factors that contribute to this predicament of poor learner performance in mathematics however the teachers' lack of both content and pedagogy have been identified as the dominant factors. According to research the majority of South African mathematics teachers lack both the pedagogy and the content of the subject they teach and this has negatively affected the learners' performance from as early as Foundation Phase (FP) (Fleisch, 2008; Taylor & Taylor, 2013). As a way to contribute to the intervention strategies to this predicament, my study sought to explore what MKfT is required by Foundation Phase teachers to develop number sense through observing an expert teacher conducting counting lessons with her grade two learners. I investigated what MKfT she employed to successfully develop number sense through counting to her grade two learners to inform both practice and teacher education. I chose to investigate FP teaching because research has shown that the trajectory of poor mathematics performance begins at the FP and worsens as the learners go up the grades (Fleisch, 2008; Graven, 2014; Spaull, 2013; Robertson & Graven, 2015). Graven, Venkat, Westaway & Tshesane (2013) state "emphasis needs to be placed on improving the fundamentals of instruction in earlier grades in order to reduce the large number of learners who lack basic knowledge of mathematics" (p. 5).

According to Graven et al. (2013), learners exit FP without a well-developed number sense, which makes it difficult for them to solve more complex mathematical concepts later in their schooling. It is argued that number sense and mental agility are critical for the development and understanding of algorithms and algebraic thinking introduced in the intermediate phase as it enables learners to think flexibly and promotes confidence while working with numbers (Graven et al., 2013, Carlyle & Mercado, 2012). Learners who lack a strong number sense have problems developing the foundation needed to do simple arithmetic (Burns, 2007). Naudé and Meier (2014) refer to number sense as a "foundational building block for all content areas in mathematics" (p. 79). I use Ball et al.'s (2008) MKfT framework to investigate what knowledge an expert Grade 2 teacher has and uses in her teaching to develop number sense through counting.

Ball's Conceptualisation of Mathematics Knowledge for Teaching

In analysing the mathematical demands of mathematics teaching there is a need to identify the mathematical knowledge that is required in the work teachers do. This is possible through studying and identifying the mathematical knowledge utilised in the regular day-to-day, demands of teaching. Ball et al. (2008) propose that doing a job analysis, where one observes teachers carrying out the work of teaching, and "asking expert mathematicians and mathematics educators to identify the core mathematical ideas and skills that teachers should have" (p. 395), may be useful to ascertain what knowledge is required for the effective teaching of mathematics.

The concept of MKfT was developed by Ball et al. (2008), building on Shulman's (1986) notion of PCK. Ball et al. (2005) conducted an interactive work session to investigate the mathematical knowledge and skills that are needed in the teaching of mathematics. Focusing on what teachers do while teaching, they managed to identify six knowledge domains that are essential in the teaching of mathematics. These are: Common Content Knowledge (CCK); Horizonal Content Knowledge (HCK); Specialised Content Knowledge (SCK); Knowledge of Content and Teaching (KCT); Knowledge of Content and Students (KCS) and Knowledge of Content and the Curriculum (KCC). Together these six domains are referred to as Mathematics Knowledge for Teaching (MKfT). Ball et al. (2008, 2009) and Kim (2013) define MKfT as the mathematical knowledge, skills, and habits of mind entailed in the work of teaching. The explanation of each of these knowledge domains and their indicators are summarised below.

Common Content Knowledge (CCK) is the general knowledge of mathematics and mathematical skills used by anybody who has done mathematics successfully at school. Teachers need this knowledge to understand the work they assign to their learners. CCK is indicated by for example, the teacher's ability to: calculate an answer correctly; understand the mathematics they teach; use terms and notations correctly; recognise when a student gives a wrong answer; or recognise when a text book is inaccurate.

Horizon Content Knowledge (HCK) is the mathematical knowledge that spans across the mathematics curriculum that helps the teacher to view mathematics as whole, but not in parts. HCK is reflected for example, in the teacher's ability to make connections across mathematics topics within a grade and across grades and to articulate how the mathematics one teaches fits into the mathematics which comes later.

Specialised Content Knowledge (SCK) relates to the special knowledge that is specifically required for the work of teaching such as to: interpret students' emerging and incomplete ideas; evaluate the plausibility of students' claims; give or evaluate mathematical explanations; use mathematical notation and language and critique its use; ability to interpret mathematical productions by students, other teachers or learning materials; evaluate mathematical explanations for common rules and procedures and to appraise and adapt the mathematical content of text books.

Knowledge of Content and Teaching (KCT) is the knowledge that combines knowledge of mathematics content and knowledge of teaching which reflects in one's ability to: sequence mathematical content; present mathematical ideas; select examples to take students deeper into mathematical content; select appropriate representations to illustrate the content; ask productive mathematical questions; recognise what is involved in using a particular representation; modify tasks to be either easier or harder; use appropriate teaching strategies; respond to students' why questions; choose and develop useable definitions and to provide suitable examples.

Knowledge of Content and Students (KCS) pertains to the knowledge that combines knowledge of mathematics content and knowledge of students. This kind of knowledge is mirrored when a teacher: anticipates what students are likely to think and do; predict what students will find interesting and motivating when choosing an example; anticipate what a student will find difficult and easy when completing a task; anticipate students' emerging and incomplete ideas and when one recognises and articulates misconceptions students carry about particular mathematics content.

Knowledge of Content and Curriculum (KCC) is the knowledge of the content requirements of the curriculum and the materials that can be used to teach that particular content. A teacher with this kind of knowledge is able to: identify the topics in the curriculum; articulate the competencies related to each topic in the mathematics curriculum; articulate and demonstrate a familiarity with the structure of the mathematics curriculum; link representations to underlying ideas and to other representations and also reflect knowledgeability of available materials (e.g. textbooks) and their purposes.

METHODOLOGY

In this research, I used an interpretive research orientation to investigate the MKfT enacted by an expert FP teacher. Cohen, Manion and Morrison (2011) view the interpretive research model as an approach that seeks to "understand and interpret the world in terms of its actors" (p, 28). I used the interpretative orientation with the aim of observing and interpreting the knowledge of teaching mathematics that Gail (pseudonym), my case study teacher, employed while developing number sense with

Grade 2 learners through counting. I used a case study approach as it provides an opportunity of gathering rich data through an in-depth study of a bounded system such as an activity, event, process, or individual (Ary, Jacobs and Razavieh 2006, Creswell 2009). In my study, Gail was my bounded system in whom I sought to understand her actions, thoughts, experiences in the totality of her environment (her Grade 2 classroom) to investigate what MKfT she enacts in her teaching of number sense through counting to Grade 2 learners. After receiving ethical approval by my university to carry out this research, I sought permission from the school, the teacher and the parents to conduct research with Gail. I explained the purpose of the research and that Gail's participation was voluntary and she could withdraw anytime she wished. Pseudonyms were used to protect the anonymity of the school, the teacher and the learners. Information gathered was treated with confidentiality

RESEARCH PROCESS

I observed Gail's fifteen mathematics lessons for four weeks focusing on counting sessions she conducted with the whole class, bottom group and the top group. Observations allowed me to collect information first-hand on the experiences of Gail as occurring in her classrooms. Observation thus offered an opportunity to gather 'live' data from naturally occurring classroom situations (Cohen et al., 2011).

Two structured interviews, based on video recordings of lessons, were conducted on a one-on-one basis with Gail in her classroom during the learners' absence. Informal interviews were also conducted during the course of the observations at the end of each counting session to enable Gail to interpret and clarify aspects of each of the counting lessons.

The observed lessons and interviews were video recorded and transcribed. The MKfT framework was employed to analyse the data collected. As stated earlier this study used the MKfT theoretical framework to investigate what aspects of MKfT Gail employed in her teaching. I worked with five of the six domains of the MKfT (that is the SCK, HCK, KCT, KCS and KCC) and compared them to the data gathered through the lesson observations, interviews and the analysis of the FP mathematics curriculum to determine the MKfT reflected in her teaching. I did not use the sixth domain of the MKfT (the CCK) because Gail's positioning as an expert teacher assumes her possession of CCK. Below I present excerpts from a selected lesson where Gail utilised her counting session to develop number sense in learners. In this particular lesson, I was interested in how Gail helped her learners to overcome a common counting error of confusing the teens and the ty numbers. According to Wright (2016) learners confuse '***teen***' & ***'-ty'*** numbers, for example they can count 23, 22, 21, 20, 90, 80, 70, 60 50 … Instead of 19, 18, 17 they say 90, 80, 70 etc.

DATA PRESENTATION AND ANALYSIS

Vignette 1: A whole class counting session from Lesson One.

The focus of this whole class counting session was counting in ones within the number range of zero to eighty. The counting session began with the learners counting in 1s while Gail moved single beads across the beadstring. She stopped the learners at various numbers and asked about the composition of those numbers (e.g. 'Who can tell me what thirty-four is made of?"). *Thereafter, the learners continued counting in 1s. As they approached the 'teen numbers', she slowed the counting down to emphasise the 'teen' sound in the 'teen' numbers (e.g. thir**t-ee-n**). Likewise, she slowed the counting down when the learners reached the decuples (A **decuple** is a multiple of 10 e.g. ten, twenty, thirty, forty etc. (e.g**. tw-en-ty**, **th-ir-ty**). She counted ahead with the learners for the next three numbers. The learners counted up to 50 and then backwards in 1s to 0. Thereafter they counted in 1s from 50 to 80.* *All the time Gail used the beadstring. After they counted forwards in 1s from 50 to 80, the learners counted backwards in 1s from 80 to 50. As the learners counted backwards Gail emphasised the move from the decuple (e.g. 70) to the next number (e.g. 69). As she did this she told the learners that they were now 'closing off Mr 70s house' and she used her hands to demonstrate this closure. She ended the counting by exploring with the learners the meaning of counting backwards and made the link between counting backwards and subtraction.*
Concepts developed in the counting session: Counting on Counting back Decomposition of numbers Relationship between counting back and subtraction

The 'teen' and 'ty' concept

At the beginning of the mathematics lesson, presented in Vignette 1, all the learners in Gail's class were seated in an orderly arrangement on the mathematics carpet. They were counting in ones. As they said each counting number name, Gail moved beads on the beadstring. Excerpt 1 provides a transcript and account of what transpired in this counting session. T is for the teacher i.e. Gail; TLL refers to the teacher with all learners speaking/counting together; LL refers to all the learners speaking/counting; while L indicates a single learner speaking/counting (although there are no examples of the latter in this excerpt).

Excerpt 1: Emphasising ‘teen’ and ‘ty’ numbers (L1V1)

	T	Okay boys! It is time for Maths. Let us go to the back carpet. Come on come on let us be quick. [Learners move to the carpet and sit down facing the teacher’s chair. The teacher stands next to the beadstring in front of the learners] Quiet boys! We are going to count forward in ones.
	TLL	[Tr pulling the beads on the beadstring one at a time as the learners count] One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve …
	TLL	[Tr reducing the tempo and emphasising the ‘teen’ sound as she counts with the learners] thir**teen**, four**teen**, fif**teen**, six**teen**, seven**teen**, eigh**teen**, nine**teen** [Tr then emphasises the**—‘**ty**’** sound as she counts with the learners] twen**ty**, twen**ty**-one, twen**ty**-two, twen**ty**-three …
	LL	[Tr keeps quiet] twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine, thirty, thirty-one thirty-two, thirty- three, thirty-four, thirty-four, thirty-five, thirty-six
	T	Stop! I was on thirty-six, I want you to listen very carefully. What two numbers do I need to make thirty-six? **Thirty**-six [Tr putting emphasis on thirty]
	LL	Thirty and six
	T	I need a thirty and a six! Good, let’s go on.
	TLL	Thirty-seven, thirty-eight, thirty-nine, forty
	T	Stop! How many tens do we have in forty? Julius?

Extracted from L1, V1

RESULTS AND DISCUSSION

In the counting session represented in Excerpt 1, Gail marks each counting word by moving a bead across the beadstring hanging above her small chalkboard at the mathematics carpet in helping learners to count meaningfully. Her counting sessions were influenced by her understanding that **“there are five principles of counting that govern meaningful counting. (FI2, V2, T73).** Gail chooses to use a beadstring to facilitate the principal of one-to-one correspondence, which is, matching each number name with a bead on the beadstring. Gail told me that her counting sessions were influenced by her knowledge that: “**Counting is not just a component of just rote**

count, spit out of your mouth and it means absolutely nothing, which is what a lot of teachers actually do. They think it's just count, count, count. They don't even know if children are actually counting the right number on the right word or whatever". She explains that counting **"must be one to one, it must be an uttering of the correct word on the correct number"**. Haylock and Cockburn (2008) concur that in learning to count learners should "learn to co-ordinate the utterance of the number word with the movement of the finger and the eye along a line of objects, matching one noise to one object until all the objects have been used up" (p. 41). The beadstring is, for Gail, an appropriate representation for developing the one-to-one principle as it affords learners an opportunity to coordinate Gail's movement of the beads with each number name. Each bead was given one count and one number name.

A further counting principle evident in Excerpt 1 is the *stable order principle*. The stable order principle upholds the consistence of the counting sequence (Gelman & Gallistel, 1978). Number names follow a stable order sequence such as one, two, three, and so on. This principle was generally emphasised in Gail's counting sessions. For example, in the excerpt 1 above Gail orders her learners to count in ones. She counts with them following the stable order of the counting numbers so that the learners can master the sequence of the number names. According to Naudé & Meier (2014) it is critical for learners to know that the number names maintain a consistent order in counting regardless of the counting strategy (be it forward or backward, counting in ones or skip counting). In particular, Gail was concerned with the learners counting in ones between ten and twenty. In Excerpt 1, we note how she chooses to count with the learners to remind them of the order of numbers between one and thirteen.

Gail's understanding of the counting principles and her ability to implement them in her teaching reflects her SCK and KCT. Knowledge of the counting principles is the SCK that is of particular importance to teachers, especially Foundation Phase teachers. The vignette above show that Gail draws on this knowledge while she teaches signifying that she is able to make the link between her SCK and KCT.

Gail, as suggested by the DBE (2011), avoided using one kind of resource all the time, but employed a variety of representations during her counting sessions such as the beadstring, dice, pairs of socks hanging in the classroom and paper hands on the wall. Each representation was chosen for a purpose and she managed to use them successfully as indicated by her class of learners being able to count correctly in relation to her movement of each bead. She showed understanding of what underlies the use of each representation. In this respect her KCS comes to the fore. Furthermore, Gail demonstrated that she is cognisant of available materials and their purposes in assisting her learners in developing in counting. In this respect, she not only demonstrated knowledge about content and teaching but also of content and curriculum as it is broadly understood.

As noted in Excerpt 1, each time the learners counted Gail started with them (as indicated in the TLL parts of the transcript) before she left them to count on their own in unison (LL). She chose to re-join the counting when she anticipated common counting errors or counting challenges. For example, in Excerpt 1 above, Gail counted with the learners up to twelve because the sequence up to twelve is arbitrary and can easily be confused or forgotten. According to Reys et al. (2007), "patterns facilitate the counting process" (p. 160) and it therefore is easier for learners to grasp the counting sequence when they identify its pattern. Gail confirmed that "**if a child can see patterns they can do maths" (FI2, V2, T103)** because **"Maths is a pattern. It's the same thing over and over".** This is confirmed by research which shows that during the early stages of learning to count, learners struggle to count in an accepted number word sequence from one to sixteen because there are no obvious patterns to the number names and their sequence (Reys et al. 2007). It is suggested that from thirteen onward learners can depend on the pattern of the number names such as ***thir***teen with ***thir*** standing for three, ***four***teen with ***four*** for four, ***fif***teen with ***fif*** for five to master the accurate counting sequence (Gifford, 2005).

The emphasis on the 'teen' numbers was not limited to this particular counting session. Gail emphasized the 'teen' numbers in a variety of other counting sessions with both the whole class and in small groups. Furthermore, Gail not only emphasised the 'teen' numbers but also those ending with 'ty' (e.g. twen**ty**). As noted in Excerpt 1, Gail counted with the learners in her class as they moved from the 'teen' numbers to the 'ty' numbers. The syllable 'ty' is emphasized as the learners count. At the beginning of the teen numbers Gail slowed down the counting speed to emphasize the 'teen' numbers, bridge the decade and counted with them the beginning of the twenties slowly to emphasize the decuple. When they got to the twenties in excerpt 1 she emphasized the '***ty***' sound once more and elaborated on the pattern that follows all decuples twen**ty-one**, twen**ty-two**, twen**ty-three** … thir**ty**, thir**ty-one, t**hir**ty-two..**.

Gifford (2005) alleges that learners often confuse the teen numbers and the decuple. She argues that learners "dovetail these two patterns together. Sixty, seventy, eighty, also sound like sixteen, seventeen, eighteen, which can be problematic to learners with hearing difficulties" (p. 79). Gail, anticipating this challenge and wanting to avoid its consequence, slowed down the counting in Excerpt 1 and emphasized the pronunciation of both the '*teen'* and '*ty*' numbers.

Central to ensuring that the learners in her class grasped the difference between the 'teen' and 'ty' numbers, Gail used questioning to encourage the learners to identify whether a number has a 'teen' or a 'ty' in it, for example, in one lesson learners were counting down from twenty, Gail stopped them at ten and asked "is it fourteen or forty?", "is it seventeen or seventy?" When the learners gave a correct response, she cautioned them to pronounce the words properly and continued with the counting by asking them to count down from twenty to ten once more. Learners eventually emphasised the '***teen***' sound when counting without her support. She assured that learners differentiated the spellings of the teen number words and the decades by asking such questions as who can tell the difference between the spelling of thirteen and thirty.

In many respects, Gail recognised that the pattern in the counting words which begins after twelve is an important aspect of learning to count. Not only did Gail realise that the patterns of the number words assist children with counting, but she also emphasised the use of mathematical language (that is, the number names) by encouraging the learners to differentiate between the ‘teen’ and ‘ty’ words. Her knowledge of the significance of patterns in learning mathematics and the emphasis on the mathematical language are indicative that Gail has knowledge related specifically to the work of teaching which is beyond the content knowledge expected of ordinary citizens (i.e. CCK). In addition, her emphasis on the ‘teen’ and ‘ty’ numbers are an example of Gail’s knowledge of the link between content and learners (KCS) as she was able to anticipate the errors that learners make and the typical challenges they have in learning to count. In assisting the learners with this possible difficulty, Gail demonstrated her KCT as she selected appropriate counting sequences (e.g. counting in 1s from 0 to 50), counted with the children when she deemed it necessary and asked productive questions in relation to the numerosity of the numbers that the learners named as they counted.

Gail demonstrated her mathematics knowledge for teaching counting during this counting session in many ways. Table 1 below provides a snapshot of Gail’s’ MKfT in relation to developing children’s knowledge of the distinction between ‘teen’ and ‘ty’ numbers. The indicators of each domain that Gail drew on during this and other counting sessions are also given in Table 1 below.

Table 1: Summary of MKfT domains employed relating to the concept of teen and ty numbers

MKfT Domains	**Indicators evident in Gail’s teaching of counting**
SCK	• Knowing that counting requires an understanding of the one-to-one principle, the stable order principle, and the cardinal value (numerosity) principle • Knowing that there is more to counting than rote counting • Knowledge that counting involves a general pattern beyond the number 12 • Knowledge of counting errors and challenges (i.e. distinguishing between ‘teen’ and ‘ty’ numbers)
KCT	• Sequences the counting sessions starting with 1s before skip counting, and counting forwards before counting backwards • Selects appropriate counting exercises (e.g. counting up to the ‘teen’ numbers before counting the ‘ty’ numbers) • Asks productive questions (e.g. what is the difference between the spelling of thirteen and thirty?)

	• Presents mathematical concepts accurately by emphasising the ‘teen’ and ‘ty’ numbers
KCS	• Anticipates that the students will find the ‘teen’ and ‘ty’ numbers difficult to distinguish • Recognises and articulates misconceptions learners carry about the ‘teen’ and ‘ty’ numbers
HCK	• Connects the topic of counting to that which the learners learned in the prior year
KCC	• Knows which instructional materials would be effective • Has a familiarity with the curriculum and the structure of the curriculum content? • Demonstrates the expectations from the mathematics curriculum • Knows what instructional materials are available, what approach they take and how effective they are

CONCLUSION

In this study, I found that KCT seemed to be the centre of MKfT. The study found KCT directly linking the teacher and the learner. Gail expressed KCC, HCK KCS, SCK through her teaching. The knowledge of the other five domains only become beneficial to the learner through KCT where Gail actually interacted with the learners and employed the knowledge of the other five domains to improve teaching and learning. For example, Gail’s knowledge about the misconceptions and errors made in their counting (KCS) was crucial to her teaching. She taught in a way that addressed possible errors prior to the learners even making the errors. For example, she knew that learners confused the ***teen*** and the ***ty*** numbers. She thus taught them in such a way that that this possible error would be addressed. She emphasised the ***teen*** sound on the teen numbers like thir***teen***, four***teen*** and the ***ty*** sound with numbers like thir***ty*** and for***ty.*** She also asked her learners what the difference was between four***teen*** and for***ty***. We thus inferred that the five domains (CCK, SCK, KCC, HK and KCS) influenced her KCT.

Although it may be difficult to determine where one knowledge domain begins and ends, Pinsky (2013) argues that “the importance of Ball et al.’s work resides not in clearly drawing the boundaries between them, but rather in establishing their existence” (p. 40). This analysis of Gail’s teaching establishes the existence of these domains and illuminates their interrelationship. Through my research, I realised that Gail was not always conscious of her MKfT. Her counting sessions were seamless and in many respects her MKfT was automated. Like an experienced driver, Gail did not stop to think about what MKfT she was drawing on at any particular moment in time.

This study found that the indicators that Ball et al. (2008) has established for each domain were not particularly useful in relation to the teaching of *counting*. In other words, while the domains were visible the indicators as stated by Ball et al. (2008) were not always evident in relation to the domains relating to counting. We surmise that those indicators may be better suited to studying the MKfT teachers draw on in teaching number operations and other mathematical concepts. Given this, we had to establish our own indicators by drawing on Ball et al.'s (2008) definitions of each of the domains. Most notably was the challenge in identifying Gail's SCK in relation to counting.

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