AN INVESTIGATION INTO THE MATHEMATICS KNOWLEDGE FOR TEACHING REQUIRED TO DEVELOP GRADE 2 LEARNERS' NUMBER SENSE THROUGH COUNTING

A thesis submitted in fulfilment of the requirements for the degree of

MASTER OF EDUCATION
(Mathematics Education)

of

RHODES UNIVERSITY

By

SAMUKELISO CHIKIWA

January 2017

DECLARATION

I, Samukeliso Chikiwa, hereby declare that the work in this thesis is my own and where ideas from other writers have been used, they are acknowledged in full using referencing according to the Rhodes University Education Guide to References. I further declare that the work in this thesis has not been submitted to any university for degree purposes.

------------------------------------	--------------------------------------
SIGNATURE	DATE

ABSTRACT

Poor learner performance in mathematics has a long-standing record in South Africa. More than two decades after attainment of democracy South Africa is still seeking ways of addressing this crisis. Research around poor mathematics points to a number of factors, however, the dominant being that South African teachers lack both mathematics content and the pedagogical knowledge to teach it effectively. Ball, Thames and Phelps (2008) refer to the knowledge to teach mathematics effectively as Mathematics Knowledge for Teaching [MKfT]. MKfT combines the knowledge of both the content with the pedagogical skills. Mathematics teachers in South Africa are said to lack MKfT to teach mathematics in ways that enhance conceptual understanding and the effect of this deficiency is felt as far back in the education system as Foundation Phase. Research suggests Foundation Phase teachers do not develop the learners' number sense well enough to equip them with essential mathematical strategies and proficiency that would help them learn mathematics with ease and understanding. This deficit expands as learners move up the grades. My qualitative research, case study approach was employed to investigate MKfT enacted in the teaching of an expert Foundation Phase teacher, which she used while developing number sense in her Grade Two learners. A key aim is to inform fellow Foundation Phase teachers and Foundation Phase teacher educators, both in-service and in-training, of the key aspects of MKfT required in developing number sense. The study found that Foundation Phase teaching requires employment of all the domains of the MKfT to develop number sense to Grade 2 learners. These domains are complexly interconnected and interdependent and the research shows that while one needs the full set to be able to teach effectively, the expertise becomes visible in the seamless and somewhat automated interweaving of these domains. Furthermore, the research will illuminate how such seamless and automated interweaving can render the individual domains difficult to discern.

ACKNOWLEDGEMENTS

The success of this study was not my individual effort. There are many people who have generously contributed to its accomplishment. I specifically acknowledge the following:

- First and foremost, I would like to thank God, the Almighty, for providing me the opportunity to carry out this research. I thank Him for life, strength, courage and wisdom to go through successfully. I praise Him for setting around me people who provided me with guidance and support that carried me through this study.

- I am indebted to my two supervisors, Lise Westaway and Prof Mellony Graven. Lise Westaway thank you very much for seeing the potential in me and working tirelessly to see me where I am right now and beyond today. I don't have enough words to appreciate all your efforts to develop me both academically and professionally and all the support that you went out of your way to provide. I believe God will reward you. Prof Graven, this work would not have been what it is without your experienced guidance and support, thank you a lot.

- My husband Doctor Clemence Chikiwa and our beautiful children Munashe, Rejoice Chikiwa; Thabo Makomborero Chikiwa and our twinkling little star Joylyn Ntsako Chikiwa, thank you so much for your unwavering support, your time and assistance as I walked through this journey. Clemence, thanks my love for setting the standard and encouraging me to go for it and push through it. I am following right behind you.

- My mother Constance Ossie Dube and my siblings Siphilisiwe and Mpilwenhle, thank you so much for your prayers and unending support and love.

- I also give my gratitude to Tawanda Wallace Mataka for all the support he provided tirelessly as I worked on this research. Thank you for helping transcribe the videos and for always being willing to put your work aside to proof read my drafts and give all the support that I needed. This journey would have been much harder without you.

- Nompilo Tshuma and family (including aunty Mpuza) thank you for all the support, particularly for collecting my children from school and taking care of them when my work tied me down. Your prayers and words of encouragement are greatly appreciated. You really made my burden lighter. You are such wonderful friends.

- To all my friends and family who were always there for me in many ways, thank you so much. Though I could not mention all of you by name, I really appreciate your support. God bless you all.

- Lastly but not least, I will render my sincere gratitude to my research participant Gail (pseudonym) for being there all the time to see this research through. Without you and your class this research would not have been successful. Thank you for not holding back any information and support that contributed to the success of this research. Thank you once more for working together with Lise Westaway to nurture my academic and professional growth. God greatly bless you.

TABLE OF CONTENTS

LIST OF FIGURES iv

LIST OF TABLES v

CHAPTER ONE 6

THE CENTRALITY OF THE TEACHER IN EXPLANATIONS OF LEARNER PERFORMANCE IN SOUTH AFRICA 6

1.1 INTRODUCTION 6

1.2 THE CRISIS OF MATHEMATICS EDUCATION IN SOUTH AFRICA 6

1.2.1 Mathematics education in South Africa: results from international research 7

1.2.2 Mathematics education in South Africa: national research 9

1.3 UNDERSTANDING WHY THE CRISIS EXISTS 11

1.3.1 Teachers' poor content knowledge as a contributor to the crisis 12

1.3.2 Poor pedagogy as a contributor to the crisis 13

1.4 REASONS UNDERPINNING TEACHERS' DEFICIT IN KNOWLEDGE FOR TEACHING MATHEMATICS 15

1.5 THE PROBLEM STATEMENT 16

1.6 THE RESEARCH GOALS AND QUESTIONS 17

1.6.1 Research goals 17

1.6.2 Research questions 17

1.7 OUTLINE OF THE THESIS 18

CHAPTER TWO 19

ELABORATING ON MATHEMATICS KNOWLEDGE FOR TEACHING AND THE IMPLICATIONS FOR MY RESEARCH 19

2.1 INTRODUCTION 19

2.2 SHULMAN'S CONCEPTUALISATION OF PEDAGOGICAL CONTENT KNOWLEDGE 21

2.3 BALL'S CONCEPTUALISATION OF MATHEMATICS KNOWLEDGE FOR TEACHING 23

2.3.1 Subject Matter Knowledge (SMK) 26

2.3.2 Pedagogical content Knowledge (PCK) 28

2.4 CHALLENGES WITH THE MKfT FRAMEWORK 32

2.5 CONCLUSION 33

CHAPTER THREE 35
CONCEPTUALISING NUMBER SENSE AND COUNTING..35
3.1 INTRODUCTION..35
3.2 CONCEPTUALISING NUMBER SENSE 36
3.2.1 The importance of number sense 36
3.2.2 How number sense can be developed 39
3.3 COUNTING...44
3.3.1 Counting perspectives...44
3.3.2 The importance of counting...45
3.3.3 The counting process 46
3.4 CONCLUDING REMARKS...57

CHAPTER FOUR...59
RESEARCH METHODOLOGY..59
4.1 INTRODUCTION..59
4.2 BACKGROUND TO THE RESEARCH DESIGN 60
4.3 RESEARCH ORIENTATION..60
4.4 CASE STUDY METHODOLOGY 62
4.5 THE SAMPLE 63
4.6 THE RESEARCH SITE..65
4.7 DATA COLLECTION PROCESS AND TECHNIQUES...65
4.7.1. Data collection process 65
4.7.2 Research Techniques 66
4.8 DATA ANALYSIS PROCESSES..71
4.9 VALIDITY..72
4.10 ETHICS 73
4.11 REFERENCING CONVENTIONS IN MY THESIS 75
4.12 CONCLUDING REMARKS 76

CHAPTER FIVE 77
DATA PRESENTATION AND ANALYSIS 77
5.1 INTRODUCTION...77
5.2 COUNTING IN GAIL’S CLASSROOM 78

5.2.1 Whole class counting 79
5.2.2 Small group counting 83
5.3 INTERPRETATION AND ANALYSIS OF BOTH THE WHOLE CLASS AND SMALL GROUP COUNTING DATA 86
5.3.1 Different mathematics concepts that were developed during the counting activities 88
5.3.2 Bridging the tens 94
5.3.3 The subtraction concept 108
5.3.4. The place value concept 114
5.4 CONCLUSION 121

CHAPTER SIX 124
CONCLUDING REMARKS 124
6.1 MY THEORETICAL FRAMEWORK: MKFT 125
6.1.1 Limitations of using MKfT theoretical framework 126
6.1.2 Key findings from my research 127
6.2 SOME INSIGHTS EMERGING FROM MY RESEARCH 131
6.3 RECOMMENDATIONS FOR FURTHER RESEARCH 131
6.4 A FINAL WORD 132

REFERENCES 132

APPENDICES 145
APPENDIX 1 – PRINCIPAL CONSENT FORM 145
APPENDIX 2 – TEACHER CONSENT FORM 147
APPENDIX 3 – INFORMATION SHEET AND PARENT CONSENT FORM 149
APPENDIX 4 – EXAMPLE OF AN INTERVIEW 153
APPENDIX 5 – WHOLE CLASS COUNTING SESSION LESSON 1 161
APPENDIX 6 – WHOLE CLASS COUNTING SESSION LESSON 2 161
APPENDIX 7 – WHOLE CLASS COUNTING SESSION LESSON 10 161
APPENDIX 8 – WHOLE CLASS COUNTING SESSION LESSON 12 161

GLOSSARY OF ABBREVIATIONS AND ACRONYMS 163

LIST OF FIGURES

2.1 Illustration of Shulman's (1987) PCK

2.2 Model of Teachers' Mathematical Knowledge for Teaching

4.1 Outline of the Research Design and Methodology

4.2 Showing the process of data collection

5.1 Resources for whole class counting

6.1 How the MKfT domains relate to each other

LIST OF TABLES

1.1 Average % marks in mathematics by grade for ANA (2011-2014)
1.2 Studies revealing teachers' poor pedagogical classroom practices
2.1 A summary of the interconnection between Shulman's PCK and Ball's MKfT
3.1 Showing the learners' incompetence in arithmetic skills resulting from poorly developed number sense.
3.2 Common Counting Errors
3.3 Counting errors related to place value
4.1 Interview referencing formats
4.2 Outline of the Research Goals, Questions, Design and Methodology
5.1 A summary of MKfT domains and their indicators
5.2 Summary of MKfT domains employed relating to the concept of teen and ty numbers
5.3 Summary of MKfT domains employed relating to the concept of bridging the tens
5.4 Summary of the MKfT domains prevalent in development of bridging the tens concept while counting back
5.5 Summary of the MKfT domains prevalent in development counting back as a strategy of subtraction
5.6 Showing the MKfT domains in action during the place value excerpt

CHAPTER ONE

THE CENTRALITY OF THE TEACHER IN EXPLANATIONS OF LEARNER PERFORMANCE IN SOUTH AFRICA

1.1 INTRODUCTION

Few would argue that the state of mathematics education in South Africa is an area of concern (Reddy, 2006; Fleisch, 2008; Spaull, 2013; Spaull & Kotze, 2015). Many studies have identified that South African mathematics teachers lack both content and pedagogy to teach, that the learners acquire learning deficits early on in their schooling, and that these backlogs are the root cause of underperformance in their later years of schooling (Fleisch, 2008; Taylor & Taylor, 2013; Graven, 2016). It is therefore suggested that interventions must first address these deficits in order to successfully raise learners' mathematical proficiency (Taylor, 2011). The present study seeks to contribute to possible intervention strategies by investigating from practice the content and pedagogical knowledge required to teach Foundation Phase [FP] mathematics in a way that addresses these learning deficits. In other words, the study seeks to frame the knowledge required by teachers in terms of the work teachers do. According to Ball, Thames and Phelps (2008), defining knowledge for teaching mathematics from practice "addresses two important problems; it provides a basis for setting priorities for what teachers are taught, and it increases the likelihood that teachers will be able to use what they are taught when they teach" (p. 5).

This chapter provides an overview of my study which sought to investigate the knowledge that an expert FP mathematics teacher draws on in her mathematics teaching practice. The chapter starts by presenting the background and context of the study, followed by an outline of the research goals and questions that guide this study. I present the summary of the research design based on the interpretivist paradigm and conclude this chapter with an overview of the structure of the thesis.

1.2 THE CRISIS OF MATHEMATICS EDUCATION IN SOUTH AFRICA

Poor performance in mathematics in South Africa is well documented (Kazima, Pillay & Adler, 2008; Spaull & Kotze, 2014; McAuliffe & Lubben, 2013; Bansilal, Brijlall & Mkhwananzi, 2014). South Africa has participated in a number of local and international educational achievement studies over the past 20 years to monitor and evaluate the quality of schooling in specific subjects such as mathematics and literacy. According to these studies South African learners are not performing well in mathematics throughout the schooling system. Spaull (2013) confirms that "as far as educational outcomes go, South Africa has the worst education system of all middle-income countries that participate in cross-national assessments of educational achievement" (p. 10). I elaborate on learner performance by drawing on various studies in the section below.

1.2.1 Mathematics education in South Africa: results from international research

Results of international assessments done with South African learners, both at primary and secondary school levels, reflects poor performance in mathematics. The Trends in International Mathematics and Science Studies [TIMSS] (Spaull, 2013; Reddy, 2006), South and East African Consortium for Monitoring Educational Quality [SACMEQ] (Fleisch, 2008; Spaull, 2013) and the Monitoring Learning Achievement [MLA] have shown that South Africa has one of the lowest achievement rates in mathematics (Fleisch, 2008).

South Africa has been participating in TIMSS evaluations in mathematics and science since 1995. According to Spaull (2013) the evaluations showed that the South African Grade 8's achievement in mathematics did not improve between 1995 and 2002. This prompted a shift to testing Grade 9 learners instead of Grade 8 learners from 2011 (Reddy, Zuze, Visser, Winnaar, Juan, Prinsloo, Arends, & Rogers, 2015). Reddy et al. (2015) explain that:

> The reason for changing the testing grade from Grade 8 to Grade 9 was because of South Africa's overall low performance in previous rounds of the study. A shift from testing Grade 8 learners to testing Grade 9 learners was judged to enable a better match between the content knowledge presented to learners in TIMSS and the curriculum coverage in South Africa (p. 3).

Despite the tests being completed by Grade 9 learners, in 2011 South Africa was still ranked last. Spaull (2013) confirms:

> South Africa's post-improvement level of performance is still the lowest of all participating countries, with the average South African Grade Nine child performing between two and three grade levels lower than the average Grade Eight child from other middle-income countries (p. 4).

In the most recent 2015 TIMSS evaluations South Africa came forty-eighth out forty-nine participating countries for the Grade 4 evaluations and forty-seventh out forty-eight countries for the Grade 8 evaluations (written by Grade 9s in South Africa), taking the third place behind

Botswana (Clerkin, Perkins & Cunningham, 2016). Poor performance in Grade 9 suggests that FP does not prepare learners well enough to meet the demands of learning in the higher grades (Graven, 2016). Reddy et al. (2015) assert that three quarters of the learners in Grade 9 in South Africa do not have the required mathematical knowledge and skills expected of them. Graven (2013) states:

> This is worrying. These learners are stuck at the shallow end of skills acquisition. The acquisition of skills in mathematics and science is cumulative. Emphasis needs to be placed on improving the fundamentals of instruction in earlier grades in order to reduce the large number of learners who lack basic knowledge of mathematics and science in grades 8 and 9 (p. 5).

Graven (2013) in agreement with Fleisch (2008) proposes that the crises of poor performance begins early at FP where the learners acquire basic skills that they need in the learning of mathematics. Fleisch (2008) argues that "the education achievement gap begins in the Foundation Phase and continues unbroken" (p. 30). Both Fleisch (2008) and Graven (2013) concur that learners do not acquire adequate skills and therefore perform more poorly as they go up the grades. This is evidenced by the poor performance by Grade 4s in the TIMSS evaluation. The Grade 4s in the TIMSS study would have exited the Foundation Phase which suggests the crisis to be having its origins in FP.

The SACMEQ studies confirm the result of the TIMSS. Fourteen countries in Southern and Eastern Africa participated in SACMEQ study. South African Grade 6 learners participated in SACMEQ II (2000) and III (2007) and were ranked at the bottom of the participating countries in both studies. As with the TIMSS results, the South African learners' performance was behind learner performance in countries much poorer, such as Tanzania, Kenya and Swaziland (Spaull, 2013). However, in the SACMEQ IV South Africa showed a great improvement where it ranked number six out of the fourteen countries that participated (DBE, 2016b). Although it was still behind some countries with considerably smaller economies, like Zimbabwe and Swaziland, DBE (2016b) reports that learners showed significant improvement, increasing by ninety-two points in mathematics. He states that only two other countries within SACMEQ had more significant increases than South Africa.

The analysis of the SACMEQ III (2007) by province, showed that Eastern Cape learners, where my research is situated, had an average score lower than four other South African provinces (Moloi & Chetty, 2010). This confirmed that the problem of learner underperformance in mathematics education is particularly acute in Eastern Cape Province. DBE (2016b) however reports that there is a substantial improvement margins in SACMEQ IV results in such provinces as Eastern Cape and Limpopo, which had low scores in SACMEQ III. Although there is this notable improvement in learner performance in the Eastern Cape, it however still remains close to the bottom of other provinces with only Limpopo behind it. This suggests there is still more work that needs to be done in Eastern Cape to help the learners to pull up to the expected levels.

In another study that compared Grade 6 learner performance in North West province of South Africa and the south-eastern region of Botswana, it was found that the South African learners performed worse than their Botswanan counterparts scoring an average of 28.6% while the Botswanan learners scored 34.6% in the pre-tests given during the study (Carnoy & Arends, 2012).

The above benchmarking tests all suggest that South African learners are consistently underperforming when compared with learners internationally. Having presented the international data, the ensuing section focuses on benchmarking tests developed and implemented in South Africa.

1.2.2 Mathematics education in South Africa: national research

National research has demonstrated that, on average, learners in South Africa perform poorly in mathematics (Adler &Venkat, 2013; Reddy, 2005; Taylor, 2008; Graven, 2013). The results of the various national benchmarking tests confirm the above results.

The National Senior Certificate [NSC] and the Annual National Assessments [ANA] are the key indicators of learner achievement in South Africa. The NSC is written at the end of Grade 12. The NSC mathematics results reflect a problem in South African mathematics education. Mathematics is the most poorly performed of all subjects and there is a downward trend in the mathematics matric pass rate results since 2013. The mathematics results dropped from 59% in 2013 to 54% in 2014 and dropped further in 2015 to 49% (Department of Basic Education [DBE], 2016a). Notable in the South African NSC examination is that more and more learners are opting to write Mathematical Literacy (Adler & Pillay, (2017). Adler & Pillay (2017) argues that the majority of those who choose to write mathematics do not perform well enough to enrol for sciences in universities. For example, in 2014 only 42% of learners wrote mathematics and only 35% scored above 40%. In 2015, 41% wrote mathematics and only 32%

scored at 40% and above. The general analysis of the 2014 NSC results by province place Eastern Cape at the bottom of all the provinces with 65%, while Gauteng led with 85%, indicating that the crisis is worse in the Eastern Cape than in other provinces (DBE, 2016a). The DBE (2015) commented that these mathematics results “have pointed to learning gaps that start much earlier in the system and we must continue to strengthen our interventions and support in earlier grades of the system” (p. 8).

The ANAs were introduced in 2011. The focus was to enable the DBE to track learner performance in the system on an annual basis so as to identify key problems in the teaching and learning of mathematics. The table below shows the achievement results in mathematics between 2011 and 2014[1].

Table 1.1 Average % marks in mathematics by grade for ANA (2011-2014)

Grade	2011	2012	2013	2014	Grade averages across 2011-2014
1	63	68	60	68	64,75
2	55	57	59	62	58,25
3	28	41	53	56	44,5
4	28	37	37	37	34,75
5	28	30	33	37	32
6	30	27	39	43	34,75
9	n/a	13	14	11	12.6 (2012-2014)

(Robertson & Graven, 2015, p. 13)

The result of analyses of the ANAs reveal the extent of the learning deficits in the South African primary schools (Graven, 2016). As noted in Table 1.1, the general downward trend in the ANA results is evidence of the poor mathematics skills among the South African learners in all the years across the grades. As learners move to the higher grades mathematics performance seems to decrease suggesting the effects of poor teaching and learning at lower grades have accumulative learning effects (National Education Evaluation and Development Unit [NEEDU], 2013; Spaull, 2013). In 2014, Grade 1 learners attained average of 68% while the Grade 9 scored an average of 11%. At this stage the majority of learners drop mathematics and choose mathematics literacy which they consider as an easier option (Adler & Pillay 2017). NEEDU (2013) reports that the ANAs mirror “dismal performance across the country in mathematics” (p. 55).

[1] No ANAs were written in 2015 and 2016.

The FP results of the ANA in the Eastern Cape is of particular significance as my research is located in the Eastern Cape. The DBE's (2016a) report on the 2014 ANA results by province indicated that the Eastern Cape FP was the second worst-performing province. Grade 1 learners scored an average of 65%, Grade 2, 58% and Grade 3, 52%. These grade averages were below the national averages of 68%, 62% and 56% respectively. This suggests the effects of poor teaching and learning are worse in Eastern Cape than the other eight provinces in South Africa.

Spaull (2013) argues that the majority of learners in South Africa are operating significantly below the level they should be as indicated in the curriculum with the majority of learners from disadvantaged backgrounds performing well below their grade levels (NEEDU, 2013). He maintains that the 2012 ANAs indicate that the South African Grade 3 learners are 1.8 years behind their international counterparts. Furthermore, Spaull and Kotze (2015) argue that most learners are already two grades behind by the time they get to Grade 4.

The results of both the international and national benchmarking tests confirm the extent of learner underperformance in mathematics education in South Africa. The decrease in the average scores in the ANAs between Grade 3 and Grade 6 signify the effects of poorly laid foundations of learning mathematics.

Poorly laid mathematical foundations are difficult to redress in later grades due to mounting demands and pressures at those levels. Graven, Venkat, Westaway and Tshesane (2013) agree that "pressure to keep up with the Intermediate Phase [IP] curriculum often means it is difficult for the teachers to address the backlog of foundational understanding of learners" (p. 138). Concurring with this, Mourshed and Barber (2007) argue that "at the primary level, students that are placed with low-performing teachers for several years in a row suffer an educational loss which is largely irreversible" (p. 12). NEEDU (2012) reports that "it is widely known that South African schools perform below expectations. But much less is known about why it should be so" (p. 6). The next sections seek to give an understanding of why there is a crisis in mathematics education in South Africa.

1.3 UNDERSTANDING WHY THE CRISIS EXISTS

The continuous low performance of learners in mathematics has led to an increase in research that seeks to understand how mathematics teacher characteristics, pedagogical practices and content knowledge contribute to these patterns of poor learner performance. Common findings across these studies relates to the presence of large numbers of South African mathematics teachers who lack fundamental understanding of mathematics and employ poor quality teaching methods (Venkat & Spaull, 2014; Carnoy et al., 2008). This section discusses the explanations given by researchers for the poor Mathematics performance in South Africa. I start with the claim that teachers lack adequate content knowledge to teach effectively.

1.3.1 Teachers' poor content knowledge as a contributor to the crisis

A number of studies have been conducted locally in an attempt to understand the crisis in learner performance in mathematics in South Africa (Hugo, Wedekind & Wilson, 2010; NEEDU, 2013; Bolowana, 2014; Venkat & Spaull, 2014). These studies agree that one of the contributing factors to poor learner performance is the teachers' lack of content knowledge.

According to Venkat and Spaull (2014), the Grade 6 teachers whose learners took part in the SACMEQ III evaluations were also given the same test as their Grade 6 learners. The analysis of the South African SACMEQ III mathematics teachers' test data revealed that 79% of Grade 6 mathematics teachers showed content knowledge competence below the Grade 6 and 7 levels and only 17% of those tested in the Eastern Cape province had adequate content knowledge to teach Grade 6. Venkat and Spaull (2014) report that the majority of teachers in Eastern Cape refused to write the test, suggesting the possibility of their lack of confidence in their content knowledge levels.

The Grade 6 comparative evaluation between South Africa and Botswana discussed earlier in the chapter showed that the South African teachers did not have adequate content knowledge, which contributed, among other factors, to their learners performing worse than their Botswanan counterparts (Carnoy & Arends, 2012). On comparing the performance of the learners to that of their teachers it was found that those teachers who performed better on the mathematics test taught mathematics more effectively as evidenced by their learners' results. This suggests a positive correlation of the teacher's content knowledge, the quality of their teaching and the learners' achievements.

The Integrated Education Project measured Grade 4 to 6 teachers' content knowledge using Grade 4 to 7 content items in four South African provinces: KwaZulu Natal [KZN], Eastern Cape, Limpopo and Northern Cape. It found that the tested teachers performed poorly, scoring an average of 32% (Mabogoane & Pereira, 2008).

Similarly, Hugo et al. (2010) conducted a study in KZN to evaluate teachers' content knowledge. Like the above mentioned studies they found that none of mathematics teachers at primary school level were able to score 100% in a test on the curriculum they teach. The same story prevailed with the Khanyisa Baseline Project that tested a sample of 39 Grade 3 teachers in 24 Limpopo province schools on Grade 6 mathematics and literacy items found that the majority of teachers scored between 29% and 50% with a lowest score of 21,7% and only one teacher scored higher than 75% (Taylor & Moyana, 2005).

While the above research is small in scale, the fact that all studies reach the same conclusion, suggests that mathematics teachers in South Africa have low mathematics content knowledge therefore one can conclude that their content knowledge base is inadequate to provide learners with a sound understanding of mathematics. This proposes a need for content intervention for both teachers practicing in the schools and those that are in pre-service training. Venkat and Spaull (2014) propose that "primary school mathematics teachers should, at the most basic level, have mastery of the content knowledge that they are required to teach" (p. 2). For, as Taylor (2008) declared, "teachers cannot teach what they do not know" (p. 24).

Another factor that contributes to poor learner performance is the teachers' lack of teaching skills. The next subsection addresses the issue of the teachers' poor pedagogy in relation to poor performance.

1.3.2 Poor pedagogy as a contributor to the crisis

Shulman (1986) asserts that a teacher has a special responsibility of teaching content knowledge to learners and therefore should have adequate content knowledge to help learners understand the subject matter. The teacher's knowledge of what to teach and how to teach it has a tremendous impact on learner achievement.

Many researchers attribute the underperformance of South African learners in mathematics to poor pedagogical practices (Bansilal et al., 2014; Taylor & Taylor, 2013; Venkat & Spaull, 2014). Pedagogy refers to the principles and methods of instruction that a teacher employs in the classroom to facilitate effective teaching and learning such as a structuring of time, space and text in a way which considers both the organization of knowledge and its transmission (Hoadley, 2012). Teachers' pedagogical practices have been found to have a positive correlation with learners' achievement (Schollar, 2008). Small scale studies have been conducted to explore how the teachers' pedagogy relates to the crisis of poor mathematics performance in South Africa.

According to the Centre for Development Enterprise [CDE] (2014), the analysis of the test of both content and pedagogy that was given to the Grade 9 teachers whose learners participated in TIMSS 2011 in South Africa revealed that 54% of Grade 9 mathematics learners were taught by teachers that had specialized in mathematics but did not have any pedagogical training. This suggests that teachers in South Africa do not only lack the content of the subject they teach but also do not have pedagogy.

The Count One Count All [COCA] study conducted in relation to how South African FP teachers developed number sense in learners found that the teachers did not have adequate pedagogy to effectively develop number sense in learners. Instead they employed poor teaching strategies that engaged learners in very concrete methods for solving problems rather than exposing them to the access to more abstract procedures for solving problems (Hoadley, 2012).

Hoadley's (2012) analysis of various small scale studies conducted in South Africa confirmed that the majority of primary school teachers generally employ poor pedagogical practices which result in poor learner performance. Table 1.2 below gives a summary of Hoadley's (2012) findings and the research studies that the findings are based on.

Table 1.2 Studies revealing teachers' poor pedagogical classroom practices

Findings	**Key studies**
Low levels of cognitive demand	Adler et al, 2002
Dominance of concrete over abstract meanings	Schollar, 2008; Ensor, 2009; Reeves et al, 2008
Lack of opportunities for reading and writing (oral discourse dominates)	Pretorius & Machet, 2004
Slow pacing	Hoadley, 2003; Ensor et al, 2002
Collectivized as opposed to individualized learning	Hoadley, 2008
The erosion of instructional time	Chisholm, 2005

Multiple complexities related to language, especially second language teaching and learning	Probyn, 2009; Setati & Adler, 2000; Desai, 2001
Lack of explicit feedback to learners	Brock-Unte and Holmarscottir, 2004
Lack of coherence	Reeves, 2005; Hoadley, 2008; Venkat and Naidoo, 2012

Adapted from Hoadley (2012, p. 198)

Several suggestions have been provided in this section to explain that South African teachers lack both content and pedagogy that results in poor learner performance in mathematics. The next section considers why teachers do not have the necessary content and pedagogical knowledge to teach.

1.4 REASONS UNDERPINNING TEACHERS' DEFICIT IN KNOWLEDGE FOR TEACHING MATHEMATICS

As highlighted earlier in this chapter, teachers' knowledge of both mathematical content and pedagogy is related to how they teach and learner achievement. However, it would be incomplete to discuss the subject of deficit in teacher knowledge and not identify the causes. If causes are identified, then it becomes easier to provide corrective intervention strategy. Research has suggested that there are a number of reasons for teachers' deficit in the knowledge required for teaching mathematics (Adler, 2005; Carnoy et al., 2008; DBE, 2009)I elaborate on these below.

Carnoy et al. (2008) claim that despite concerns about teachers' insufficient content knowledge and poor pedagogical practices, the vast majority of South African teachers have appropriate teaching qualifications. The DBE (2009) offers an explanation arguing that teacher' content knowledge and pedagogical practices are poor as the universities and teacher training institutions have not equipped teachers sufficiently to teach mathematics. This inadequate training prevents teachers from achieving the expected education outcomes (DBE, 2009).

Adler (2005) puts the blame on the legacy of apartheid. She points out some challenges faced by the universities in the endeavour to educate teachers. Firstly, she suggests that there is not enough knowledge about the mathematical preparation needed to prepare teachers. She suggests that research be conducted into the Mathematics Knowledge for Teaching (MKfT) required by teachers in order to inform teacher education programmes. Secondly, she maintains that there is a lack of ongoing support that enables mathematics teachers to do a skilful job after they leave the university. Thirdly, universities are challenged to provide a large number of adequately and appropriately prepared mathematics teachers at a time when there are fewer people who are taking up advanced study of mathematics and very few people choosing teaching as a profession. Adler (2005) is concerned that this does not only threaten the availability of well-qualified mathematics teachers for the South African schools, who could improve the development of learners understanding of the subject mathematics, but also the provision of scientists and engineers that the country so much needs as well. My study responds directly to Adler's (2005) first concern that there is not enough knowledge on what teachers need to know in order to teach mathematics competently. My study seeks to contribute to research that informs teacher education as to the content and pedagogical knowledge (MKfT) required to teach mathematics in the FP.

Adler and Davis (2006) argue that it is a challenge for teacher education to embrace the content and pedagogical knowledge necessary to teach mathematics under the legacy of apartheid because most students who enrol for teacher training programs have limited mathematics content knowledge resulting from being exposed to poor teaching by inadequately qualified teachers (Department of Education [DoE], 2004). The DoE (2004) argues that if these students become teachers, when they have not received adequate training that equipped them with both content and pedagogical knowledge, they may continue to perpetuate poor teaching practices leading to a revolving cycle of poor achievement. It is therefore necessary that teacher education programs facilitate the breakdown of this deficit cycle by producing teachers that have sufficient knowledge of mathematical content and pedagogy. The DoE (2004) asserts that without appropriate training at tertiary level, teachers will carry forward the poor teaching practices they brought from school to their own teaching. Hence this study seeks to investigate what knowledge is needed for teaching mathematics at FP so as to inform in-service teachers and teacher education.

1.5 THE PROBLEM STATEMENT

As indicated above there is grave concern about poor learner mathematics performance in South Africa that is evident from the FP. Research has identified the sources of the crisis as the lack of both content and pedagogical skills by South African mathematics teachers. There appears to be a dearth of research that examines the content and pedagogical knowledge of *competent* teachers in South Africa. These teachers possess particular knowledge that is not common with the majority of the South African mathematics teachers. There is thus a need to investigate, learn and understand from such teachers what knowledge they employ, and how they employ it in their teaching, that results in effective teaching and learning and which in turn produces positive learning outcomes. For this reason, my research attempts to understand the content and pedagogical knowledge that an expert FP teacher employs, and the ways in which she employs them in her teaching, so as to illuminate the mathematics knowledge for teaching that FP teachers need to address the concerns highlighted in this chapter about teachers' poor teaching practices.

1.6 THE RESEARCH GOALS AND QUESTIONS

1.6.1 Research goals

My primary research goals relate to understanding the Mathematics Knowledge for Teaching (MKfT) an expert[2] FP teacher draws on when developing children's number sense. The second is to inform the initial teacher education of Bachelor of Education (FP) [B Ed (FP)] and Post Graduate Certificate in Education (Foundation Phase) [PGCE (FP)] students at my own institution firstly and then beyond. The third goal is to contribute to a relatively under-researched area of study in FP teacher education in South Africa, which is what MKfT is required in the FP mathematics teaching.

1.6.2 Research questions

In pursuing the goals given above my research sought to answer one key question. The following question formed the basis of my study:

> What MKfT in relation to development of number sense through counting does an expert Grade 2 teacher have and use in her teaching?

The crisis of poor learner performance in mathematics as outlined earlier in this chapter starts as early as FP. In this chapter I argued that two major contributing factors to poor learner performance in mathematics are the teachers' insufficient content knowledge and poor pedagogical practices.

[2] Defined in Chapter Four

Research in South Africa tends to focus on teachers whose learners are underperforming. Literature appears silent about a number of competent teachers in South Africa whose learners score highly in the various national and international benchmarking tests. Indeed, as Fleisch (2008) and other researchers indicate, there are two different education systems operating in South Africa. One is successful and its results compare favourably to international performance and the other is less functional, with learners performing a grade or so behind their international counterparts (Spaull, 2013).

Many teachers in this well-functioning system possess some special knowledge that is not common to the majority of the South African mathematics teachers. However, there may be some of the teachers in the less functional system who possess strong MKfT. There is need therefore to investigate, learn and understand from such teachers what knowledge they employ in their teaching that result in effective teaching and learning which in turn produces positive learning outcomes.

In this study, one teacher has been identified from the well-functional system, as discussed in Chapter 4, and investigated as to what knowledge she employs in developing number sense successfully, so as to inform teacher education and teacher development of teachers who are already in service. Further research is proposed with teachers from the less functional system. My next section outlines and summarizes the chapters in my research study.

1.7 OUTLINE OF THE THESIS

My research thesis consists of six chapters structured as follows:

Chapter One provides the context and the background for my study. It presents the crisis of the mathematics education in South Africa as reflected both in the national and international research. The reasons behind this predicament, as evidenced by research, is elaborated as being the teachers' lack of both mathematics content and poor pedagogical practices. In this chapter, I also outlined my research goals and posed my research question.

Chapter Two reviews the Mathematics Knowledge for Teaching (MKfT) theoretical framework by Ball, Hill and Bass (2005) and Ball, Thames and Phelps (2008) that I used as a lens for my research. I discuss how this MKfT framework is informed by Shulman's (1986) Pedagogical Content Knowledge (PCK) and how I used the framework in my research.

Chapter Three explains the importance of well-developed number sense for mathematics learning as one of the intervention strategies to the crisis of poor performance in mathematics in South Africa. I elaborate on, through literature, how counting is the basis for number sense and can be used to help learners to develop number sense.

Chapter Four describes the methodology followed in this study. In my research I used an interpretive research paradigm to investigate the MKfT needed by a FP teacher to develop number sense through counting. I used a qualitative case study approach where an expert FP teacher was observed developing Grade 2 learners' number sense through counting. The observation of lessons was followed by interviews about the observed lessons.

In **Chapter Five** I present and analyse the empirical data using Ball and colleagues' MKfT framework as the analytical tool in order to illuminate the key findings relating to my research questions.

Chapter Six concludes the study by discussing findings and focusing on the key contributions of the study. Additionally, I discuss the implications of the study and engage with the limitations and opportunities for further research.

CHAPTER TWO

ELABORATING ON MATHEMATICS KNOWLEDGE FOR TEACHING AND THE IMPLICATIONS FOR MY RESEARCH

2.1 INTRODUCTION

In Chapter One I reviewed research that showed that South African learners are underperforming in mathematics throughout the schooling system. Research that tries to explain the underperformance of learners in South Africa suggests that teachers generally lack both content knowledge and pedagogical skills to teach mathematics in an effective and meaningful way (Schollar, 2008; Spaull, 2013; Spaull & Kotze, 2014; Graven et al., 2013; and Graven, 2016). Askew (2008) argues that the majority of teachers currently teaching at primary schools "express, a lack of confidence in their mathematical knowledge" (p. 16), and such teachers are not managing the demands of teaching mathematics. Green, Parker, Deacon and Hall (2011) suggest that South Africa requires teachers equipped with both the necessary content and pedagogical knowledge to teach mathematics in FP. They write:

> If the provision of sufficient numbers of quality foundation phase teachers is not achieved, it is likely that very little headway will be made in relation to improved Literacy and Numeracy levels at primary school level, and this will have a domino effect on learning and achievement at all levels of the system (p. 119).

In addition, Human, van der Walt and Posthuma (2015) argue that "teachers, who have been educated well, perform better in the classroom than those whose training did not prepare them adequately for the task" (p. 3). This suggests that teacher education institutions need to equip teachers with both relevant content knowledge and pedagogical knowledge in order to address poor mathematics performance in South Africa. However, the work of teaching mathematics is very complex. Adler (2005) claims that it entails decompression, or unpacking, of mathematical ideas which many teachers find challenging. The current challenge for teacher education institutions is that there is "no mathematics knowledge and practice standards have as yet been defined for the preparation of Foundation Phase student teachers in South Africa" (Human et al., 2015, p. 1).

According to Human et al. (2015), teacher education and policy makers still need to address such questions as "what constitutes the professional knowledge required for teaching mathematics effectively and can it be represented in a well-considered heuristic" (p. 54). Based on their experience in this field, Hill et al. (2005) suggest the need to learn from practice for "little improvement is possible without direct attention to the practice of teaching" (p. 14). Drawing on current debates in the field of mathematics education that focus on the knowledge teachers need to teach mathematics, this study therefore sought to investigate what Mathematical Knowledge for Teaching (MKfT) was employed by an expert FP teacher to develop number sense through counting as a way of learning from practice. The aim is to ultimately inform both pre-service and in-service foundation phase teacher education.

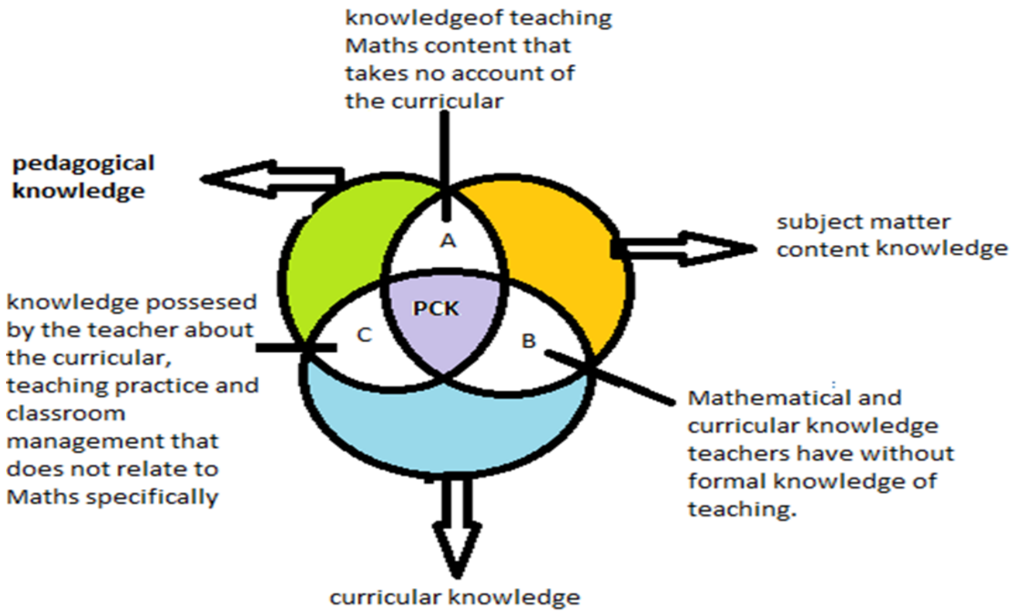
My study was guided by the work of two dominant theorists, Lee Shulman and Deborah Ball, who claim there is knowledge required specifically for teaching mathematics. The MKfT framework by Ball et al. (2008) was chosen for this study because it originated as a result of empirical research specifically on mathematics teaching and because of its focus on different dimensions of knowledge (Wilkie, 2015). In this chapter, I deliberate on the frameworks of each of the above theorists, that is, Shulman's Pedagogical Content Knowledge (PCK) (Shulman, 1986; 1987) and Ball et al.'s Mathematical Knowledge for Teaching (MKfT) (Ball et al., 2008).

I begin with the work of Shulman (1986, 1987) as he first alerted teacher educators to guard against separating content knowledge and pedagogy as both are needed to enable teachers to carry out their work of teaching effectively. I follow with Ball et al.'s MKfT, as this framework is based on Shulman's conceptualisation of PCK. This study used Ball et al.'s (2008) MKfT as both a theoretical and analytical framework.

2.2 SHULMAN'S CONCEPTUALISATION OF PEDAGOGICAL CONTENT KNOWLEDGE

Shulman (1986) identified a significant factor in teacher education programmes that researchers and practitioners had previously overlooked in the study of classroom teaching. He claimed that teacher training programs treated subject knowledge and pedagogy as mutually exclusive thus they focused either on subject matter or pedagogy. Shulman (1986) attempted to address the dichotomy of training programs separating content and pedagogy by introducing a special domain of teacher knowledge which he referred to as *pedagogical content knowledge* [PCK]. Shulman (1986) defines PCK as "the ways of representing and formulating the subject making it comprehensible to others" (p. 9). PCK therefore bridges content knowledge and the practice of teaching into an understanding of how particular aspects of subject matter are organized, adapted, and represented for instruction. In other words, PCK is also referred to as the specific amalgam of knowledge that results when teachers combine what they know about mathematics with teaching and learning (Shulman, 1986). It serves to blend content and pedagogy to enable transformation of content into pedagogically powerful forms. Content knowledge is defined as the aggregate information that grows in teachers and how it is organized (Shulman 1986). According to Shulman (1986) knowledge for teaching should include subject matter content knowledge, PCK and curricular knowledge. PCK exists at the intersection of content, curriculum and pedagogy (Shulman, 1986). This intersection is considered central to all teaching. It is critical for the teaching of any given topic as it enables a deep understanding of how particular aspects of subject matter can be organized, adapted, and represented for instruction (Park & Oliver (2008)). Figure 2.1 below illustrates my interpretation of the components of PCK and how they are interrelated.

Figure 2.1 Illustration of Shulman's (1987) PCK



Subject matter content knowledge refers to the knowledge and understanding of the subject matter structures, such as the substantive and the syntactic structures. *Substantive structures* are the ways in which concepts and principles are organized within a discipline, while *syntactic structures* refer to mathematical processes (i.e. to generalize, reason, explain and defend thinking, conjecture etc.). It is essential that the teachers' subject matter content knowledge is established and developed in such a way that is useful for them as they do their work of teaching (Shulman & Grossman, 1988). Venkat (2015) argues that a teacher who knows mathematics should be able to use the mathematical concepts in a mathematical way.

Curricular knowledge is the knowledge of programs designed for the teaching of particular subjects and topics at a given level and knowledge of the available variety of instructional materials that relate to the teaching and learning of those programs. In South Africa, the Curriculum Assessment Policy Statements [CAPS] currently provide the basis of that curricular knowledge. However, it is important for teacher education courses to educate both pre-service and in-service teachers on how to access the curricular knowledge from curricular documents.

Pedagogical Content Knowledge is the knowledge of the forms and strategies teachers use to guide their learners into a meaningful understanding of the concepts of the subject they teach. Woolfolk (2010) argues that good teachers use pedagogical knowledge and skills to help learners understand abstract concepts. Likewise, Park and Oliver (2008) claim:

> PCK is teachers' understanding and enactment of how to help a group of students understand specific subject matter using multiple instructional strategies, representations, and assessments while working within the contextual, cultural, and social limitations in the learning environment (p. 264).

Hill, Rowan & Ball (2005) argue that Shulman's (1986) PCK model is not a fixed model and suggest there was a need to refine the model for teacher education and development that would meet the specific needs for teaching mathematics. Ball, Thames and Phelps (2008) and Ball, Sleep, Boerst and Bass (2009), decided to investigate what teachers need to know to be able to effectively carry out their work of teaching mathematics. Although Ball et al. (2008) were interested in primary school education, they were influenced by the ideas of Shulman's (1986) PCK, whose work was on secondary school teacher education. Ball et al. (2008) developed a practice-based theoretical framework which they named the Mathematics Knowledge for Teaching (MKfT). Their central goal was to define a framework that made explicit the knowledge teachers needed to carry out the work of teaching mathematics effectively. They argue that mathematics knowledge for teaching must take account of both regularities and uncertainties of practice and must equip teachers to know in the contexts of the real problems they solve. The next section focuses on Ball et al (2008)'s conceptualisation of MKfT.

2.3 BALL'S CONCEPTUALISATION OF MATHEMATICS KNOWLEDGE FOR TEACHING

In analysing the mathematical demands of mathematics teaching there is need to identify mathematical knowledge that is required in the work teachers do. This is possible through studying and identifying the mathematical knowledge entailed in the regular day-to-day, demands of teaching. Ball et al. (2008) propose that doing a job analysis, where one observes teachers carrying out the work of teaching, and "asking expert mathematicians and mathematics educators to identify the core mathematical ideas and skills that teachers should have" (p. 395), may be useful to ascertain what knowledge is required for the effective teaching of mathematics.

The concept of MKfT was introduced by Ball et al. (2008). Building on Shulman's (1986) notion of PCK as deliberated earlier in this chapter, Hill et al. (2005) conducted an interactive work session to investigate the mathematical knowledge and skills that are needed in the teaching of mathematics. Focusing on what teachers do while teaching, they managed to identify six domains. Together these six domains are referred to as Mathematics Knowledge for Teaching (MKfT). Hill, et al. (2005), Ball et al. (2008) and Kim (2013) define MKfT as the mathematical knowledge, skills, and habits of mind entailed in the work of teaching. It is the crucial mathematical knowledge that teachers apply during teaching to improve the teaching and learning of mathematics (National Mathematics Advisory Panel, 2008). Ball et al. (2008) argue that "it seemed obvious that teachers need to know the topics and the procedures that they teach…but they "decided to focus on how teachers need to know that content (p. 395). In addition, they "wanted to determine what else teachers need to know about mathematics and how and where teachers might use such mathematical knowledge in practice" (p. 395). In other words, Ball et al. (2008) suggest that MKfT gives teachers a better understating of what to teach, and when and how to teach it, hence they argue that MKfT "is concerned with the tasks involved in teaching and the mathematical demands of these tasks" (p. 395).

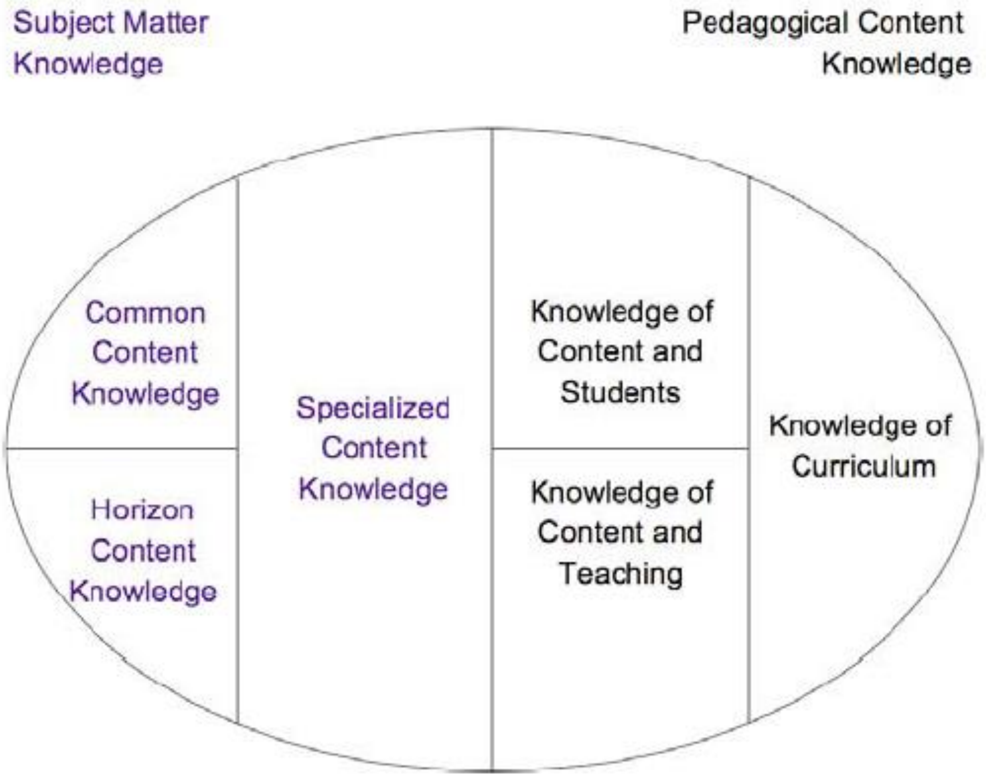
The work of teaching involves such tasks as: presenting mathematical ideas; selecting appropriate representations and recognising what is involved in using a particular representation; providing mathematical explanations for common rules and procedures; and scrutinising and understanding unusual solution methods to problems. Hill et al. (2005) and Rockoff, Jacob, Kane and Staiger (2011) propose that the MKfT of teachers can be linked to attainment of positive learner results. Given the state of learner underperformance in mathematics in South Africa, and the explanations provided as to why learners are not achieving in Chapter One, an examination of the MKfT of FP teachers may be able to provide useful knowledge on what pre-service and in-service teachers need to know and do to curb the crisis. Hurrel (2013) states that MKfT contributes to quality instruction, and

> it therefore would not seem unreasonable to suggest that if we want to improve teacher effectiveness the development of MKfT is an important factor. At the very least, familiarity with this construct would allow teachers to reflect on various domains that require development to foster PCK and allow them the opportunity to strengthen any areas in which they may feel they are deficit (p. 62).

Although the MKfT framework serves diverse purposes, it is influential in the field of mathematics teacher education (Ball et al., 2009; Graeber & Tirosh, 2008) as it makes it possible to identify the areas of a teacher's mathematical knowledge that support learner achievement (Hill et al., 2005). The DBE and the Department of Higher Education and Training [DHET] (2011) argue that although "…a wide variety of factors interact to impact on the quality of the education system in South Africa, teachers' poor subject matter knowledge and pedagogical content knowledge are important contributors" (p. 4). MKfT is therefore a refinement of Shulman' s (1987) PCK to provide a clearer understanding of what subject matter knowledge and pedagogical knowledge teachers need to carry out their work of teaching effectively. According to Ball et al. (2008) the work of teaching is "everything that teachers must do to support the learning of their students […] involve mathematical ideas, skills of mathematical reasoning, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency" (p.395).

Ball et al. (2008) developed the MKfT framework drawing on Shulman's two categories, PCK and Subject Matter Knowledge [SMK]. They extended the SMK to include three domains: Common Content Knowledge [CCK], Horizon Content Knowledge [HCK] and Specialized Content Knowledge [SCK] to illuminate what "effective teaching require[s] in terms of content understanding" (p. 394). The domain of PCK was split into Knowledge of Content and Teaching [KCT], Knowledge of Content and Students [KCS] and Knowledge of Content and Curriculum [KCC]. These emerged in response to the question: "What do teachers need to know and be able to do in order to teach effectively?" (p. 394). The MKfT framework and its domains are shown in Figure 2.2 below. Each of these domains is discussed in detail below.

Figure 2.2 Model of Teachers' Mathematical Knowledge for Teaching (Ball et al., 2008, p.403)



2.3.1 Subject Matter Knowledge (SMK)

As noted above SMK consists of three domains: the Common Content Knowledge [CCK], the Horizon Content Knowledge [HCK] and the Specialised Content Knowledge [SCK]. Each of these domains are discussed in the sections that follow.

2.3.1.1 Common content knowledge (CCK)

Common Content Knowledge (CCK) is “the mathematical knowledge and skills used in the settings other than teaching” (Ball et al., 2008, p. 399). It is the mathematical knowledge and skills that any person who passed mathematics at school should know, such as knowing how to calculate with whole numbers and fractions. It relates to the mathematics content knowledge that is used in everyday life, by anybody, regardless of age and profession. It is used in the work of teaching mathematics in the same way that it is used in other professions or occupations that also use mathematics, such as accounting, economics, statistics and others (Ball et al., 2008).

Although CCK is not specific to the work of teaching, teachers need this type of knowledge to be able to understand the work that they assign their learners to do and to be able to identify correct or incorrect answers, identify errors in learning materials such as textbooks and be able to use terms and notation correctly in speech and writing. This concurs with Flores, Escuddero and Carrillo’s (2012) definition of CCK as “the knowledge required in order to solve such tasks as are given to pupils” (p. 3). In agreement, Nolan, Dempsey, Lovatta and Castel (2015) propose that teachers must know the content of the subject they teach (e.g. place value, addition and subtraction) thoroughly to be able to present it efficiently, to make the concepts accessible to a wide variety of learners and to engage learners in challenging work. In this research, I do not investigate Gail’s CCK on the hypothesis that her selection as an expert teacher assumes she has adequate CCK. The next domain that Ball et al. (2008) identified relating to SMK was HCK. It is elaborated on in the next section.

2.3.1.2 Horizon Content Knowledge (HCK)

Horizon Content Knowledge (HCK) is the mathematical knowledge that is not directly deployed in instruction in a particular content area, but mathematical knowledge that spans across grades. It relates to the knowledge of how different mathematics topics in the curriculum are interconnected and informs the teacher as to how to teach the topics in a way that facilitates learners to realise the connections (Jakobsen, Thames & Ribeiro, 2013). HCK is the teacher's mathematical perspective on what lies in all directions, behind as well as ahead, for their learners (Mosvold & Fauskenger, 2014). HCK helps the teacher to understand the mathematical concepts and how they connect with each other. It informs the teacher of the topics covered that lay foundation to what is currently being taught and how the current topic lays foundations for other topics that are still to come.

For example, a Grade 2 teacher should understand that teaching learners the composition of numbers is based on learners' understanding of the number names and symbols, knowledge of the relative magnitude and numerosity of numbers etc. The composition of numbers is required for them to be able to decompose and recompose numbers as they solve problems. They need that knowledge to understand place value and to deal with rounding off later in later grades. Hence Ball and Bass (2009), define HCK as knowledge that "supports a kind of awareness, sensibility [and] disposition that informs, orients and culturally frames instructional practice" (p. 5).

HCK provides a wide picture of the mathematical environment and an ability to see the connections to topics in the mathematics discipline that is being taught and the ones that learners may or may not meet in the future (Ball et al., 2008). In this study, HCK comprises the teacher's ability to view mathematics as a whole and to establish the connections in mathematics as a subject and how it connects to other subjects.

The last domain of SMK is the Specialized Content Knowledge (SCK) and it is argued in the next section.

2.3.1.3 Specialised Content Knowledge (SCK)

The third domain developed from Shulman's (1986) Subject Matter Knowledge is SCK. SCK is defined as "the mathematical knowledge and skill unique to the work of teaching" (Ball et al., 2008, p.400). It is not typically needed for any purposes other than teaching. Ball et al. (2008) argue that the work of teaching involves an

> uncanny kind of unpacking of mathematics that is not needed— or even desirable––in settings other than teaching. Many of the everyday tasks of teaching are distinctive to this special work requires unique mathematical understanding and reasoning that is uncommon to other professions (p. 400).

This suggests that the work of teaching requires unique mathematical understanding, reasoning and skill, such as looking for patterns in the errors made by the learner, or sizing up whether nonstandard procedures are valid and generalizable (Nolan, Dempsey, Lovatta & O'Shea, 2015). Ball et al. (2008) suggest "teaching requires knowledge beyond that being taught to students" (p.400) and therefore teachers "must hold unpacked mathematical knowledge because teaching involves making features of particular content visible and learnable by students" (p.400). For example, teachers need not only know how to subtract (CCK), but they also need to know the difference between 'comparison' and 'take-away' models of subtraction (SCK) (Ball et al., 2008).

Research acknowledges that SCK is an area of interest in the work of teaching (Hill et al., 2005; Ball et al, 2008; Wilkie, 2015) and makes a teacher an effective professional, different from other individuals who have good understanding of CCK. However, Flores et al. (2012) contends that research does not specify the nature of the knowledge itself but rather what it does. They argue that "the definitions of SCK tend to be phrased in terms of what having this knowledge enables one to do: responding to students' why questions, […] choosing and developing useable definitions, modifying tasks to be either easier or harder" (p. 3).

According to Flores et al. (2012), after analysing the context in which SCK is used and how it is applied they concluded that SCK includes classroom sequences, or episodes that show how the teacher interacts with mathematics. In the next section I discuss the domains that were refined from PCK

2.3.2 Pedagogical content Knowledge (PCK)

Emergent from Shulman's (1987) PCK are three domains in Ball et al.'s MKfT. These are the Knowledge of Content and Teaching [KCT], Knowledge of Content and Students [KCS] and the Knowledge of Content and the Curriculum [KCC]. These too are elaborated individually in this section

2.3.2.1 Knowledge of Content and Teaching (KCT)

According to Ball et al. (2008), KCT intertwines knowing about mathematics and knowing about teaching. They claim that the majority of mathematical tasks of teaching require the teacher to integrate their mathematical knowledge with the instructional design. According to Wilkie (2015), KCT "includes knowledge about how to choose appropriate representations and examples, how to build on students' thinking, and how to address student errors effectively" (p. 249). Ball et al. (2008) propose that mathematical tasks require a sound mathematical knowledge in order to design instruction. For instance, the teacher needs to know what teaching strategies to employ where and when, what resources to use and what representations and examples to employ so that students can learn with understanding.

KCT involves contingent teaching actions, where, for example, a teacher decides which student contributions to pursue and which to put on hold or ignore. It also includes the teacher's questioning skills. In this study, KCT was seen to be utilized by the case study teacher as described in this section. In addition, I included the knowledge of the exploratory use of concrete materials with learners that facilitates learning with understanding. This inclusion was done alongside questioning techniques that elicited learners' noticing of functional features of different representations, and teachers' ability to respond to or address the learners' errors or misconceptions. I also considered the ability to progress to explicit strategies and to use several teaching approaches to developing students' mathematical thinking through examples, representations, and questioning.

2.3.2.2 Knowledge of Content and Students (KCS)

KCS can be described as the knowledge that integrates knowing about students and knowing about mathematics in a way that enables teachers to relate to learners in such a manner that enhances their learning (Nolan et al., 2015; Ball et al., 2008). According to Ball et al. (2008), KCS "implies an understanding of students' thinking and what makes the learning of particular concepts easy or difficult, but does not include knowledge of teaching moves (p. 378)". Wilkie (2015) argues that with this knowledge teachers are able to "attend to how students typically learn a concept, and to common mistakes and misconceptions" (p. 250). Thus, KCS enables the teacher to anticipate what learners are likely to think, what common errors learners possibly make, what learners will find interesting, motivating or confusing in the work assigned to learners (Ball et al. 2008).

According to Ball et al. (2008), "teachers must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language" (p. 401). The teacher should be able to interpret what learners are trying to communicate. As indicated in Chapter One, South African learners do not only struggle with mathematics but also have challenges with literacy (Spaull, 2013; NEEDU, 2013; Reddy et al., 2015). These learners may not be able to explicitly express themselves and the teacher should be able to understand and interpret the meaning of their poor expressions through KCS. KCS also includes knowing the misconceptions learners have about mathematics and the different topics one teaches.

Lastly, I will focus on the sixth domain of MKfT which is the KCC.

2.3.2.3 Knowledge of Content and Curriculum (KCC)

The curriculum can be defined as the full range of programs that are designed for the teaching of a particular subject and its different topics at a given grade (Petrou & Goulding, 2011). The curriculum includes the variety of instructional materials available in relation to these programs (e.g. the national workbooks). KCC is therefore the knowledge that pertains to the knowledge, evaluation, adaptation and use of these materials in the teaching and learning of different mathematical concepts (Ball et al., 2008).

I provide a summary of the MKfT domains in Table 2.1 below, showing the interconnectedness of Ball and colleagues and Shulman's work.

Table 2.1 A summary of the interconnection between Shulman's PCK and Ball's MKfT

Ball et al. (2008) MKfT domains	**Definition of the MKfT domains**	**MKfT domain indicators**	**Shulman's**
Common Content Knowledge (CCK)	General knowledge of mathematics and mathematical skills used by anybody who has done mathematics successfully at school. Teachers need this knowledge to understand the work they assign to their learners	• calculate an answer correctly • understand the mathematics you teach • recognise when a student gives a wrong answer • recognise when a text book is inaccurate or gives an inaccurate definition use terms and notations correctly	SMK
Horizon Content Knowledge (HCK)	Mathematical knowledge that spans across the mathematics curriculum that helps the teacher to view mathematics as whole, but not in parts	• make connections across mathematics topics within a grade and across grades • articulate how the mathematics you teach fits into the mathematics which comes later	

Specialised Content Knowledge (SCK)	Special knowledge that is specifically required for the work of teaching.	• interpret students’ emerging and incomplete ideas • evaluate the plausibility of students’ claims give or evaluate mathematical explanations • use mathematical notation and language and critique its use • ability to interpret mathematical productions by learners, other teachers or learning materials • evaluate mathematical explanations for common rules and procedures • appraise and adapt the mathematical content of text books	
Knowledge of Content and Teaching (KCT)	Knowledge that combines knowledge of mathematics content and knowledge of teaching	• sequence mathematical content • present mathematical ideas • select examples to take students deeper into mathematical content • select appropriate representations to illustrate the content • ask productive mathematical questions • recognise what is involved in using a particular representation • modify tasks to be either easier or harder • use appropriate teaching strategies • respond to students’ why questions • choose and develop useable definitions • provide suitable examples	PCK
Knowledge of Content and Students (KCS)	Knowledge that combines knowledge of mathematics content and knowledge of students	• anticipate what students are likely to think and do • predict what students will find interesting and motivating when choosing an example	

		• anticipate what a student will find difficult and easy when completing a task • anticipate students' emerging and incomplete ideas • recognise and articulate misconceptions students carry about particular mathematics content	
Knowledge of Content and Curriculum (KCC)	Knowledge of the content requirements of the curriculum and the materials that can be used to teach that particular content	• articulate the topics in the curriculum • articulate the competencies related to each topic in the mathematics curriculum • articulate and demonstrate a familiarity with the structure of the mathematics curriculum • link representations to underlying ideas and to other representations • knowledgeability of available materials (e.g. textbooks) and their purposes	

The above summary concludes the domain indicators that are used in the analysis of the case study teacher teaching counting (discussed in Chapter Four).

Teachers need to know and understand the requirements of the curriculum so that they can fully meet the demands of each topic at a given grade. According to Petrou and Goulding (2011) in the countries which have official curriculum documentation and assessment systems like South Africa, the teachers' KCC should not only include an awareness of possible resources and materials to use during the teaching and learning process but should also embrace the appropriate learning activities that will enhance effective learning of the prescribed content areas. In this study, KCC was used to refer to the knowledge of content taught at a particular grade and the materials that relate to the teaching of that particular content.

2.4 CHALLENGES WITH THE MKfT FRAMEWORK

The MKfT framework, as discussed earlier, is a refinement of Shulman's (1987) PCK and has significant implications for mathematics teacher education. Depaepe, Verschaffel and

Kelchtermans (2013) described MKfT as the most influential reconceptualization of teachers' PCK within mathematics education. However, Hurrel (2013) highlights that the line between the MKfT domains are too blurred and it becomes difficult to discern where one domain ends and where the other one begins. For example, it is difficult to differentiate between CCK and SCK, between KCT and SCK and KCS. Ball et al. (2008) testify that:

> The lines between our four types of knowledge can be subtle. For instance, recognizing a wrong answer is common content knowledge (CCK), while sizing up the nature of the error may be either specialized content knowledge (SCK) or knowledge of content and students (KCS) depending on whether a teacher draws predominantly from her knowledge of mathematics and her ability to carry out a kind of mathematical analysis or instead draws from experience with students and familiarity with common student errors. Deciding how best to remediate the error may require knowledge of content and teaching (KCT) (p.400).

Hurrel (2013) further argues that "the model does not display the possibilities of all the interactions between the domains" (p. 59). The representation of the domains in the diagram suggests the importance of one domain of knowledge over others. For example, SCK appears to employ a larger area of the MKfT model (see Figure 2.2) suggesting it is more important than KCT which occupies a smaller area. Hurrel (2013) argues that there is no evidence in any reading, that any one domain was more important than any other.

Despite the above challenges, I have discovered the advantages provided by this seminal framework to outweigh the challenges

2.5 CONCLUSION

In an attempt to curb the low student performance in mathematics in South Africa, South African teacher education institutions "face an enormous challenge of providing large numbers of adequately and appropriately trained mathematics teachers, at a time when few are choosing teaching as a profession" (Adler, 2005, p. 3). Adler (2005) claims that it is a challenge to get students who have strong mathematical background to train as mathematics teachers, more still as FP teachers. Training institutions have a mandate to equip pre-service teachers with the both the mathematical content knowledge and pedagogical knowledge they need for teaching. This is imperative despite the mostly weak mathematical backgrounds of students who have chosen to study as FP teachers. Education institutions are therefore expected to equip the student teachers with the full set of knowledges as reflected in the MKfT model above.

MKfT is a framework that is currently being researched in South Africa in various forms informing teacher education programmes for secondary school teachers (Kazima et al., 2008; Adler & Venkat 2013). Having looked at PCK and MKfT and how these influence the teaching of mathematics, the next chapter focuses on the key concepts informing my research, that is, the development of number sense broadly and counting in particular.

CHAPTER THREE

CONCEPTUALISING NUMBER SENSE AND COUNTING

3.1 INTRODUCTION

It has been established in Chapter One that among many other factors behind the learner poor performance in mathematics in South Africa is that the learners exit FP with poorly developed number sense. Research also points to poorly developed number sense in FP as a reason for the crisis of poor learner performance in mathematics in South Africa (Schollar, 2008; Hoadley, 2012; Graven et al., 2013; Graven, 2016). In a study conducted on Grade 3 and 4 by Graven and Heyd-Metzuyanim (2014) in Grahamstown, a region in Eastern Cape where my research took place, they found that the majority of learners in Grades 3 and 4 have not yet developed the advanced skills of solving mathematical problems such as breaking down and building up numbers but still relied on tallying and finger counting to solve such problems as 55+67. According to Graven and Heyd-Metzuyanim (2014), such behaviour is an indication of “lack of progression towards more efficient methods away from concrete counting in South African Schools” (p. 31). Graven (2016) argues that learners exit FP with inefficient arithmetic strategies and as a result fail to cope with the arithmetic demands of the higher grades. Schollar (2008), based on findings of the 2004 Primary Mathematics Research Project, confirms that:

> the fundamental cause of poor learner performance across our education system was a failure to extend the ability of learners from counting to true calculating in their primary schooling ... Learners are routinely promoted from one Grade to the next without having mastered the content and foundational competences of preceding Grades, resulting in a large cognitive back- log that progressively inhibits the acquisition of more complex competencies. The consequence is that every class has become, in effect, a “multi-Grade” class (p. 1).

This inability to draw on a range of efficient calculation strategies suggests that many learners in schools in South Africa have not developed the necessary number sense to cope with curriculum demands in the primary school. This study aims to contribute to an under-researched body of research in primary numeracy education in South Africa (Graven et al., 2013). The study focuses on the identified causes of poor performance in mathematics in South Africa and in particular teachers’ lack of content knowledge, particularly in relation to number sense and pedagogical skills.

3.2 CONCEPTUALISING NUMBER SENSE

Different researchers have aired different but harmonising conceptions about number sense that has led to a deeper understanding of what number sense is. Graven et al. (2013) suggest that number sense is more than just the ability to count, and name, identify and write numbers. Rather they argue number sense includes having a sense of what numbers are, understanding their relationship to one another, being able to perform mental math calculations efficiently and effectively, and being able to use numbers in real world situations. Reys, Lindquist & Smith (2007) confirm that number sense refers to a person's general understanding of number and operations along with the ability to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for solving complex problems.

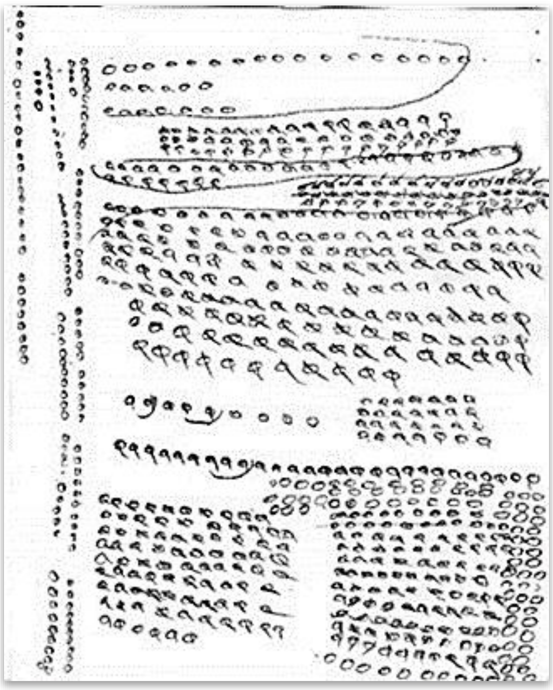
The Department of Basic Education (DBE) (2011) in South Africa identified number sense as characterized by developing an understanding of the meaning of different numbers, their relative sizes, the relationships between them, knowledge of different ways of representing numbers, and operations involving numbers. Number sense can therefore be described as a well organised conceptual framework of number information (Bobis, 1996). This framework entails deep knowledge and understanding of numbers that enables learners to be fluid and flexible with numbers such that they can use numbers to solve problems, identify inaccurate answers, understand how numbers can be decomposed and recomposed in different ways, see connections among operations, calculate mentally, and make reasonable estimates (Burns, 2007).

3.2.1 The importance of number sense

As an intervention to poor mathematics performance by South African learners, the South African Numeracy Chair Projects [SANCP], located at Rhodes University and the University of the Witwatersrand, work (through research and development) to develop mathematical fluency in primary school learners by focusing on the development of number sense. After examining CAPS and the ANAs, the extent of how number sense influences mathematics teaching and learning at FP and IP, the SANCP team concluded that "number sense and mental agility are critical for the development and understanding of algorithms and algebraic thinking introduced in the intermediate phase" (Graven et al., 2013, p. 131).

The report by the National Mathematics Advisory Panel [NMAP] (2008) confirms that poor number sense interferes with learning algorithms and number facts and prevents the use of a variety of strategies to solve arithmetic problems. For example, in South Africa, many children solve addition of two two-digit numbers by unit counting or by uniting when using the standard algorithm (Table 3.1) (Askew, 2012; Graven et al., 2013).

Table 3.1 Showing the learners' incompetence in arithmetic skills resulting from poorly developed number sense.

Unitising when using the standard algorithm	**Grade 5 solution strategy with unit counting**
$$\begin{array}{rcc} & 5 & 8 \\ + & 2 & 3 \\ \hline & 7 & 11 \end{array}$$ The child says: 8 + 3 = 11 and 5 + 2 = 7 In that way each sum '8+3' and '5+2' is seen as separate. Graven et al. (2013)	 Schollar (2008)

The above table exposes the critical role played by number sense in the development of effective and efficient strategies for mathematical understanding throughout mathematics learning. Learners who lack a strong number sense have problems developing the foundation needed to do simple arithmetic (Burns, 2007). Vinjevold and Crouch (2001) confirm that in a study carried out by the District Development Support Project [DDSP] it was found that many Grade 3 learners who took part in this study could not do problems that required the addition of two double digit numbers with carrying[3], such as 86+39. This suggests that many learners leave FP depending predominantly on the use of concrete objects, such as their fingers, and simple counting techniques to calculate and solve problems. Learners exit FP without a well-developed number sense, which makes it difficult for them to solve more complex mathematical concepts later in their schooling. Carlyle and Mercado (2012) state that number sense would enable learners to think flexibly and promotes confidence while working with numbers. Hence Naudé & Meier (2014) refers to it as a "foundational building block for all content areas in mathematics" (p. 79).

The learners in the above examples (Table 3.1) displayed a lack of advanced arithmetic skills that comes with a lack of well-developed number sense. Learners in Grade 5 are expected to have developed more efficient methods to perform calculations than the ones they employed in the examples above. It is not expected that at Grade 5 level learners are dependent on concrete procedures to calculate or solve problems that require abstract methods such as decomposing and recomposing numbers (discussed later in the chapter) or carrying. Graven (2016) argues that learners exit FP with inefficient arithmetic strategies and as a result fail to cope with the arithmetic demands in the higher grades. These efficient strategies are developed through developing number sense. Marmasse, Bletsas & Marti (2000) corroborate suggesting that

> children with a better number sense are able to decompose numbers into smaller groups, usually around powers of 10 or 5, depending on the kind of the problem, or regroup them later, simplifying their problem-solving strategies. Number regrouping and decomposition (derived facts) accelerate problem solving and improve number understanding (p. 6).

The concerns with children's use of inefficient strategies for calculating, as identified above, suggests that the problems of poor mathematics performance in South Africa may be a result of learners' poor number sense. It is such a conclusion that may lead one to conclude that teachers' lack of knowledge of how to develop learners' number sense interferes with learners' learning mathematics in a meaningful way.

In an attempt to address the problem of poor performance in mathematics as a result of poorly developed number sense, the DBE (2011) emphasizes the development of number sense in the CAPS for FP mathematics. The DBE (2011) confirms that "number is the most important topic

[3] I use the term 'carrying' instead of 'regrouping' as the former is the dominant term used in South Africa.

in Foundation Phase Mathematics" (p. 37) and has therefore devoted most of the time of mathematics learning to development of number sense through the Numbers, Number Operations and Relationships content area in comparison to the other content areas. Of the seven hours allocated to mathematics per week, 60% has been allotted to the development of number sense. The other four content areas share the remaining 40%.

In the following section I deliberate on the development of number sense in FP mathematics classrooms.

3.2.2 How number sense can be developed

Number sense develops gradually over time as a result of exploring numbers, visualizing numbers in a variety of contexts, and relating to numbers in different ways (Burns, 2007). The SANCP (Graven, 2016) suggests that the development of number sense begins in pre-primary where learners develop a feel for numbers and enjoyment for working with them as they count verbally, count objects, and add and take away small numbers. Number sense develops as mathematical knowledge in children[4]. From my reading of research, I have identified two different conflicting viewpoints on how mathematical knowledge develops in children. I refer to them as (1) the conventional view of the development of children's understanding of number, and (2) the progressive view of the development of children's number sense. Thorndike (as cited in Clements & Sarama, 2009), proposed that "it seems probable that little is gained by using any of the child's time for arithmetic before Grade 2, though there are many arithmetic facts that he [*sic*] can learn in Grade 1" (p. 198). By contrast, Vygotsky (as cited in Clements & Sarama, 2009) argued that "children have their own preschool arithmetic which only myopic psychologists could ignore" (p. 84). In this next section, I will discuss these changing views on how young children's mathematical knowledge develops.

3.2.2.1 A conventional view of the development of children's knowledge of number

Theorists holding a conventional view of the development of number knowledge (Thorndike, 1922; Piaget, 1965) believed children are born with no ability or capacity to engage in logical thinking. Thorndike (1922) viewed pre-schoolers as 'blank slates' or 'empty vessels' who would only pass time through idle play until they began school. It was only at school where

[4] When I use the term 'children', I am referring to children who are not yet in school and those that are at school. It is a generic term. When I use the term 'learner' I am referring to children in school only.

they would start ‘real’ mathematical learning under the discipline of the teacher (Baroody & Wilkins, 1999). According to Baroody and Wilkins (1999), this conventional view was based on three assumptions: (1) children were uninformed and helpless; (2) learning was a passive process and (3) children were not naturally interested in learning mathematics. The first assumption was based on the work of Thorndike (1922). Thorndike (1922) viewed young children to be so mathematically incompetent that he concluded it was waste of time to engage children in arithmetic before Grade 2.

The second assumption was based on the first. Since children were uninformed and helpless, learning was a passive process. Teachers were believed to be the ‘knowers of all’ and the learners were expected to sit and listen carefully to what the teacher said and just memorize the facts even without understanding them. The learners would only have to reproduce these facts through memory when required to. A ‘jug and mug’ relationship existed between the learners and their teacher. The teacher was considered all knowledgeable and the learners were empty mugs that waited to be filled with the knowledge from the teacher (Thorndike, 1922).

The third assumption was that children are not naturally interested in learning mathematics and it was therefore necessary for teachers to motivate them to learn. In this respect learning was promoted through rewards and punishment.

From this perspective, the development of children’s knowledge of number was solely dependent on the teacher. The learners’ responsibility was to memorize all the number-related knowledge taught by the teacher. Contrary to the conventional view is the progressive approach to developing children’s knowledge of number.

3.2.2.2 The progressive approach to developing knowledge of number

The progressive approach purports that the development of mathematical knowledge begins well before school (Ginsburg, 1977; Griffin, 2004). Within this approach researchers affirm that children are born with the capacity to develop mathematical reasoning and therefore need to be actively involved in their learning process (Baroody, 1987; Koehler & Grouws, 1992; Dehaene, 1997; Kamii, 1997; Griffin, 2004). These assumptions are based on the work of pioneering neuro-psychologists (e.g. Dehaene, 1997) and mathematics educational researchers (e.g. Gelman & Gellistel, 1978). I present the neuro-psychologists’ perception in relation to development of number sense first.

According to Dehaene (1997) "every human being is endowed with a primal number sense, an intuition about numerical relations. Whatever is different in adult brains is the result of successful education, strategies, and memorization" (p. 21). Pioneer researchers in cognitive neuro-psychology and infant cognition concur that human infants are born with brain structures that are specifically attuned to numerical quantities. These structures are partially independent of the brain structures that support verbal processing (Dehaene, 1997). Infants use these in-born structures to distinguish one set of objects from another in the first few days of life (Dehaene, 1997).). They are able, for example, to recognize the number of people in their presence even though they have not yet developed the language to describe it. They can realize there is one person here and two people there.

Similarly, research by mathematics education researchers, such as Gelman and Gellistel (1978), confirm that children as young as two years of age can instantaneously recognise the number of objects in a small group before they can actually count with understanding. As children grow older, their natural quantitative competencies expand and they develop a language which makes it possible for them to describe quantities through numbers. Griffin and Case (as cited in Griffin, 2004) confirm that by the age of 4, children would have constructed two schemas: one for comparing quantity and another for counting. Griffin (2004) further suggests that

> at age 5 or 6, children experience a revolution in thought as they merge these two schemas into a single, superordinate conceptual structure for number. This new concept closely connects number with quantity and enables children to use the counting numbers without needing the presence of physical objects to make a variety of quantity judgments, such as determining how many objects they would have altogether if they had 4 of something and received 3 more. With this new conceptual structure, which researchers believe provides the basis for all higher-level mathematics learning, children have acquired the conceptual foundation for number sense (p. 2).

In other words, children are not born with number sense but with the capacity to develop it. As their schema for comparing quantity merges with that of counting, a new brain structure is formed that helps them to develop number sense. Number sense will then gradually develop over time as children explore numbers, visualise them in a variety of contexts, and relate them in ways that are not limited by traditional algorithms (Sood & Jitendra, 2007). Teachers are therefore tasked with the responsibility to expose learners to situations and tasks that will enforce the development of this number sense.

There are a number of ways in which teachers could develop their learners' number sense. Griffin (2004) proposes that teachers need to understand that the discipline of school mathematics is comprised of three worlds. These three worlds represent the actual quantities that exist in space and time, the counting numbers in the spoken language, and formal symbols written as numerals and operation signs. According to Griffin (2004) number sense develops when a rich set of relationships among these three worlds is constructed. Learners should first link the real quantities (that is, objects) with the counting numbers to be able to connect this integrated knowledge to the world of formal symbols and gain an understanding of the meaning of numbers. He substantiates that teachers should provide learners with the opportunities to discover and to construct relationships among these three worlds at higher levels of complexity to attain number sense by providing rich activities for making connections, exploring and discussing concepts. Teachers should also ensure the concepts are appropriately sequenced.

Tsao and Lin (2012) clarify Griffin's ideas suggesting that teachers should provide opportunities for learners to: (1) work and play with concrete materials; (2) build up and break down numbers; (3) use different arrangements and representations of number; (4) work with large numbers and their representations using number lines; (5) solve realistic problems using a variety of approaches; (6) discuss and share their discoveries and solutions; (7) explore number patterns and relationships; and (8) measure, estimate measures and calculate with a purpose.

Teachers need to understand that their learners are not born with number sense but with the capacity to do so and they therefore have the responsibility to promote number sense development by providing rich mathematical tasks connected to each learner‘s real-life experiences and encouraging them to connect the tasks to their own experiences and their previous learning (Back, Sayers & Andrews, 2013). Baroody and Wilkins (1999), in support argue that

> most children naturally seek out opportunities to acquire new information and practice new skills. They have a natural interest in hearing and rehearsing again and again the string of words that adults call numbers. They repeatedly practice counting sets of real or pictured objects. Children are also curious about numbers and often ask questions to fill in gaps in their knowledge (p. 25).

Baroody and Wilkins (1999) assert that children have an inborn curiosity to learn mathematics and to understand the world of numbers. It then lies with the teacher to utilize this curiosity by exposing learners to learning situations that will perpetuate this readiness and a love for learning. Counting is one of the most important mathematical skill that is formally introduced to learners at foundation phase as a way of developing number sense and for problem solving. Naudé & Meier (2014) writes that "learning to count is a great achievement in a child's life. Although counting itself does not equate to understanding of a number, it is often seen as the starting point of developing number sense" (p.79). Marmasse et al. (2000) emphasise the importance of formal education provided by teachers in the development of number sense

> it is apparent that the child's concept of numbers and arithmetic gradually changes, affecting the observable skills. The strongest influence on arithmetical development is formal education, which can lead to the development of skills that would not have emerged in a more natural environment, without formal instruction (p.11).

The importance of teacher involvement in assisting learners to develop number sense cannot be under-estimated. Reys et al. (2007) agrees that there is no other subject that is susceptible to such extremes of good and poor teaching as mathematics. He claims that poor performance is a result of poor teaching which emanates from failure to bring out the excitement of 'creating mathematics' in learners.

Naudé & Meier (2014) suggests that teachers can make mathematics fun and unthreatening by engaging the use of friendly and familiar numbers to help learners understand how numbers relate to one another. She reiterates that this will result in learners who are eager to learn and are confident to tackle any mathematical problem. Building on this, Reys et al. (2007) proposes that teachers need to first develop love for mathematics before they can teach it effectively. He argues "a *sine qua non* for making mathematics exciting to pupils is for the teacher to be excited about it first. If he is not, no amount of pedagogy training will make up for the deficit" (p. 4). This suggests the need for mathematics teachers to develop a positive attitude towards mathematics and present problems to their learners that are related to their experiences both inside and outside the classroom (Burns, 1997). Burns (1997) further suggests that since number sense develops over time, learners should therefore be exposed to regular opportunities to manipulate and reason with numbers, hear others' thoughts and opinions, and crystalize their own thinking.

Counting is regarded as the first step to developing number sense (Naudé & Meier, 2014). Children generally start meeting and understanding numbers when they learn to count.

According to Marmasse et al. (2000), “counting is an important exercise for children. It helps them explore the relationships between numbers. Reflecting on number ordinality and realizing that smaller numbers are included within bigger numbers helps them modify their problem-solving strategies” (p.5). For this reason, my research focuses on the MKfT required to develop the learners’ number sense through counting. Counting is one of the most important mathematical skill that is formally introduced to learners at FP as a way of developing number sense and for problem solving. Naudé & Meier (2014) writes that “learning to count is a great achievement in a child’s life. Although counting itself does not equate understanding of a number, it is often seen as the starting point of developing number sense” (p.79).

In the next section I will discuss counting as an important concept in the development of number sense. I will define counting, its importance and how it should be developed in a meaningful way. In so doing I provide a rationale for my focus on the teaching of counting in this research.

3.3 COUNTING

Counting is one of the first mathematical concepts that children learn and it is an important developmental milestone in most cultures of the world. As Chrossely (2007) confirms, counting forms the inception of mathematical elements in all cultures. Counting lays the foundation for many mathematical concepts and, as such, plays a crucial role in the development of number sense. Research suggests two different perspectives on counting. The first perspective views counting as reciting number names and identifying the quantity of a given collection of objects. The second perspective is an approach that perceives counting as a method of solving mathematical problems. These two perspectives will be examined in the section below.

3.3.1 Counting perspectives

The first perspective considers counting as involving number words and the sequence of number words, one-one-correspondence, and cardinality (Education Development Centre [ECD], 2015). In other words, counting involves both the ability to recite number names and the identification of the units of items in a collection and assigning a number name to each unit (Chrossely, 2007; Reys et al., 2007) argues that “true counting is the process whereby a correspondence is set up between the objects of the collection to be counted and certain symbols, verbal or written” (p. 33). Important for Chrossely (2007) is the view that “counting only occurs when we have something to count. Putting things together into a group both gives us the opportunity to count the objects and also provides the necessity to count" (p. 43).

The second perspective by Wright (2008) offers a different interpretation of counting. He views counting as a problem-solving strategy. He identifies the process of calling numbers by names in ascending or descending order as forward or backward number word sequences [FNWS or BNWS] and argues that "counting refers to situations where the child uses the FNWS or BNWS to solve problems" (p. 196). In my research, I embrace both of these perspectives. Counting will be considered as reciting of number names (verbal counting), identifying the objects in a collection by number names in order to find the muchness of a collection (rational counting) and also as a strategy of solving mathematical problems (advanced counting). Each of these kinds of counting will be discussed later in the chapter. Before doing that, I will focus on the importance of counting.

3.3.2 The importance of counting

Counting is central to everyday life experiences. Even young children are exposed to the need to count. For instance, a pre-schooler might have to look for her two shoes or a pair of socks, or compare sweets or cookies given to them and those given to siblings, and respond to questions involving number (e.g. 'How old are you now?'). It is therefore important to teach them the meaning of numbers.

The National Association for the Education of Young Children [NAEYC] and NCTM (2002), in a joint position statement emphasize the need to develop understanding of the meanings of whole numbers and recognition of the number of objects in small groups without physical counting (perceptual subitising), and by counting to find the muchness of a collection among pre-schoolers. In agreement, the DBE (2012) confirms that children are often introduced to number through counting and therefore credits counting as an important mathematical skill used throughout the FP for solving mathematical problems. According to the DBE (2011), "counting enables learners to develop number concept, mental mathematics, estimation, calculation skills and recognition of patterns" (p. 9). This suggests learning to count with understanding is central to building of number sense and to the mathematics learning as a whole. Askew (2012) suggests that learning to count: (1) helps children to develop the language of numbers and makes the children understand better the meanings of songs and rhymes they sang before coming to school; (2) leads to an understanding of the muchness of numbers; and (3) provides a tool for solving mathematical equations and word problems.

Having discussed the importance of learning to count I now move on to deliberate on the process of counting.

3.3.3 The counting process

Askew (2012) alleged that "the individual child is at the centre of learning mathematics, learning mathematics is a process of acquiring knowledge and acquiring this knowledge is a well ordered process" (p. 3). Learning to count is a process of acquiring knowledge about numbers and using that knowledge to solve problems. Similarly, it is a well ordered process that is regulated through some principles. This section views the process of counting. I do this by examining the principles that govern this process of counting first.

3.3.3.1 The principles of counting

Gellman and Gallistel (1978) identified five principles they claimed young children displayed in learning to count which are now commonly known as the counting principles. These include the stable order principle, one-to-one correspondence, cardinality principle, order irrelevance, and the abstraction principle. The first three of these principles are regarded by Clements and Sarama (2009) as the 'how to count' principles. In other words, they give guidance on how counting should be done. The latter two are identified as the 'what to count' principles. Clements and Sarama (2009) have included a sixth principle which they identified as the *movement is magnitude* principle. The principles will be briefly discussed in this section. Clements and Sarama (2009) claim there is adequate research evidence that children understand all these principles explicitly and implicitly by the age of five.

The *stable order principle* is a counting principle that depicts that counting follows an ordered sequence that does not change, regardless of where the counting starts from for example, 1, 2, 3, 4, or 16, 17, and 18. Marmasse et al. (2000) argue that counting involves more than the ability to assign arbitrary tags to the objects in an array. It requires the learner to know the order of the sequence of numbers so that when the counting is done one can be able to determine the correct number of the objects in the collection. Gelman and Gallistel (1978) claim that young counters understood the stable order principle in a way that if they master a wrong sequence of counting numbers they maintain it because they understand that counting requires using the same sequence of number words without skipping or repetition.

The *one to one principle* emphasizes the significance of assigning only one counting name (number word, alphabet element, or other) to each counted object in the collection. For example, the learner is never expected to count "one, one, two" while counting three objects. To follow this principle, a learner has to coordinate the process of partitioning and tagging the objects. Haylock and Cockburn (2008) suggest that as learners learn to count, they should "learn to co-ordinate the utterance of the number word with the movement of the finger and the eye along a line of objects, matching one noise to one object until all the objects have been used up" (p. 41). This means that every item being counted needs to be tagged from the 'to-be-counted category' of the collection giving it one name following the ordered sequence of the number names until all the objects are tagged. The number name given to the last object determines the muchness of the collection.

Marmasse et al. (2000) argue that the purpose of counting is to determine the muchness of the collection. This is referred to as *the cardinality principle*. It is important therefore that as the learners count, they should understand they are looking for a muchness of a collection, they are counting for a reason. Bruce and Threlfall (2004) suggest learners should be taught to answer the 'how many?' question as they count. The cardinality principle depicts an understanding that the last number word of a collection of counted objects has a special meaning as it represents the set as a whole and the muchness or numerosity of this collection of items. According to Jordan and Montani (1997) insufficient understanding of the cardinality principle results in learners developing learning difficulties. Those who understand the cardinality of numbers can depend on their understanding of cardinality and counting proficiency to solve different problems (Gifford, 2005). They also understand that quantity can be represented verbally, physically or symbolically (Naudé & Meier (2014).

The *order irrelevance* principle underpins the need for learners to understand that the order of enumeration (from left to write, right to left, top to bottom or any other way) is irrelevant as long as at the end of the counting the muchness of the collection is attained. Haylock and Cockburn (2008) argue that teaching order irrelevance is a "sophisticated piece of learning" (p. 42) and teachers should be careful of learners who may rigidly follow taught procedures and fail to identify correct but unusual ways of counting (Wynn, 1990). Gifford (2005) therefore proposes that children should be encouraged to explore different ways of counting to help them recognise that some different counting procedures may yield the same results.

The *abstraction principle* illuminates an understanding that the quantity is not determined by the features of objects being counted, such as size, shape or colour. Marmasse et al. (2000), and Cotton (2010) suggest that it is significant for learners to realize that counting could be applied to heterogeneous objects like toys of different kinds, colour, or shape. They further support the importance of granting learners an opportunity to demonstrate skills of counting in actions or sounds.

The *movement is magnitude* principle suggests that as one moves up the counting sequence the quantity increases.

While children develop these principles of counting, they move through three stages of counting (Griffin, 2004). It is the stages of counting that I focus on now.

3.3.3.2 The three stages of counting

According to Griffin (2004) counting is the first mathematical pattern children encounter. However, the ability to count accurately develops over a long period of time and most children follow a natural developmental progression to be able to count meaningfully. Learning to count begins when the toddler starts making the connection between the inherent sense of 'how many there are' and the language we use to count (Griffin, 2004). Again Gifford (2005) argues that children learn some number words as soon as they start to talk. As children grow older, their natural quantitative competencies expand and they develop a language which makes it possible for them to describe the quantities through numbers. The ability to count cannot forced on learners but develops as an individual child personally construct the idea of counting in order to understand what is to count and how to count (Naudé & Meier, 2014).

Counting may start off as nothing more than just a song or rhyme or a pattern of sounds uttered without any apparent purpose (Ginsburg, 1977). With time the child extends this skill to the task of determining the number of items in a collection then learn how to use the counting sequence to create their own collections and to determine the number in successively larger collections. The child then learns to use the acquired knowledge or counting skills to solve problems (Baroody & Wilkins, 1999). Thus the process of counting develops through three stages (Saddler, 2009): rote counting; rational counting; and advanced counting. These three stages are deliberated below.

Stage 1: Rote Counting

The process of counting starts off as oral counting commonly referred to as verbal counting or rote counting. For the purposes of clarity and context of the research these terms are going to be used interchangeably. Children develop oral counting skill very early in life; sometimes even before a child is two years of age (Gelman & Gallistel, 1978; Fuson, 1988). Rote counting involves reciting the number names in an ordered counting sequence from memory (Askew, 2012) as stipulated by the stable order counting principle discussed above. Gifford (2005) claims that when children start to count they have no understanding of the number name sequence. In agreement, Fuson (1988) suggests that they may not even realize that the counting sequence is composed of distinct words and children may memorize '*onetwothree*' as a single sound chunk, then later realize that the number-word sequence is composed of a chain of distinct sounds 'one; two; three'. They may learn the number words but may not necessarily sequence them properly. With more practice and exposure they finally learn the accurate sequence of numbers. The string of correct sequence starts building up and grows longer and longer with more and more practice and exposure (Fuson, 1988).

The DBE (2012) and Naudé & Meier (2014) state that rote counting is essential for learners. Through rote counting learners develop the knowledge of number names, their sequence and the pattern that is within number names. The DBE (2012) emphasises the need for learners to gain lots of experience with rote counting before they are introduced to rational counting. The Centre of Innovation in Education (2011) suggests that if teachers and parents would take advantage of natural counting opportunities there would be less need to contrive special counting activities. Children can memorize counting sequences through counting songs and rhymes.

Naudé & Meier (2014) however, argues that teachers should make a planned effort to develop learners' verbal counting in the classroom as "some learners may know some number names but not necessarily the right sequence - and they will benefit when they hear and participate without being put on the spot" (p. 81). She further suggests that teachers can assist learners to develop verbal counting through exposing them to: frequent and repeated opportunities for verbal counting through rhymes, songs, actions and games; kinaesthetic experiences such as moving and clapping while reciting the number names to assist learners in internalising the counting sequence; and point counting using the number line or number grid in order for learners to visualise the number symbols and the number sequence.

The DBE (2012) however argue “although rote counting plays an important role in developing the social knowledge about the number names, children who can rote count do not necessarily associate meaning with the word” (pp. 27-28). The DBE (2012) suggests learners need to be exposed to rational counting to understand the meaning of the counting words (number names). The following section discusses rational counting.

Stage 2: Rational Counting

Askew (2012) simply defines rational counting as the act of counting physical objects. It involves counting the physical objects and matching them with the number names so as to determine the cardinal value. Being able to count involves both procedural skills and conceptual understanding (Fuson, 1988). The learners must first be able to follow the procedure of saying the number words in the correct order (that is, verbal counting), and then demonstrate one-to-one correspondence by saying only one of the counting words as they point to each item or object. Learners must conceptually understand that when counting is correctly done, the final number said represents the muchness or quantity of the set of collection.

The main purpose of rational counting is to assist learners to gain understanding of cardinality (Fuson, 1988). An understanding of cardinality and the connection to counting is however not an easy task for some children. Teachers are therefore advised to frequently ask their learners ‘how many are they?’ at the end of each count so that they may understand that the last number name called out represents the quantity of the collection which is cardinality (Fosnot & Dolk, 2001).

Baroody and Wilkins (1999) propose that counting is not an easy task as it requires a child to simultaneously know: (1) the number-word sequence; (2) that each object in a set is given one counting word (one-to-one correspondence); (3) to keep track of counted and uncounted objects so that each object is labelled once and only once; and (4) monitor and stop the counting process at the requested number to be able to enumerate the collection of objects correctly. At the initial stages of counting learners find it challenging to coordinate the skills of tagging the object at the same time saying the correct number word (Baroody & Wilkins, 1999). Fuson (1988) suggests that using the traditional way of counting at the initial stages of counting can help learners coordinate the counting skills mentioned above. Traditional counting consists of continually increasing the elements of the set (mentally or verbally), in the same order while marking or moving away those elements to avoid counting the same element more than once, until no unmarked elements are left to determine the value of the set (Fuson, 1988). They may for example point to the first item in a collection and label it "One, two, three," because they have difficulty simultaneously pointing and controlling their number-word sequence. However, with time children learn to coordinate these skills to ensure one-to-one correspondence, starting with smaller collections. Reys, Lindquist and Smith (2007) suggest teachers should encourage their learners who have not yet developed the order irrelevance principle to arrange the counting objects in a way that will facilitate easy and fluent counting.

It is important to note that with rational counting learners go through four levels of development. The first, level is the concrete level of understanding where learners are heavily dependent on physical objects to represent quantities. According to Naudé & Meier (2014) most learners in Grade 1 will be at this level and some may even remain at this level till Grade 2. They propose the learners' prior knowledge of counting' quantity and the practice opportunities they have determines how long they will remain in this level. The second level is the semi-concrete level of understanding which develops once a learner has had sufficient experience with concrete objects. It is at this level that they begin to represent objects with pictures. In other words, they use representations of objects (Naudé & Meier, 2014). These representations are usually direct representations meaning that the children draw the actual object. The third level is the semi abstract level where the learners have moved on from using direct representations to using other representations such as tallies (an example of this is evident in table 3.1). For example, if the problem is about the wheels of a bicycle they no longer draw the wheels but make two tallies to represent the wheels of each bicycle. The fourth and last level is the abstract level where learners have graduated from using any tallies to understanding that symbols are representations of quantity and numbers.

When learners have gone through all the four stages of development they are then ready to see connections and relationships in numbers and use these for problem solving. They are then able to do the last of the three stages of counting, which is, advanced counting.

Stage 3: Advanced counting

Advanced counting relates to Wright's (2016) understanding of counting. Wright (2016) proposes that counting is 'facility' with number word sequences (Forward Number Word Sequence [FNWS] and Backward Number Word Sequence [BNWS]) to solve mathematical problems. Wright (2008) defines counting as "situations where the child uses a FNWS or a BNWS to solve a problem – typically a problem about quantities or items of some description" (p. 196). For the purposes of my research I will refer, as Wright (2016) does, to this kind of counting as advanced counting.

Reys et al. (2007) contend that after learners have mastered rational counting they should be able to see patterns and relationships in numbers that will help them to solve both real life and mathematical problems. They propose that learners should be encouraged to develop from using simple problem solving strategies such as count all to use more efficient and sophisticated strategies such as counting on, counting backwards, and skip counting to solve mathematical problems. Counting all is when a learner is required to add 3+5, they count 3 counters, then count the other 5, put the two groups of counters together then count all of them to determine the answer. It is regarded as the first and easiest level of counting (Naudé & Meier, 2014).

Counting on is a counting strategy where a learner starts from any number and gives the correct number names as counting proceeds (for example, 3+5 is calculated by starting at 5 then count on three numbers e.g. 5_6, 7, 8) (Reys et al., 2007). Learners do not usually have challenges with this strategy (Naudé & Meier, 2014). Naudé & Meier (2014) explain that counting is as an essential strategy for developing counting and it leads learners to many other valuable strategies.

Counting backward is when learners count back from any point giving the correct number names (for example, 5 - 3 is calculated by counting backward 3 starting from such as 5, 4, 3, 2). Research argue that many children find it difficult to count backward just as much as many adults find it difficult to recite the alphabet backward (Fuson & Hall, 1982; Wright, 1991; Reys et al., 2007). According to Fuson and Hall (1982), learners usually find counting backward a slow and difficult process that is highly dependent on the existing knowledge and fluency in the forward counting sequence. Wright (2016) identified the following as the difficulties learners face as they count backward; (a) learners may count down to the wrong decuple[5] for

[5] Multiples of ten

example 45, 44, 43, 42***, 41, 30, 39***, 38 …; (b) they may omit the decuple (45, 44, 43, 42, ***41, 39, 38*** …); c) they may omit the repeated digit (72, 71, 70, 69, 68, 67, 65, 64, …) (omitted 66); or (d) confuses '***teen***' & ***'-ty'*** for example 23, 22, 21, 20, 90, 80, 70, 60 50, … Instead of 19, 18, 17 they say 90, 80, 70. However, Wright (1991) proposes some strategies that teachers should use to enable learners to count back with less difficulties. Reys et al. (2007) suggest that the calculator can provide a valuable instructional tool to help children improve their ability to count backward. The calculator digits are listed from nine going down to naught. Reys et al. (2007) however suggest that the instruction in counting should expose learners to a lot of practice in forward and backward counting, making use of calculators, calendars, number grids and number lines. They assert that counting backward assists learners to establish sequences and relate numbers to each other in different ways. Counting back is a strategy for early subtraction where a learner counts back the number to be subtracted, starting from the number to subtract from (Wright, 1991; Naudé & Meier, 2014). An example is, 17-5, where a learner moves backwards five counts starting from seventeen.

Counting on and counting back can be used as early strategies to solve simple addition and subtraction problems (Naudé & Meier, 2014). Thus, Wright (2016) suggests that counting should not be regarded as a topic distinct from early addition and subtraction. Instead it should be viewed as integrating these concepts as early as possible.

Another advanced counting strategy is skip counting where learners instead of counting by one give correct number names counting by values other than one. Skip counting provides learners with readiness for multiplication and division (Reys et al., 2007; Clements & Sarama, 2009; Saddler, 2009; Naudé & Meier, 2014). Reys et al. (2007) further suggests that skip counting coupled with counting on and counting backwards equips learners with excellent preparation for counting change when dealing with money. Gifford (2005) however, argues that the ability to use counting to solve problems is dependent on the learners' counting proficiency and their understanding of cardinality.

Having identified the principles and stages in learning to count, I next consider the challenges children face with learning to count.

3.3.3.3 Counting challenges with the number words and their sequence

The process of learning to count and understand numbers and their sequence is not an easy one. Researchers propound that during the early stages of learning to count, learners face challenges that are related to the structure of the number system (Fuson, 1988; Gifford, 2005; Reys et al., 2007). It is argued that learners struggle to master the counting sequence of the arbitrary numbers because mastering the counting sequence is dependent on the child's memory and there is no pattern to aid the memory. Reys et al. (2007) posits that it becomes easier for children to grasp a counting sequence when they identify its pattern. They propose "patterns facilitate the counting process" (p. 160).

Researchers note that number names are arbitrary up to twelve and learners have to rely on memory to master the number name and sequence (Fuson, 1988; Baroody & Wilkins, 1999; Gifford, 2005; Reys et al. 2007). They propose that from thirteen onward counting becomes easier as learners can depend on the pattern of the number names such as ***thir***teen with ***thir*** standing for three, ***fou***rteen with ***four*** for four, ***fif***teen with ***fif*** for five.

Gifford (2005) also proposes that counting up to sixteen forms the most challenging part in learning the counting sequence as it does not have any identified pattern but requires learners to recall from memory the sequence of arbitrary words. Fuson (1988) maintains that the pattern before sixteen is however not as clear as from ***six***teen, ***seven***teen, ***eight***een, and ***nine***teen.

Gifford (2005) claim that the counting sequence after ten easily confuse beginner counters. She proposes that "it would be more logical to count ***one***-teen, ***two***-teen ***three***-teen, ***four***-teen, ***five***-teen, ***six***-teen" and so on, as this is suggestive of the order of the counting numbers (p. 79). She argues that Asian speakers master counting faster than the English speakers because the Asian way of counting follows the arbitrary pattern of counting in ones after each ten such as *ten-one, ten- two*, *ten-three* for 11,12, 13. The isiZulu way of counting does the same, e.g. *ishumi lanye* (ten and one) for eleven, *ishumi nambili* (ten and two) for 12, *amashumi amabili nantathu* (two tens and three) for 23. Gifford (2005) also argues that the written symbols offer more confusing clues as sixteen is written as 16 which suggest *ten six*, 17 suggest *ten-seven* yet they are counted as *six-teen, seven-teen.* Fuson (1988) claims that sometimes when learners count, they develop their own counting rules that may have errors as they endeavour to progress counting from ten such as saying "*ten-one, ten- two*" or...*nine- teen, ten-teen, eleven-teen*. She asserts that even in these 'rule-governed errors' the learners still maintain the stable order principle. The next section considers errors that children commonly make as they learn to count.

3.3.3.4 Common counting errors

As learners demonstrate their knowledge about counting and cardinality across a variety of tasks and situations they may encounter a number of challenges and make errors that indicate they are still developing an understanding of counting. Clements and Sarama (2014) suggest the errors summarised in Table 3.2 below seem to be the most common among learners as they develop their counting proficiency. Reys et al. (2007) suggest errors on Table 3.2 to be counting errors that are related to place value.

Table 3.2 Common Counting Errors

Concept /Skill	Typical Errors
Number words and sequence	When reciting number words or using them in counting situations, a learner may: • omit numbers • repeats numbers
One-to-one correspondence	When counting a set of items, a learner may: • skip an item and not include it in the counting sequence • assign more than one number word to a single item • point to two or more items while saying one number word
Cardinality	After counting, when asked how many there are in the set, a learner may: • give the wrong number through guessing • recount to determine the number of items
Comparing number/ Number conservation	When asked to compare two sets and identify which has more, for example Set 1 –••••• Set 2 – • • • • • answers ‘Set 2’ based purely on perceptual cues

	• answers ‘Set 2’ even after prompted to count

Adapted from Education Development Centre [EDC] (2015, p. 3)

While the EDC (2015) propose a number of errors that are associated with counting, Reys et al. (2007) identify counting errors that are specifically related to place value. These are given in Table 3.3.

Table 3.3 Counting errors related to place value

Concept/Skill	**Characteristic of the error**
Counting	Becoming confused when counting teen numbers because of the lack of pattern in the numbers from eleven to nineteen
Bridging the decade or hundred	Making the transition to the next decade when counting for example counting aloud thirty-eight, thirty-nine, thirty-ten or writing 38, 39, 3010
Reversing digits when writing numbers	For example, writing 52 as 25 and not recognising any difference in these numbers.
Writing numbers that were read aloud	For example, writing one hundred and sixty-four as 100604

Adapted from Reys et al. (2007 p. 183)

Learners commonly make the errors that are related to counting highlighted in Table 3.2 and those related to place value as shown in Table 3.3 above as they go through the process of learning to count. I will elaborate on errors that relate to place value because the one that relates to counting is clear and self-explanatory on the table given.

As stated above, patterns make counting easier and understandable for young counters. Lack of patterns leads to confusion and errors. As presented in Table 3.3, Reys et al. (2007) allege that learners confuse ‘teen’ numbers because of the lack of pattern in the numbers from eleven to nineteen. Learners also struggle to transit from one decade to the next because of the unclear pattern but find it easier to count thirty-eight, thirty-nine, thirty-ten because there seems to be a pattern of counting numbers *eight*, *nine*, and the next number should be *ten* (Reys et al., 2007). Learners often do not know what number comes after the ninth digit of each decade and may actually proceed to an incorrect decade, for example, 57, 58, ***59, 30***, 31, 32 (Wright, 2016). Gifford (2005) confirms that "children commonly get stuck at 29 and may say 'twenty-ten, twenty-eleven" (p.79) following the pattern of the memorized arbitrary number sequence. The problem of failing to bridge the decades is common with young learners who are still trying to master the sequence (Gifford, 2005; Wright, 2012). Gifford (2005) suggests that if learners get stuck during the counting process it is appropriate for the teacher to tell them the next number in the counting sequence so that they do not completely lose their confidence. She also suggests teachers should assist learners to learn the numbers that bridge the tens in pairs such as '29, 30'; '39, 40'; '49, 50' individually through exposing them to the 100 square where these numbers appear together at the end of the lines. Wright (2012) explains that when learners have consolidated their knowledge of counting "rehearsal mode instruction is useful to habituate the sequence" (p. 32). He suggests that the teacher should lead regular brisk counting lessons to habituate the correct counting sequence and assess the learners regularly to check for mastery.

Gifford (2005) alleges that learners often confuse the teen numbers and the decuples. She argues that learners "dovetail these two patterns together. Sixty, seventy, eighty, also sound like sixteen, seventeen, eighteen, which can be problematic to children with hearing difficulties" (p. 79). Teachers are therefore advised to listen carefully as learners count so that they can help their learners rectify the ***teen*** and ***ty*** sound as they count (Reys et al. 2007).

Learning to count is a complex process but it is in many ways the foundation of the development of number sense.

3.4 CONCLUDING REMARKS

Number sense is an important element in the learning of mathematics with understanding. It is considered an essential building block in the learning of mathematics. Without a well-developed number sense learners struggle to cope with the demands of learning mathematics later in life. This is reflected in the current crisis of poor learner performance South Africa is going through. Counting is the significant starting point for the development of number sense. Counting is a well-ordered process that is governed through principles. This process goes through three stages, identified as rote counting, rational and advanced counting. However, there are some common challenges that learners meet during this process of learning to count such as mastering the sequence of the arbitrary counting number names, confusing the teen and the decuples, bridging the decades etc. Because of these challenges, learners often make a number of errors that are associated with both counting and place value as shown in this chapter. Teachers are advised to bear in mind that number sense develops when students connect numbers to their own real-life experiences and therefore should engage learners in interesting and meaningful activities that will facilitate effective learning.

My research aims at establishing what MKfT teachers require in order to develop their learners' number sense through counting. The next chapter discusses the methodology that was used to gather data for the investigation of the MKfT enacted in an expert foundation phase teacher that she used to develop number sense through counting to her Grade 2 learners.

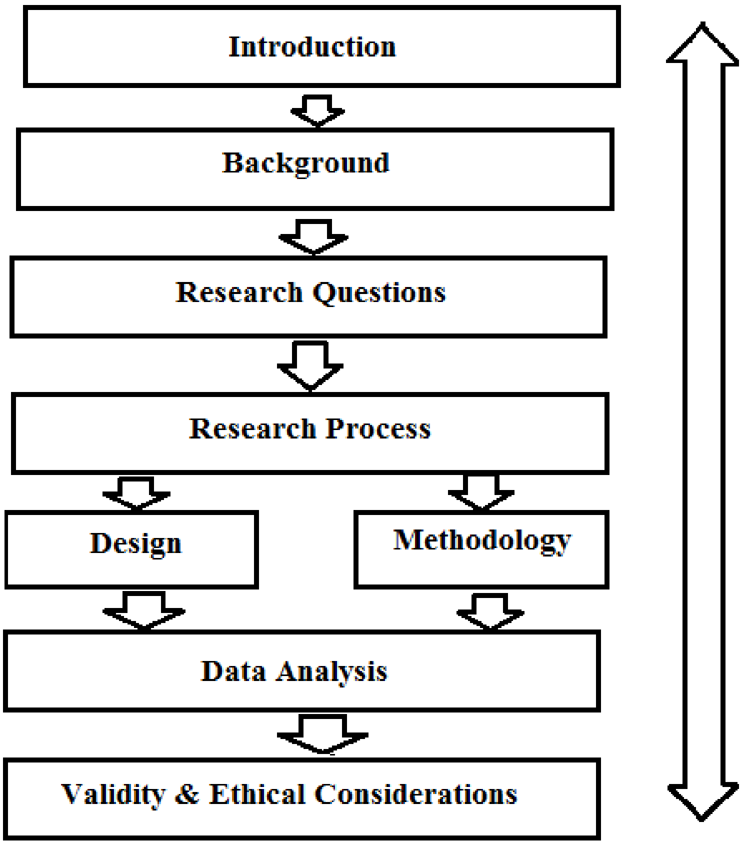
CHAPTER FOUR

RESEARCH METHODOLOGY

4.1 INTRODUCTION

This chapter discusses the research design and the methodological processes that were followed in my research study. It includes an outline of the research approach, the research goal, the main questions that guided this study, the research sample, the research methods, the data collection methods and how the data was analysed. Quality criteria (validity and reliability) and ethical considerations are also included. Figure 4.1 outlines the presentation of the chapter.

Figure 4.1 Outline of the Research Design and Methodology



Appendices 1 to 3 provide the consent documentation and information sheet referred to in this chapter. Appendix 4 provides an example of an interview, as described in this chapter.

4.2 BACKGROUND TO THE RESEARCH DESIGN

The original intention of my research was to explore what Mathematics Knowledge for Teaching (MKfT) was developed in the 2016 Post Graduate Certificate in Education (Foundation Phase) [PGCE-FP] students at a university in Eastern Cape, South Africa through their mathematics method course. I intended to investigate the MKfT their mathematics education method course lecturer employed while lecturing multiplicative reasoning. However, due to threats of student protest over various issues, including protests against rape and the 'fees must fall', there was no guarantee that the university would open at the scheduled time in February 2016, nor that courses would run in a form that would permit the smooth running of my study. With my supervisors' advice, I decided to investigate the MKfT of the same PGCE-FP part-time lecturer but within her own school context. Thus, my focus shifted to the MKfT she drew on while teaching her Grade 2 class at a local school. The redirection was guided by an assumption that the MKfT she employed in the classroom would be similar to the MKfT she would promote and develop in the FP pre-service teachers for use in their classrooms as mathematics teachers.

There are two primary research goals that have informed my study. The first relates to the MKfT required by a FP teacher, in the case of my research a Grade 2 teacher, so as to inform both pre-service and in-service teacher education. The second goal is to contribute to a relatively under-researched area of study in FP teacher education in South Africa, which is what MKfT is required in the FP mathematics teaching, particularly in relation to the development of children's number sense through counting.

As indicated in Chapter One part of my rationale for selecting a considered expert teacher as my case study for this investigation is to focus on what MKfT is required for expert teaching and thus to move away from research that focused on the absence of sufficient MKfT as indicated in much national literature reviewed in Chapter Two. My research sought to answer '*What MKfT in relation to development of number sense through counting does an expert Grade 2 teacher have and use in her teaching?*'

4.3 RESEARCH ORIENTATION

In my research, I used an interpretive research orientation to investigate the MKfT enacted by an expert foundation phase teacher. This orientation defines knowledge as dependent upon human perception and thus never free from influences such as culture, history and belief (Henning, 2004; 2005). Cohen, Manion and Morrison (2005) view the interpretive research model as an approach that seeks to "understand and interpret the world in terms of its actors" (p, 28). Interpretivism emphasises experience and interpretation (Henning, 2004) hence I used it with the aim of observing and interpreting the knowledge of teaching mathematics that Gail (my case study teacher) employed while developing number sense with Grade 2 learners through counting.

Mack (2010) alleges that interpretive research is observed from inside, hence it permits the researcher to mingle with people and observe from the inside through the direct experience of people to demystify and explain the phenomena under study (Bevir & Kedar, 2008). The purpose of this study was to gain a deep understanding of the MKfT that an expert FP teacher uses during teaching. In my research, I observed and interviewed the research participant to gain understanding of the MKfT required for teaching FP learners. The participant was a Grade 2 teacher who also works as a part time PGCE (FP) mathematics method lecturer. I worked with her at the university as her assistant while she mentored me in lecturing this course. Working with her gave me ample opportunities to observe and talk to her about the MKfT she employed in her teaching/lecturing.

I used a qualitative research approach in this study. Qualitative research supports the interpretivist orientation as it enables the researcher to understand and explore the richness, depth, context and complexity within which participants in the research site operate (Mason, 2006). Denzin and Lincoln (2005) describe qualitative research methodology as involving "an interpretive naturalistic approach to the world" (p. 3). Qualitative research is naturalistic as it attempts to study the everyday life of individuals, different groups of people and communities in their natural setting. It attempts to make sense of or interpret phenomena in terms of the meanings people bring to them. Data is gathered more in verbal form rather than in numerical form in a qualitative research study. There is far more emphasis on values and context within this type of research.

Research suggests there is some overlap between qualitative and interpretive research practices (Bevir & Kedar, 2008; Yanow & Schwartz-Shea, 2006). Interpretive research is unique in its approach to research design, concept formation, data analysis, and standards of assessment. The qualitative research approach supported the interpretivist orientation by enabling me to understand and explore the richness, depth, context and complexity within which the participant on the research site operate (Mason, 2006) through observations, personal interviews, account of individuals and personal constructs (Creswell, 2009). In this interpretive research, I used a case study research methodology to investigate the MKfT employed in the teaching of mathematics by one expert teacher in a Grade 2 class. In the section that follows I will deliberate on case study as a research methodology.

4.4 CASE STUDY METHODOLOGY

Yin (2003) defines a case study as an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly defined. In my research the case under study was the MKfT required for teaching mathematics at FP level. A Grade 2 teacher at a local school who also does part-time mathematics education lecturing to pre-service FP teachers at a university was investigated to establish what knowledge is required for teaching mathematics at foundation level effectively. Ary, Jacobs and Razavieh (2006) and Creswell (2009) assert that the greatest advantage of a case study is that it provides an opportunity of gathering rich data through an in-depth study of a bounded system (such as an activity, event, process, or individual). In my study, Gail was my bounded system in whom I sought to understand her actions, thoughts, experiences and other behaviour in the totality of her environment to investigate what MKfT is enacted by her in her teaching of number sense through counting to Grade 2 learners. Case studies are anchored in providing rich, detailed, in-depth real life accounts and examinations of the phenomena of interest in a given situation.

The case study approach was also favoured in my study because it permitted the use of multiple sources of data (Yin, 2009). Creswell (2013) explains that:

> Case study is a qualitative approach in which an investigator explores a real life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information (for example, observations, interviews, audio-visual material, and documents and reports), and reports a case description and case themes (p. 97).

A variety of evidence from different sources, such as CAPS documents, lesson observations, field notes and interviews were gathered, resulting in an in-depth exploration of the case. These multiple sources of data were triangulated for the purpose of illuminating MKfT from different angles (Johansson, 2004). This provide a holistic view and rounded picture of what MKfT Gail had and used as the selected expert teacher. In the next section I will discuss how and why I selected Gail as the case teacher in this study after giving a brief understanding of what sampling is.

A case study emphasises detailed contextual analysis of a limited number of events or conditions and their relationships (Noor, 2008) and hence was considered suitable for this study. Ary, Jacobs and Razavieh, (2006) state that the greatest advantage of a case study is the possibility of in-depth study of a case in seeking to understand in-depth, the individual's actions, thoughts, experiences and other behaviour in the totality of that individual's environment. A case study approach permitted me to gain a detailed, comprehensive and all-inclusive view of the MKfT that was embedded in the selected expert teacher.

4.5 THE SAMPLE

In this study, I used opportunity sampling to select Gail as the participant of my research. Opportunity sampling, sometimes referred to as convenience sampling, is a sampling technique where the researcher chooses a sample that is easy to reach or convenient to work with yet fitting the criteria the researcher is looking for and available at the time the study is being carried out (Cohen, Manion & Morrison, 2011; Bertram & Christiansen, 2014). Gail was selected conveniently to represent the community of expert FP mathematics teachers due to her knowledge and experience (discussed later in this section) coupled with her availability and willingness to share her knowledge. As reflected earlier I serve as her assistant in lecturing the PGCE (FP) mathematics method course at the university where she is part-time lecturing. As my mentor lecturer and because we had developed a trusting relationship she was easily accessible and willing to take part in the research hence she fitted the criteria of person I needed for my research (Cohen et al., 2011).

Gail was the sole participant in my research[6] and according to Patton, (1990) "there are no rules for sample size in qualitative inquiry" (p. 184). In support, Morse (2000) argues that sample size in qualitative research depends on "the quality of data, the scope of the study, the nature of the topic, the amount of useful information obtained from each participant, the number of interviews per participant, the qualitative method and the study design used" (p. 3). Thus, the key question for my sample was whether it will provide access to enough rich data, and with the right focus, enable me to address my research question. Sandelowski (1995) advises that

[6] Classroom learners were not the focus of the study. Even while classroom observations involved learners they were not my research participants as such.

researchers need to evaluate the quality of the information they want to collect in light of the use to which it will be put, and the research method, sampling and analytical strategy employed. Determining an adequate sample size in qualitative research is important and hence ultimately a matter of judgement, logic and experience (Tongco, 2007).

In many respects, Gail was an opportunity sample because of our existing relationship in which I was learning about FP teaching and lecturing from her. However, she met my criteria of an expert teacher and I chose Gail because of her long service and wide experience in teaching FP, particularly in Grade 2. Furthermore, she is locally considered an expert in FP teaching as evidenced by her selection to hold the position of Head of the Foundation Phase at her school. Her mathematics teaching abilities have been recognised as evidenced by: her selection to be the Head of Department [HOD] of FP mathematics at her school; her role as the FP mathematics cluster leader; and her role as facilitator of mathematics education workshops in the district, in the province and nationally.

Gail’s successful mathematics teaching at FP has resulted in her being requested to do part-time lecturing in mathematics Education method course to PGCE (FP) students at a university. Her involvement in both the classroom and teacher education made her the most suitable participant for my study that sought to explore the MKfT needed for effective teaching of mathematics at FP level.

It was convenient and cost effective for me to work with Gail as my research participant as her school is not far from where I live and work. Furthermore, Gail was willing and able to share her knowledge about mathematics teaching. She has been a reliable guide to the culture of effective teaching of mathematics at FP through the experience she has had in the teaching of FP mathematics (Bernard, 2002; Lyon & Hardesty, 2005). Selecting Gail as my research participant enabled me to ensure information-rich data that manifest the phenomenon of interest intensely and whose study would illuminate the questions under study as they are being explored (Patton, 2002). Information-rich cases according to Patton (1990), are “… those from which one can learn a great deal about issues of central importance to the purpose of the research” (p.169). In my study, the participant chosen, presented herself as a suitable participant from whom rich data can be collected. Coyne (1997) alludes that sample selection in qualitative research has a profound effect on the ultimate quality of the research. Hence my careful consideration in choosing Gail as the participant of this study. In the next section I will deliberate on how data was collected.

4.6 THE RESEARCH SITE

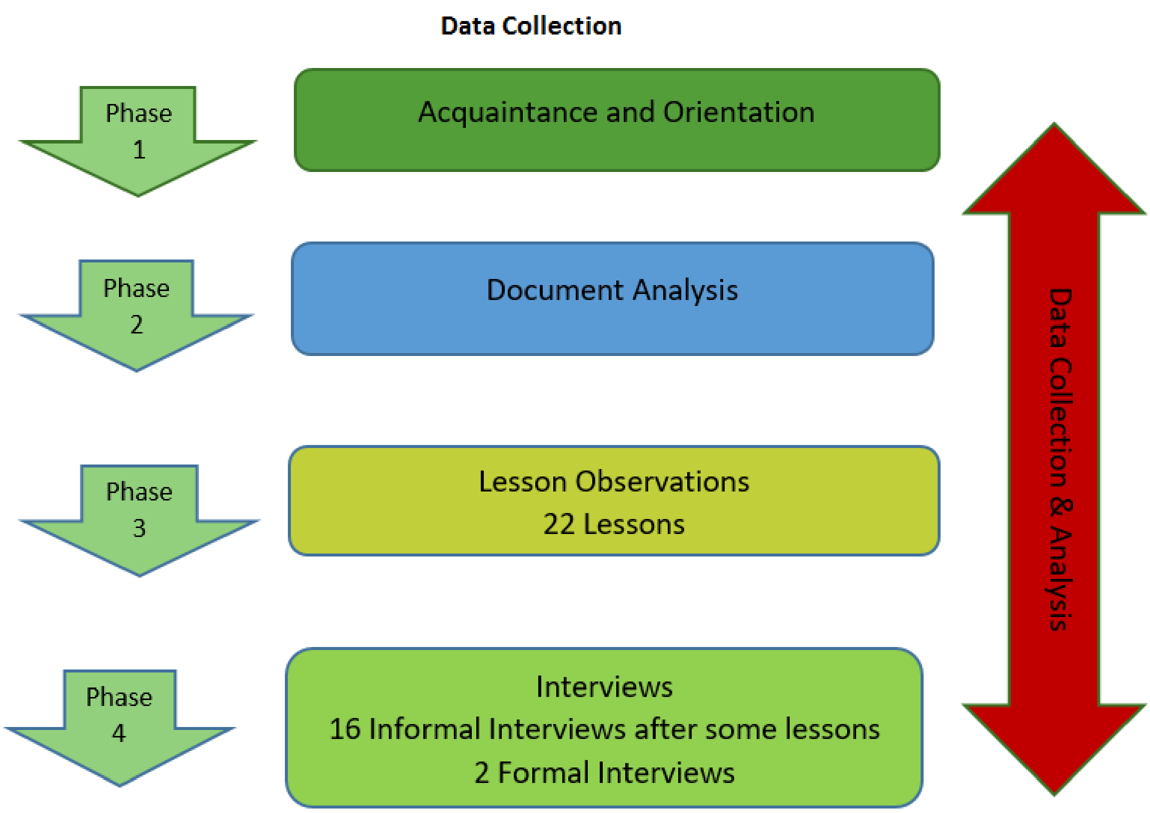
The research was conducted at a former Model C school in the Eastern Cape that runs from Grades R to 12. The school is well resourced and has a good academic record. It is a fee paying, multi-racial and multicultural school. Gail has been teaching Grade 2 at this school for the past 32 years. Her classroom is well organised and well-resourced. In addition to the resources recommended by the curriculum and provided by the school, she goes out of her way to buy or make more resources which she uses during her teaching.

4.7 DATA COLLECTION PROCESS AND TECHNIQUES

4.7.1. Data collection process

The process of collection data was in four stages as shown in Figure 4.2.

Figure 4.2 Showing the process of data collection



As reflected in Figure 4.2 above, the data collection process was in four phases. In phase 1 I had an initial interview with Gail to get to know her better in the context of her school and to gather some background data. It was a semi-structured interview where I had listed a number of questions beforehand that guided the interview. The interview was not recorded as I considered that I should first build a level of trust in terms of research relationship. However, I jotted down the important information that I needed during that interview. After the interview, she showed me around her classroom. We then organised a day when I visited her class to get introduced to the learners and to talk to them about my research and how I will carry it out. I explained that I will video record Gail (not them) as she was teaching and allowed them to look through the camera to exhaust their curiosity about the camera so that it was less disturbing during the learning process. I distributed the letters of consent for parents that explained the purpose of study, how data collection would be done, and issues of anonymity and confidentiality.

Phase 2 involved the analysis of the FP curriculum to get acquainted with the expectations of teaching and learning of mathematics at Grade 2 level. Phase 3 involved the actual observation of lessons and video recording them and Phase 4 involved the interviews. The last two phases will be discussed in detail later in the sections that follow. In the next section I will give a general view of the research techniques.

4.7.2 Research Techniques

In the process of exploring and understanding MKfT employed by an expert in teaching mathematics at FP level, research techniques that capture 'insider' knowledge, as stipulated by the interpretivist methodology, were used to collect data (Henning, 2004). These were document analysis, observations and semi-structured interviews. Multiple data gathering techniques were used based on the interpretive assumption that an event or action can only be fully understood and explained in terms of multiple interacting factors, events and processes. In addition, the assumption that the world in which teaching occurs is made up of multifaceted realities that are best studied as a whole, recognising the crucial significance of the context in which experiences occur and meanings are made (Schwartz-Shea & Yanow, 2012). This section discusses each of these data generation tools, elaborating how each was used in my research.

4.7.2.1 Document analysis

Document analysis is a qualitative data collection method in which documents are interpreted by the researcher to give voice and meaning around the topics (Bogdan & Biklen, 1998; Hopkins, 2008). Document analysis requires that data be examined and interpreted in order to elicit meaning, gain understanding, and develop empirical knowledge (Bowen, 2009). According to Bogdan and Biklen (1998) the analysis of documents is one of the central sources of qualitative data. However, they suggest document analysis should be used along with interviews and observation to collect rich qualitative data.

In my study, document analysis was conducted to understand and familiarise myself with the FP Curriculum and Assessment Policy Statement for mathematics (CAPS). The CAPS document for FP mathematics was analysed to ascertain aspects of the 'official' MKfT, embedded in the curriculum, which mathematics teachers can employ to develop number sense through counting with Grade 2 learners. Hopkins (2008) acknowledges that documents provide research with background information and understanding of the issues that would not otherwise be available, however the analysis of the curriculum put me in the danger of judging Gail and reading what she does in the classroom primarily through the policy perspective.

One of the major advantages of the document analysis for my study was its 'stability'. Unlike other sources of data, my presence as the researcher did not alter what was studied in the CAPS document. In qualitative research, drawing upon multiple sources of evidence is expected in order to seek convergence and corroboration through the use of different data sources and methods (Bowen, 2009). While document analysis provided an important background to the policy context in which Gail was working, observations provided essential data on MKfT used in teaching.

4.7.2.2 Observation

Observation is a fundamental way of finding out about the world around us. It entails the systematic noting and recording of events, settings, routines, behaviours and artefacts in the social setting chosen for the study (Marshall & Rossman, 1989; Marshall, 2006) through direct contact with the person or a group of persons. Such recordings were used to help me "understand and interpret the world in terms of its actors" (Cohen et al., 2005, p. 28) as purported by the interpretivist paradigm. Patton (2002) defines observation as "descriptions of activities, behaviours, actions, conversations, interpersonal interactions, organisations or community processes or any other aspect of observable human experience" (p. 29).

I chose to conduct observations in Gail's class for various reasons including primarily that my research question required that I do so. Drawing on Randolph (2007), observation allowed me to collect information first-hand on the experiences of Gail as occurring in her classrooms.

Cohen, Manion and Morrison (2000) assert that observations allow the gathering of live data from live situations and establish insight into the extent to which the teachers teach. Observation thus offered me an opportunity to gather ‘live’ data from naturally occurring classroom situations (Cohen et al., 2011). It provided me with an opportunity to describe the situation under study using my senses to have a ‘written photograph’ (Kawulich, 2005) of the MKfT enacted by Gail in developing her Grade 2 learners’ number sense through counting.

Kawulich (2005) advises that during observation one must have an open, non-judgmental attitude and be interested in learning more about others. It also requires one to be a careful observer and a good listener. For the most, I was a non-participant observer in my research. In other words, as Simpson and Tuson (2003) suggest, I did not manipulate the classroom situations or research participants. During observations, I focused my attention on observing from a distance as the teacher interacted with her class. This permitted me to gain knowledge and understanding about my research context (the classroom). It also allowed me to fully observe what was transpiring in Gail’s classroom. Robson (2002) confirms that “observations provide a reality check” (p. 310). However, at one time I found myself actively involved in the teaching and learning process. A learner seated next to me was struggling to count down six from sixty-four and I helped him use his fingers to count.

Observations of lessons were video recorded and analysed to find out what MKfT was utilised as Gail developed number sense through counting. According to Wright (2003), the process of video recording serves a number of fundamental and important purposes. Besides being a distinctive approach to gathering data, it also provides permanent records of what happened in the classroom. Video recording gave me an opportunity to go through the observed lessons over and over again, picking up non-verbal information that I could have missed during lesson observation. These include: (1) anxiety, where I would find the teacher pushing the struggling learners to count correctly and anxiously push them towards the answers; (2) frustrations were observed where Gail became frustrated with learner behaviour (for example at one point when one learner could not count through because he had diverted his attention from counting into playing with his pencil while they had been told to put their pencils behind them); and (3) body language, where Gail used non-verbal language such as actions to illustrate a mathematical idea (Cooper & Schindler, 2001). Gail used her body language regularly in her teaching. For example, she helped her learners to link counting backward and subtraction through taking steps backward.

Watching the videos repeatedly became essential in that I was investigating MKfT which is not immediately accessible through observation. Going back to the videos allowed me to critically analyse Gail's teaching to figure out which of the MKfT domains was being employed at each particular time. The video recorded lessons were analysed and the clarity on them was sought through interviewing Gail. The next section focuses on the interviews.

4.7.2.3 Interviews

In my research, interviews were understood as arranged conversations with a particular research focus between Gail and me. These were initiated for the purposes of gathering research-related information (Cohen et al., 2011). I used the interviews to gather information that I could have missed in observation, to check the accuracy of the observation (Maxwell, 2013) and to get Gail to interpret and clarify some information that I observed during her teaching (Merriam 2009). Interviews provided an opportunity for me to probe Gail's opinions, experiences and interpretations of her classroom life. In turn they also provided Gail with an avenue to discuss and express her opinions and interpretations on the actions and activities that took place during the process of counting. According to Marshall and Rossman (2006) interviews allow for greater depth in the collection of data than other methods. They enabled me extensive opportunities for asking questions and probing responses (Merriam, 2009), prompting and rephrasing (Klenke, 2008), pressing for clarity and elucidation, and checking for confirmation (Cohen et al, 2011). This helped to deepen the responses Gail gave to the questions and thus to increase the richness of the data.

Semi-structured interviews were favoured because they allowed me to prepare questions ahead of time (Pope & Myers, 2001). They have a flexible and fluid structure, organized around an interview guide that contain topics, themes, and areas to be covered during the course of the interview (Yin, 2003). The interview structure ensured flexibility in how and in what sequence questions are asked (Ary et al., 2006), and how particular areas might be followed up (Creswell, 2009), allowing me to shape and direct the interview to meet the interests of this study.

The interviews were conducted on a one-on-one basis with Gail in her classroom during the period her class went for computer lessons. Using her classroom facilitated privacy and provided a comfortable and familiar space for the interviewee. Her classroom was convenient in that she could refer to some resources that she utilised in developing the learners' number sense through counting. For example, when asked in the interview why she had conducted her counting lesson using a beadstring she pointed at the beadstring that still showed where the counting ended and explained her reasons for using it.

Two formal, pre-planned interviews, based on video recordings of lessons, were conducted. During these interviews, Gail and I observed selected video-recorded lessons. I had previously watched these lessons in order to make notes of what required clarification. As we watched the video I would stop it at particular points of interest and ask Gail to clarify what was happening in the lesson and explain why she was doing ‘it’ in that particular way. The interviews were guided by questions that I had previously constructed on each of these videos.

Through these interviews, I managed to access rich data on the MKfT Gail employed to develop number sense through counting. However, there were some problems in conducting these interviews as they seemed to be too long for Gail. With Gail holding many responsibilities in the school, interviews were normally interrupted as either learners or staff members would come to enquire or report about some issues. To overcome this, we resorted to shorter interviews that made it possible to get Gail to interpret and clarify each of the counting lessons. Gail seemed to prefer these short informal interviews as she would sometimes start explaining things before I even asked her. I preferred them too as they seemed to flow more naturally than running a formal interview. Doyle (2016) refers to the informal interviews as effective research tools because they were less stressful and less structured than a traditional interview. However, their limitation was that I did not have time to carefully analyse the lesson and write down the questions in a formal way before the discussion. That being said, I did make notes of aspects that I wanted to discuss with her after reflecting on her lessons. Given that I spent four weeks in her class I was able to ask her for clarification during the course of my visits each day.

Open-ended questions were used during interviews for their ability to mitigate the potential of bias and subjectivity in interviews (Patton, 2002; Merriam, 2009). As noted and acknowledged by Oduol (2014), open-ended questions allow participants and researchers to reflect on the experiences discussed and to respond to new ideas that emerge in the interviews. Cohen et al. (2011) acknowledges that open-ended questions can result in unexpected or unanticipated answers that may suggest unthought-of-relationships.

The formal interviews were both video and audio recorded to ensure the capturing of all the data during the interview. Cohen and Crabtree (2006) advise that it is generally best to tape-record interviews because semi-structured interviews often contain open-ended questions and discussions that may diverge from the interview guide. I used a paper-based interview guide to help follow the plan and order of the interview and to ensure interview items were all covered (Wright, Martland, Stafford & Stanger, 2006). Transcription of these interview tapes was done later for analysis.

The videos of interviews and observations of the lessons were transcribed. The transcribed videos of observations and interviews were given to two people to verify the accuracy of the translations. One of these people hold a Masters of Education (English Language) and the other a Masters of Education (mathematics). During the verification process, it was noted that three videos had missing translations of some of the sections. These were rectified. Also, some language issues were noted and corrected in the transcriptions.

I observed and listened carefully to the videos of the interviews and lessons on counting, writing down notes on what I saw and heard that helped me develop ideas on what category of the MKfT was reflected (Maxwell, 2013). I then read the interviews and lesson transcriptions together with the documents, comparing with my analytic memos and colour coded the data according to the domains of MKfT.

All the collected data was analysed so as to create meaning of it. The following section discusses the data analysis process, starting with a brief description of what data analysis is.

4.8 DATA ANALYSIS PROCESSES

Data analysis is the process of systematically inspecting and modelling data with the intention of making sense of it and to discover useful information that one can use to draw conclusions and support decision-making (Bloomberg & Volpe, 2008). Merriam (2009) describes it as a process of making meaning of what people have said and what the researcher has seen and read. Data analysis in this study involved breaking down the data into manageable patterns or categories in order to understand and make sense of the data. Data analysis was approached from Mouton's (2001) perspective where "it is the process of bringing order, structure and meaning to the data collected by breaking it up into manageable themes, patterns, trends and relationships" (p. 108). I used predetermined categories from the MKfT literature. These were the six domains identified by Ball et al. (2008).

Merriam (2009) and Cohen et al. (2011) assert that data collection, recording and analysis ought to be done concurrently as interrelated simultaneous procedures rather than individual processes done in a linear form. In this study the data was initially analysed simultaneously with data collection, followed by an intensive analysis after all data collection was completed. Merriam (2009) argues that "without ongoing analysis the data can be unfocused, repetitious and overwhelming in the sheer volume of material that needs to be processed" (p. 171). In my study, analysis of data was done for the 22 lessons in a recursive, iterative and dynamic way as it started during data collection (Merriam, 2009). All of the 22 lessons taught by Gail were subjected to the same rigorous analytical processes using the six MKfT's domain as categories. As highlighted in Chapter Two, I did not investigate Gail's CCK because her selection as an expert teacher assumes she has adequate common content knowledge.

I employed the etic method to analyse the data I collected. In the etic method, the researcher focuses on an existing theory and tries to apply it to a new setting or population to see if the theory fits (Morris, Leung, Ames & Lickel, 1999). In my case, I used the MKfT theoretical framework to investigate what MKfT Gail employed in her teaching. I worked with five of the six domains of the MKfT discussed in Chapter Two (that is the SCK, HCK, KCT, KCS and KCC) and compared them to the data gathered through the lesson observation, interviews and the analysis of the FP mathematics curriculum to determine the MKfT reflected in her teaching. In Chapter Two I explained why I did not use the sixth domain of the MKfT (the CCK).

I also used analytical memos in which I documented my thoughts, reflections and points of interest related to the data. These were kept throughout the data collection and analysis process. Each of the domains were later discussed in reference to data that was gathered through document analysis, lesson observation and semi-structured interviews. Such triangulation of data gathering methods helped to ascertain validity of the results of this study. Finally, the conclusions and recommendations were drawn.

4.9 VALIDITY

Charmaz (2006) defines validity as the degree to which research accurately represents that which it was intended to research. McMillan and Schumacher (2010) on the other hand propose that validity refers to the "truthfulness of findings and conclusions… and the degree to which explanations are accurate" (p. 104). I increased the validity of the data by using multiple sources of evidence during data collection, as Maxwell and Loomis (2003) suggest that triangulation (collecting converging evidence from different sources) enhances validity. Yin (2011) argues that "a valid study is one that has properly collected and interpreted its data" (p.

78). In my research, the process of triangulation added value to the validity of the study as themes and ultimately conclusions were based on multiple sources of data. The techniques (interviews, observations and document analysis) used in my research assisted in ensuring triangulation.

In this thesis, I use the direct words from the participant without correcting grammar or tense to support validity of data. Member checking, described by Lincoln and Guba (1985) as "the most crucial technique for establishing credibility" (p. 314) in a study, was also done. Gail was asked to read and check transcripts of classroom observations and interviews in which she participated. Preliminary analysis was to given Gail for verification of the emerging themes and inferences (Merriam, 2009) I formed during my dialogues with her. Gail was asked to check whether the words in the transcripts matched what she actually intended. Member checking in this study provided Gail with an opportunity to assess my description and transcription of data and preliminary results. I used member checking to also confirm aspects of the data and to provide Gail an opportunity to correct errors and challenge my interpretations of what she did and said during the teaching of counting. Gail confirmed that the data was accurate.

4.10 ETHICS

Ethics are norms and standards of conduct that distinguish between right and wrong and they help to determine the difference between the acceptable and non-acceptable behaviours (Burgess, 1989). Merriam (2009) advises that "ensuring validity and reliability in research involves conducting the investigation in an ethical manner" (p. 209). Ethical issues can arise at any stage of the research process (Bloomberg & Volpe, 2008), thus there is a need to take precaution to minimize any associated risk when doing research (Cohen et al., 2011). The key to ethics in my research was to minimise the harm and maximise the benefits. Ethics were to ensure that the right procedures were followed during the course of my research.

According to Banegas and de Castro (2015), ethical considerations are an important aspect of research as they prevent falsifying or fabrication of data and therefore promoting generation of true data and correct knowledge. Banegas and de Castro (2015) propose that ethical considerations involve collaboration, anonymity and confidentiality. In my study, collaboration was ensured by first seeking the consent from the school, the parents and the participant. The second aspect was to explain to them the purpose of the research and how it would be conducted. They were given letters that sought permission and that explained the research aim and how the research was to be conducted. Gail as the participant was given a week to study the contents of the letter and respond. The letter also emphasized that her participation was voluntary and that she could withdraw at any point of research.

Researching teaching and learning in primary schools is a very sensitive matter and might result in ethical issues. The Department of Education, parents and teacher organizations in South Africa are wary of who observes teachers during teaching and for what purpose. To overcome potential challenges, I adhered to the Faculty of Education, Rhodes University Research Ethics Guidelines. I stated explicitly that this study was about understanding the MKfT required by a FP teacher, and that it was not about evaluating or judging the quality of her teaching. I ensured that the participant had received a full disclosure of the nature of my study, the risks, benefits and alternatives. I also provided opportunities for her to ask questions at any stage of this study. It was crucial that I clearly explained the goal of my study and provided detailed accounts of data I wished to collect, the processes I would engage with and what benefit it would bring about without injuring or damaging participant's dignity (Cohen et al., 2011).

As noted by Merriam (2009), the standard collection of data techniques of observation and interviewing present their own ethical dilemmas. Thus, for this research, permission was sought from the Rhodes University Faculty of Education's Higher Degrees Committee, the school, Gail and the parents of the learners in Gail's class, prior to any collection of data. Informed consent was sought from the principal of the school, the teacher (Gail) and the parents of the children that were in Gail's class. Flick (2011, p. 217) encourages that "studies should involve only people who have been informed about being studied and are participating voluntarily". Informed consent is defined by Emanuel et al. (2000) as the provision of information to participants about the purpose of the research, its procedures, potential risks, benefits, and alternatives. This was done to make the principal, the teacher and parents understand this information so that they could make voluntary decisions to participate in this research or not. Thus, all the stakeholders were informed about the process and reason for my study. They all consented except one parent, who asked that his son not be video recorded. I made sure not to focus the camera on this learner when recording either class or small group counting.

Confidentiality in research studies refers to the obligation of an individual or organization to safeguard entrusted information (Creswell, 2009). I explained to Gail that it was my ethical duty to protect information she gave me from unauthorized access, use, disclosure, modification, loss or theft. I clearly pointed out that all data which was to be collected would be treated with confidentiality.

Gail's right to be free from intrusion or interference by other individuals or organizations was ensured in this study. Her identity and that of the school has remain anonymous and hence the use of pseudonyms. I refer to the participant by her pseudonym, Gail. However, within the local community within which Gail teaches and lectures it is possible that should they read this thesis they might be able to identify Gail as a participant. However, since both Gail, myself and my supervisors did not consider the possibility for harm from such a situation, given the overall positioning of Gail as an expert that we can learn from I considered the harm to negligible.

All the research material I collected for the purposes of this study during this period was kept in a safe and secure place and remained the case after the research terminated (Creswell, 2007). Physical, administrative and technical safeguards were ensured to protect data from unauthorized access, loss or modification.

4.11 REFERENCING CONVENTIONS IN MY THESIS

The following referencing conventions are used to refer to my empirical work: the particular interview, lesson, video and turns.

Table 4.1 Interview referencing formats

Interview referencing format	
FI	Formal Interview
R	Researcher
G	Gail
Lesson referencing format	
L1, V1, T.4	Lesson 1, Video 1 and turns 6-8
L4, V2, TT.6-8	Lesson 4, Video 2 and turns 6-8

Gail's comments in the interviews are written in bold in Chapter Five. Actions in the classroom when Gail is teaching appear in italics.

4.12 CONCLUDING REMARKS

This chapter is summarized in Table 4.2 below.

Table 4.2 Outline of the Research Goals, Questions, Design and Methodology

<table>
<tr><td colspan="2">Research Goals</td><td colspan="3">Explore what MKfT is required in teaching FP mathematics so as to inform practice in this area and teacher education programs both at Bachelor of Education (BED) FP and PGCE (FP) level.

Contribute to the relatively under-researched area of study in the FP teacher education.</td></tr>
<tr><td colspan="2">Key Research Questions</td><td colspan="3">What MKfT, (SCK; HCK; KCT; KCS and KCC) in relation to development of number sense through counting, does an expert Grade 2 teacher utilize?</td></tr>
<tr><td colspan="2">Research Design of Study</td><td colspan="3">Case study (interpretive)</td></tr>
<tr><td colspan="2">Nature of data collected</td><td colspan="3">Qualitative</td></tr>
<tr><td rowspan="3">ACTION PLAN</td><td>Data Collection Instruments</td><td>Lesson observations</td><td>Interviews</td><td>Document analysis</td></tr>
<tr><td>Data Source</td><td>Lesson videos and scripts</td><td>Videos of interviews and

Transcription of the interviews</td><td>FP mathematics Curriculum (CAPS)</td></tr>
<tr><td>Data Analysis</td><td colspan="2">Etic methodology</td><td></td></tr>
<tr><td colspan="2">Ethical Considerations</td><td colspan="3">Confidentiality and anonymity, informed consent</td></tr>
</table>

The next chapter discusses data presentation and analysis.

CHAPTER FIVE

DATA PRESENTATION AND ANALYSIS

5.1 INTRODUCTION

This study illuminates claims that teachers' Mathematics Knowledge for Teaching (MKfT) plays a significant role in teaching mathematics successfully. In this chapter I present and analyse data that was collected to investigate the MKfT that an effective Grade 2 mathematics teacher employs in the development of number sense through counting. The presentation and analysis of the empirical work is in two parts. Part one gives a brief background of the data collection environment and the second part presents how Gail's teaching of counting illuminates different aspects of the MKfT.

This chapter presents the data that was collected in the classroom in a quest to investigate Gail's MKfT. According to Ball et al. (2008), mathematics teaching requires a teacher to possess a particular kind of knowledge that distinguishes them from any other person who knows and understands mathematics. Teachers need knowledge that enables them to effectively carry out their work of teaching mathematics, as discussed in Chapter Three. Poor learner performance in mathematics in South Africa is generally attributed to teachers' lack of both content knowledge and pedagogy, as shown in Chapter 1. Human et al. (2015) propose that prior to interventions to develop teachers' MKfT, it is necessary to study what teachers do in their classrooms and what knowledge and skills inform their practice. This research selected one mathematics teacher at FP level to investigate what knowledge she draws on as she teaches her Grade 2 class.

As discussed in Chapter 4, data was collected in a Grade 2 class during the beginning of the first term. Learners were revising Grade 1 work both as per the curriculum requirements and for Gail to assess **"what knowledge they** (the learners) **have"** (FI2, V2, T58) from their previous learning experiences. During this revision period Gail assessed her learners and mapped her way forward with them. She told me "**We are doing revision ... And then of course I am building in all my things…" (FI2, V2, T52).**

I noted in Gail's class that she not only focused on counting per se, but also developed other areas relating to number sense development through counting. During the counting sessions she revised such number concepts as place value, building up and breaking down numbers, doubling and halving, addition and subtraction. The following section discusses how Gail conducted her counting session[7]. The section below presents the data on the counting sessions that were captured during data collection.

5.2 COUNTING IN GAIL'S CLASSROOM

There is extensive research evidence suggesting that counting should constitute the basis of the early years' number curriculum (Thompson, 1994; Maclellan, 1997; Graven at al., (2013)). Gail dedicated a significant amount of time to counting with her class. She spent a minimum of ten minutes on counting every day during the time that I was in her class. Counting in Gail's Grade 2 class was done in two ways: whole class counting and small group counting. Whole class counting always took place at the beginning of the mathematics lesson on the mathematics carpet at the back of the class. During this time the learners were seated on the carpet while Gail sat in front of them next to the resources she intended to use for the counting session. At the end of the whole class counting she would assign the class independent work, send them to their desks and remain with one of the three groups in her class for small group counting.

Gail divided her class into three groups according to her assessment of their mathematical competence. These groups were given animal names to avoid labelling and discriminatory language. In this study, however, I will use the colours to retain the learners' anonymity and also to avoid discriminating and labelling learners according to ability. She had the Brown Group, which was composed of learners who were deemed less competent in mathematics, the Red Group for the learners deemed to be average performers and the Green Group for learners that were perceived as competent in mathematics. Each day Gail worked with two groups one of which was always the Brown Group. Her emphasis on the Brown Group relates to: her concern that these children are less competent mathematically and need more time on task with her direct support; and her belief that "**It is very easy to teach a clever child, your teaching ability lays with the bottom group, that is where your teaching ability lays [nodding] it's very easy to teach a top child" (FI2, V2, T103).** In this study my focus was on the Brown and the Green groups.

[7] Gail's mathematics lessons included whole class counting and small group work. In this chapter, a **session** refers to the part of a lesson relating to particular activity of counting. This took place with the whole class and in small groups.

I will present and analyse the data related to whole class and small group counting jointly as the categories that emerged from the two were similar. In the next section I present two vignettes of how the whole class counting was generally facilitated by Gail, and follow this with a vignette of Gail's facilitation of small group counting. I examine the data by identifying key counting concepts that emerged during my data analysis.

5.2.1 Whole class counting

In this section I provide a general view on how whole class counting was facilitated in Gail's class. Two typical whole class counting sessions are used in this section. In the analysis, I will complement these two vignettes with examples from other counting sessions that I observed. I will highlight what is typical of Gail's teaching in order to investigate what MKfT underpinned her way of facilitating whole class counting.

For the most, the whole class counting sessions involved all the learners counting in unison following an ordered sequence. However, at times Gail chose a particular learner or group of learners to count while the rest of the class listened. While the learners were counting Gail facilitated the means for engaging the one-to-one correspondence counting principle. She did this as she moved beads on the beadstring, pointed at the mentioned numbers on the number chart, pointed at paper hands on the wall, encouraged the learners to clap hands, click fingers, stamp their feet or anything else that would facilitate the principle of one-one correspondence. Agreeing with Bruce and Threlfall's (2004) proposal that counting should not be just saying words but be made meaningful, Gail explained **"I can use the numbers set, I can use the number grid, I can use anything as long as they are actually uttering the word to the correct number" (FI2, V1, T27).** In this way, she ensured a more meaningful learning experience of counting**.**

Figure 5.1 shows some of the resources Gail used to facilitate learners' ability to count during the whole class counting. Evident in this photograph are the beadstring, the number chart, and number line paper hands.

Figure 5.1 Resources for whole class counting

Gail gave the learners instructions on what she expected the learners to count. By that I mean, she told the learners to count in ones, or skip count in twos, fives or tens. She also told them whether to count forward or backwards, the starting point and the end point. The learners mostly counted aloud and in unison, though she sometimes asked an individual learner or a small group of learners to count. This she said she did to encourage concentration or when she wanted to highlight an aspect of counting to the learners.

As noted earlier, Gail's counting sessions served a number of purposes, such as revision of work covered in Grade 1, assessment of the learners' knowledge levels and introduction to the work to be done. She explained "**I don't just say 'Right! Today I am doing problem-solving, tomorrow I am just going to count'. All this builds in. I am giving them what we call conceptual knowledge, basis of mathematical knowledge. I am building up their network in logical mathematical knowledge" (FI2, V1, T17).** During the counting session, Gail stopped the counting periodically to assess or develop a concept through questioning. This also helped to keep the learners attentive as they knew they could be asked any question at any time. Gail asserted: "**The reason why you also break is because you want to keep them focussed. You are working with the whole group, and you're going to get day dreamers. So instead of just counting and being boring you need to throw in these other things to keep them focused (FI2, V1, T17).**

While I have given a synopsis of Gail's whole class counting above, I will now provide vignettes of two sessions in different lessons that typically represent how Gail facilitated whole class counting. Refer to Appendix 5 for the lesson.

Vignette 5.1 A whole class counting session from Lesson One (L1, V1)

The focus of this whole class counting session was counting in ones within the number range of zero to eighty. The counting session began with the learners counting in 1s while Gail moved single beads across the beadstring. She stopped the learners at various numbers and asked about the composition of those numbers (e.g. 'Who can tell me what thirty-four is made of?").

*Thereafter, the learners continued counting in 1s. As they approached the 'teen numbers', she slowed the counting down to emphasise the 'teen' sound in the 'teen' numbers (e.g. thir**t-ee-n**). Likewise, she slowed the counting down when the learners reached the decuples[8] (e.g. **tw-en-ty**, **th-ir-ty**). She counted ahead with the learners for the next three numbers. The learners counted up to 50 and then backwards in 1s to 0. Thereafter they counted in 1s from 50 to 80.*

All the time Gail used the beadstring. After they counted forwards in 1s from 50 to 80, the learners counted backwards in 1s from 80 to 50. As the learners counted backwards Gail emphasised the move from the decuple (e.g. 70) to the next number (e.g. 69). As she did this she told the learners that they were now 'closing off Mr 70s house' and she used her hands to demonstrate this closure. She ended the counting by exploring with the learners the meaning of counting backwards and made the link between counting backwards and subtraction.

Concepts developed in the counting session:

Counting on

Counting back

Decomposition of numbers

Relationship between counting back and subtraction

[8] A **decuple** is a multiple of 10 e.g. ten, twenty, thirty, forty etc.

Vignette 5.2 is taken from Gail's second counting session. Refer to Appendix 6 for this lesson.

Vignette 5.2 A whole class counting session taken from Lesson Two (L2, V1)

The focus of this counting session was skip counting. The counting session began with an action song about counting in 2s. Thereafter the learners, together with Gail, counted in 2s up to 50 while she pointed to each of her fingers. The learners counted backwards from 50 to 0. Once they had completed that, they counted in 2s from 50 to 90 and then backwards from 90 to 50. As the learners counted backwards Gail discouraged learners from saying 'close Mr Ten's house' but allowed them to use their hands to signal the closure of each 'ten's house'.

When the learners had counted back to 50 she stopped them and asked how many 2s were in various numbers (e.g. "How many twos are there in 10? in 6?" etc.). Each answer was followed by a "How do you know?" She encouraged them to use their fingers to determine the number of 2s in each of the given numbers. She tried to lead them into doubling. When they failed to understand she made them count the 48 socks that hung on the line in the classroom in 1s. She asked them if counting in 1s was the best way to count the number of socks. She asked them to find a quicker way of counting the socks. The learners suggested counting in 2s and they then counted in 2s while Gail pointed to the pairs of socks. She then used the pairs of socks to lead the learners into understanding the concept of 'pairs' and construct an understanding of doubling and halving. Having done this, she moved onto counting in 5s. The learners were asked to count the fingers on paper hands displayed on the wall. They counted in 5s up to 100 then she stopped them and asked them to count the fingers individually on the two paper hands. As the learners counted in 5 she pointed at the hands with a pointer. She asked them how many fingers were in a given number of hands or how many hands would be required to match a given number of fingers (e.g. How many fingers are in three hands? or Thirty figure make how many hands?). The learners then counted in 5s up to 100 while Gail pointed at the paper hands. She asked them to identify the pattern in the multiples of 5 numbers and wrote these multiples on the board in a strategic manner as shown in Table 5.3 below. She wrote the numbers in such a way that the numbers ending in 5 were in one column and the numbers ending with a 0 were in another column The learners were quick to identify the pattern (i.e. in the first column the numbers ended with 5 and in the second there were multiples of 10. They then added and

<table>
<tr><td colspan="2">subtracted 5 to/from given numbers (e.g. 15 count on 5 or 20 take away 5). The whole class counting session ended.</td></tr>
<tr><td>Concepts developed in Lesson Two:
• Skip counting forwards
• Skip counting backwards
• Doubling and halving
• Relationship of counting back and subtraction
• Relationship of counting on as addition
• Patterns of numbers in skip counting</td><td>Gail writes the numbers strategically on the board as the learners count in fives.<table><tr><td>5</td><td>10</td></tr><tr><td>15</td><td>20</td></tr><tr><td>25</td><td>30</td></tr><tr><td>35</td><td>40</td></tr><tr><td>45</td><td>50</td></tr></table></td></tr>
</table>

Vignettes 5.1 and 5.2 above provided typical examples of whole class counting sessions in Gail's lessons. Gail started each mathematics lesson with whole class counting so "**that weak kids can learn from the stronger children" (FI2, V1, T39).** She emphasized that she engaged whole group counting because there is **"conceptual knowledge that I need to get through to them to discover through my mental mathematics and learning from each other" (FI2, V1, T45).** Gail spent about fifteen to twenty minutes of the one hour allocated to mathematics learning on whole class counting and the rest she would use for group teaching as mentioned earlier in the chapter. At the end of the whole class counting session she would send all the learners to their desks, assign them work that needed to be done, then call the group she wanted to work with to the carpet. In the next section I will give a general view of the small group counting followed by two vignettes from small group counting. The first vignette is from the Brown Group and the second from the Green Group.

5.2.2 Small group counting

Unlike whole class counting, there appeared to be no typical method to facilitating the small group counting. The manner in which the small group counting session was facilitated depended on what Gail intended to teach them. However, in all cases Gail would call the group she intended to work with to the carpet, seat them in a circle such that she was also part of the group. She would then distribute the resources to be used during the counting activity and give them the instructions of what is to be done during that particular time. Most often the activities she used to develop the mathematics concept she intended her learners to grasp took the form of games.

Below I present two sample vignettes of how small group counting was facilitated. Refer to Appendix 7 for Vignette 5.3 and Appendix 8 for Vignette 5.4.

Vignette 5.3 Small group counting session 1 – Green Group (L4, V2)

Gail seated the learners in a circle and gave each learner a number grids. She asked them to skip count in 2s from 52 to 70, and then from 61 to 99. Gail asked the learners to point to a given number (e.g. 78) and tell her what two number made 78, (i.e. 70 and 8). She then asked the learners to identify numbers that were two or three before or after 78. She assisted the learners who did not find it easy to give expected answers. She asked learners to add or subtract using counting on or counting back (e.g. 60 count on 7 or 98 count back 10). Sometimes she asked the learners to explain how they got the answers. *Learners then counted on ten and counted back on ten from given numbers (e.g. 23 to 93 or 52 to 12). One learner discovered and explained the quickest way of counting on or backward in ten using the number grid as moving up or down the grid. Gail asked the learner to demonstrate to others how it works and encouraged the learners to use that strategy. They did a few more examples then Gail collected the grids.* *Gail put some blocks and a dice in the middle of the circle. She explained to the learners that they must take turns to throw the dice and count out the equivalent number of blocks from the pile in the middle of the circle. She then told them to stack the blocks into groups of 5. The learners played four rounds then she asked them to stop and share their blocks so that all of them have staked the blocks in 5s. She asked the learners to identify the number of blocks they need to complete their groups of 5. They counted all the blocks in 5s. Gail asked them to count their blocks starting from 100 and they did so up to 255. She made reference of their counting to the paper hands on the wall. She asked them various problems orally (e.g. How many fives in twenty-five?). She asked them to put two hands together and count in 10s. They did more oral calculations where they counted on in 10 or counted back in 10 (e.g. 53, count back 10).*
Concepts developed: • skip counting

- place value
- before and after
- more or less
- number recognition
- counting on (addition)
- counting back (subtraction)
- grouping (informal multiplication and division)

Vignette 5.4 Small group counting session 2 – Brown Group (L10V1)

The children were seated on the mat in a circle. The group threw the dice in turns and counted on in 1s the number of dice dots they get on each throw accumulatively up to 83. The counting flowed fluently up until 39 when a learner could not easily proceed to 40.

It then became a noticeable trend that learners do not easily count through the 9th number of each decade into the next decade. Gail would assist them by asking questions (e.g. what number comes after 39?) The learners then count backward from 83 to 29 still being guided by the dice. The learners struggled to count backward. Gail continually used her teaching skills to guide and help them to count down. She would ask them questions (e.g. what number comes before 71?)

When they got to 29 Gail stopped the counting. She pulled out a box of counters and poured down them in the middle of the group and asked learners to pick three groups of two. They were then asked to count in 2s from 2 to 48. Learners struggled to count between 12 and 48 especially when they had to proceed from the 8th number of each decade to the next decuple (e.g. 48, 50). Gail asked all the learners to take out a given number of counters, then increase or decrease it to another number (e.g. take out 5 counters, increase them to 7 or decrease them to 2). Learners had to explain how many more they counted on or they had to return to the pile. For example, she would ask "What must I add to 7 to make 11? Show me your way". They did simple problems of addition and doubling each time explaining themselves.

Concepts developed:

- Counting forwards and backwards
- Bridging the ten

• Before and after • Skip counting • More and less (how many do you have to take away?) • Relationship of counting back and subtraction and of counting on and addition • Addition • Doubling

5.3 INTERPRETATION AND ANALYSIS OF BOTH THE WHOLE CLASS AND SMALL GROUP COUNTING DATA

The data in this study seeks to investigate the MKfT Gail employed to develop the learners number sense through counting was analysed using the MKfT framework as explained in chapter 2. Wilkie (2015) alleges there is a substantial, strong, and positive correlation between the teacher's level of MKfT and their quality of mathematical instruction. In this section I present a summary of the domains of the MKfT framework and the indicators of each domain developed from Chapter Three that I used to analyse the knowledge Gail drew on during her teaching before I started interpreting and analysing the data on counting. I then present a number of counting excerpts that were typical of her teaching and that illuminated the MKfT evident as Gail developed her Grade 2 learners' number sense in relation to counting. Table 5.1 outlines the indicators that would be used during the analysis to determine which knowledge domain of the MKfT was employed by Gail during her teaching and where and how she employed it

Table 5.1 A summary of MKfT domains and their indicators

MKfT domains	MKfT domain indicators
Common Content Knowledge (CCK)	• calculate an answer correctly • understand the mathematics you teach • recognise when a student gives a wrong answer • recognise when a text book is inaccurate or gives an inaccurate definition use terms and notations correctly
Horizon Content	• make connections across mathematics topics within a grade and across grades

Knowledge (HCK)	• articulate how the mathematics you teach fits into the mathematics which comes later
Specialised Content Knowledge (SCK)	• interpret students' emerging and incomplete ideas • evaluate the plausibility of students' claims give or evaluate mathematical explanations • use mathematical notation and language and critique its use • ability to interpret mathematical productions by learners, other teachers or learning materials • evaluate mathematical explanations for common rules and procedures • appraise and adapt the mathematical content of text books
Knowledge of Content and Teaching (KCT)	• sequence mathematical content • present mathematical ideas • select examples to take students deeper into mathematical content • select appropriate representation[9]s to illustrate the content • ask productive mathematical questions • recognise what is involved in using a particular representation • modify tasks to be either easier or harder • use appropriate teaching strategies • respond to students' why questions • choose and develop useable definitions • provide suitable examples
Knowledge of Content and Students (KCS)	• anticipate what students are likely to think and do • predict what students will find interesting and motivating when choosing an example • anticipate what a student will find difficult and easy when completing a task • anticipate students' emerging and incomplete ideas • recognise and articulate misconceptions students carry about particular mathematics content
Knowledge of Content	• articulate the topics in the curriculum

[9] In this research a representation is anything that Gail uses to enforce conceptual understanding such as pictures, objects, images, illustrations or demonstration

and Curriculum (KCC)	• articulate the competencies related to each topic in the mathematics curriculum • articulate and demonstrate a familiarity with the structure of the mathematics curriculum • link representations to underlying ideas and to other representations • knowledgeability of available materials (e.g. textbooks) and their purposes

Summarised from Ball et al. (2008)

In the next section I present different concepts that were developed during the counting sessions.

5.3.1 Different mathematics concepts that were developed during the counting activities

In this section I discuss the mathematical concepts that emerged during the counting sessions. I selected five concepts that occurred during Gail's counting sessions to answer my research question: 'What aspects of MKfT are evident in Gail's teaching?' These concepts are: (1) the 'teen' and 'ty' numbers; (2) bridging of tens; (3) counting back; (4) subtraction and (5) place value. I present an excerpt from a counting session related to each concept, discuss it with support from other counting sessions and literature, then identify the MKfT aspects (domains) that were evident during that counting session. I begin my data presentation and analysis of the counting sessions with the MKfT Gail draws on when dealing with *'teen'* and *'ty'* numbers.

5.3.1.1 The 'teen' and 'ty' concept

At the beginning of the mathematics lesson, presented in Vignette 5.1, all the learners in Gail's class were seated in an orderly arrangement on the mathematics carpet. They were counting in ones. As they said each counting number name, Gail moved beads on the beadstring. Excerpt 5.1 provides a transcript and account of what transpired in this counting session.

Excerpt 5.1 Emphasising 'teen' and 'ty' numbers (L1V1)

1.	T	Okay boys! It is time for Maths. Let us go to the back carpet. Come on come on let us be quick. *[Learners move to the carpet and sit down facing*

		the teacher's chair. The teacher stands next to the beadstring in front of the learners] Quiet boys! We are going to count forward in ones.
2.	TLL	[*Tr pulling the beads on the beadstring one at a time as the learners count*] One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve …
3.	TLL	[*Tr reducing the tempo and emphasising the 'teen' sound as she counts with the learners*] thir**teen**, four**teen**, fif**teen**, six**teen**, seven**teen**, eigh**teen**, nine**teen** [*Tr then emphasises the—**'ty'** sound as she counts with the learners*] twen**ty**, twen**ty**-one, twen**ty**-two, twen**ty**-three …
4.	LL	[*Tr keeps quiet*] twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine, thirty, thirty-one thirty-two, thirty- three, thirty-four, thirty-four, thirty-five, thirty-six
5.	T	Stop! I was on thirty-six, I want you to listen very carefully. What two numbers do I need to make thirty-six? **Thirty**-six [*Tr putting emphasis on thirty*]
6.	LL	Thirty and six
7.	T	I need a thirty and a six! Good, Let's go on.
8.	TLL	Thirty-seven, thirty-eight, thirty-nine, forty
9.	T	Stop! How many tens do we have in forty? Julius?

Extracted from L1, V1

In the counting session represented in Excerpt 5.1, Gail marks each counting word by moving a bead across the beadstring hanging above her small chalkboard at the mathematics carpet in helping learners to count meaningfully. Her counting sessions were influenced by her understanding that **"there are five principles of counting that govern meaningful counting. I am not even going to those, but then it's a lecture that I will do (FI2, V2, T73).** Gail chooses to use a beadstring to facilitate the principal of one-to-one correspondence, which is, matching each number name with a bead on the beadstring. Gail told me that her counting sessions were influenced by her knowledge that: "**Counting is not just a component of just rote count, spit out of your mouth and it means absolutely nothing, which is what a lot of teachers actually do. They think it's just count, count, count. They don't even know if children are actually counting the right number on the right word or whatever" (FI2, V2, T15).** She explains that counting **"must be one to one, it must be an uttering the correct word on the correct number"** (FI2, V2, T23). Haylock and Cockburn (2008) concur that in learning to count learners should "learn to co-ordinate the utterance of the number word with the movement of the finger and the eye along a line of objects, matching one noise to one object until all the objects have been used up" (p. 41). The beadstring is, for Gail, an appropriate representation for developing the one-to-one principle as it affords learners an opportunity to coordinate Gail's movement of the beads with each number name. Each bead was given one count and one number name.

A further counting principle evident in Excerpt 5.1 is the *stable order principle*. The stable order principle upholds the consistence of the counting sequence (Gelman & Gallistel, 1978). Number names follow a stable order sequence such as one, two, three, and so on. This principle was generally emphasised in Gail's counting sessions. According to Naudé & Meier, (2014) it is critical for learners to know that the number names maintain a consistent order in counting regardless of the counting strategy (be it forward or backward, counting in ones or skip counting). In particular, Gail was concerned with the learners counting in ones between ten and twenty. In Excerpt 5.1 we note how she chooses to count with the learners to remind them of the order of numbers between one and thirteen, which have been described in Chapter Three as arbitrary (e.g. eleven, twelve) (Fuson, 1988; Reys et al., 2007; Gifford, 2005).

Her understanding of the counting principles and her ability to implement them in her teaching reflects her SCK and KCT. Knowledge of the counting principles is the SCK that is of particular importance to teachers, especially Foundation Phase teachers. The vignettes above show that Gail draws on this knowledge while she teaches signifying that she is able to make the link between her SCK and KCT.

Anticipating the possibility of her learners associating counting with the one representation commonly used, Gail avoided using the beadstring all the time, but employed a variety of representations during her counting sessions. For example, while she used a beadstring in lesson one (L1, V1), in lesson two (L2V1) she made use of the pairs of socks hanging in the classroom and the paper hands on the wall (Figure 5.1). In lesson ten (L10V2), the learners counted the dots on dice and wooden blocks to incorporate one-to-one correspondence counting. She showed understanding of what underlies the use of each representation. Each representation was chosen for a purpose and she managed to use them successfully. The DBE (2011) encourages teachers to employ a variety of representations to motivate their learners to count. The use of different kinds of manipulatives can also reduce the monotony in counting sessions, yet the learners will be still be rehearsing the same counting skills and number names and their stable order sequence. In so doing, Gail demonstrated that she has knowledge of both content and learners as she is aware that her Grade 2 learners will find the variation through using various representations and materials more interesting. In this respect her KCS comes to the fore. Furthermore, Gail demonstrates that she is cognisant of available materials and their purposes in assisting her learners in developing in counting. In this respect, she not only demonstrated knowledge about content and teaching but also of content and curriculum as it is broadly understood (Table 5.1).

As noted in Excerpt 5.1, each time the learners counted Gail started with them before she left them to count on their own. She chose to re-join the counting when she anticipated common counting errors or counting challenges. For example, in Excerpt 5.1 above, Gail counted with the learners up to twelve because the sequence up to twelve is arbitrary and can easily be confused or forgotten. According to Reys et al. (2007), "patterns facilitate the counting process" (p. 160). It becomes easier for learners to grasp the counting sequence when they identify its pattern. Gail confirms that "**if a child can see patterns they can do maths" (FI2, V2, T103)** because **"Maths is a pattern. It's the same thing over and over" (FI2, V2, T105).** This is confirmed by research which shows that during the early stages of learning to count, learners struggle to count in an accepted number word sequence from one to sixteen because there are no obvious patterns to the number names and their sequence (Fuson, 1988; Reys et al 2007; Gifford, 2005). As discussed in Chapter Three they suggest that from thirteen onward learners can depend on the pattern of the number names such as ***thir***teen with ***thir*** standing for three, ***four***teen with ***four*** for four, ***fif***teen with ***fif*** for five to master the accurate counting sequence.

The emphasis on the 'teen' numbers was not limited to this particular counting session. Gail emphasized the 'teen' numbers in a variety of counting sessions with both the whole class and in small groups. Furthermore, Gail not only emphasised the 'teen' numbers but also those ending with 'ty' (e.g. twen**ty**). As noted in Excerpt 5.1, Gail's counts with the learners in her class as they move from the 'teen' numbers to the 'ty' numbers. The syllable 'ty' is emphasized as the learners count. At the beginning of the teen numbers Gail slowed down the counting speed to emphasize the 'teen' numbers, bridge the decade[10] and counted with them the beginning of the twenties slowly to emphasize the decuple. When they got to the twenties in

[10] I use this term in the same way that Wright (2007) uses it. That is to indicate counting across the decuple (e.g. 57, 58, 59, **60**, 61, 62) and not in relation to bridging the ten when adding or subtracting (e.g. 7 + 6 = 7 + 3 + 3)

excerpt 5.1 she emphasized the '***ty***' sound once more and elaborated on the pattern that follows all decuples twen**ty**-one, twen**ty**-two, twen**ty**-three … thir**ty**, thir**ty**-one.

Gifford (2005) alleges that learners often confuse the teen numbers and the decuple. She argues that learners "dovetail these two patterns together. Sixty, seventy, eighty, also sound like sixteen, seventeen, eighteen, which can be problematic to learners with hearing difficulties" (p. 79). Gail, anticipating this challenge and wanting to avoid its consequence, slowed down the counting in Excerpt 5.1 and emphasized the pronunciation of both the '*teen*' and '*ty*' numbers.

Central to ensuring that the learners in her class grasped the difference between the 'teen' and 'ty' numbers, Gail used questioning to encourage the learners to identify whether a number has a 'teen' or a 'ty' in it. In L5, V1, T93-99, for example, learners were counting down from twenty, Gail stopped them at ten and asked "is it fourteen or forty?", "is it seventeen or seventy?" When the learners gave a correct response, she cautioned them to pronounce the words properly and continued with the counting by asking them to count down from twenty to ten once more. Learners eventually emphasised the '***teen***' sound when counting without her support. In L12, V1, T108 again Gail asked her learners after counting in tens from ten to one hundred. "Who can tell me what is different between the spelling of thirteen and thirty?" Here she actually asked the learners to differentiate how, for example, thirteen is different from thirty in written word form as a way of ensuring that they know the difference between 'teen' and 'ty' numbers.

In many respects, Gail recognises that the pattern in the counting words which begins after twelve is an important aspect of learning to count. Not only does Gail realise that the patterns of the number words assist children with counting, but she also emphasises the use of mathematical language (i.e. the number names) by encouraging the learners to differentiate between the 'teen' and 'ty' words. Her knowledge of the significance of patterns in learning mathematics and the emphasis on the mathematical language are indicative that Gail has knowledge related specifically to the work of teaching which is beyond the content knowledge expected of ordinary citizens. In addition, her emphasis on the 'teen' and 'ty' numbers are an example of Gail's knowledge of the link between content and learners as she was able to anticipate the errors that learners make and the typical challenges they have in learning to count. In assisting the learners with this possible difficulty, Gail demonstrated her KCT as she selected appropriate counting sequences (e.g. counting in 1s from 0 to 50), counted with the children when she deemed it necessary and asked productive questions in relation to the numerosity of the numbers that the learners named as they counted.

Gail demonstrated her mathematics knowledge for teaching counting during this counting session in many ways. Table 5.2 below provides a snapshot of Gail's' MKfT in relation to developing children's knowledge of the distinction between 'teen' and 'ty' numbers. The Indicators of each domain that Gail drew on during this and other counting sessions are also given in Table 5.2 below.

Table 5.2 Summary of MKfT domains employed relating to the concept of teen and ty numbers

MKfT Domains	Indicators evident in Gail's teaching of counting
SCK	• Knowing that counting requires an understanding of the one-to-one principle, the stable order principle, and the cardinal value (numerosity) principle • Knowing that there is more to counting than rote counting • Knowledge that counting involves a general pattern beyond the number 12 • Knowledge of counting errors and challenges (i.e. distinguishing between 'teen' and 'ty' numbers)
KCT	• Sequences the counting sessions staring with 1s before skip counting, and counting forwards before counting backwards • Selects appropriate counting exercises (e.g. counting up to the 'teen' numbers before counting the 'ty' numbers) • Asks productive questions (e.g. what is the difference between the spelling of thirteen and thirty?) • Presents mathematical concepts accurately by emphasising the 'teen' and 'ty' numbers
KCS	• Anticipates that the students will find the 'teen' and 'ty' numbers difficult to distinguish • Recognises and articulates misconceptions learners carry about the 'teen' and 'ty' numbers

HCK	• Demonstrates how the mathematics she teaches fits into the mathematics that comes later. • Connects the topic of counting to that which the learners learned in the prior year
KCC	• Knows which instructional materials would be effective • Has a familiarity with the curriculum and the structure of the curriculum content? • Demonstrates the expectations from the mathematics curriculum • Knows what instructional materials are available, what approach they take and how effective they are

An ability to demonstrate these five domains (SCK, KCT, KCS, HCK and KCC) in one counting session may be considered as an indication that Gail was conscious of what she intended her learners to grasp during her teaching. It also reflects Gail's understanding of how each of these domains is crucial to meaningful learning in the classroom. Another concept that emerged during Gail's counting session was the concept of *bridging of the tens*. This is discussed in the next section.

5.3.2 Bridging the tens

The bridging of the tens in this study refers to the ability to count across the decades such as twenty-eight, twenty-nine, thirty, thirty, thirty-one or forty-one, forty, thirty-nine etc. The bridging of the tens concept was deliberated in two parts. The first part focused on bridging the tens while counting forward and the second part on bridging the tens while counting backwards.

5.3.2.1 Bridging the tens while counting forward

Seven minutes into the small group counting sessions described in Vignette 5.4, Gail is working with the Brown Group. As stated earlier, the Brown Group is composed of the learners deemed to be less mathematically competent. The Brown Group learners were playing a dice game. Each learner had a turn to throw the dice and then count on the number that their dice landed on onto the previous learners' total scores. They counted their throws cumulatively. The learners seemed to struggle with counting past the ninth digit of each decade. Excerpt 5.2 shows the challenge Brown Group learners experienced in bridging the tens.

Excerpt 5.2 Showing the challenges learners face bridging the decades

19.	T	Right I am going to start on twenty-nine. *[Throwing the dice and gets a two]* thirty-one *[passing the dice]*
20.	L	*[Plays and gets six*] thirty-two, thirty-three, thirty-four, thirty-five, thirty-six, thirty-seven.
21.	T	We are on thirty-seven.
22.	LL	[*one learner plays and gets one*], thirty-eight
23.	L	Why are you telling me? Thirty-eight
24.	T	Good, pass the dice on
25.	L	[*plays and gets six but is stuck*]
26.	T	What comes after, thirty-eight?
27.	L	[*Mumbles*]
28.	T	What is the next number after thirty- eight? *[Flipping one finger up at a time]*
29.	L	Thirty- nine [*silence*]
30.	T	What number comes after thirty- nine?
31.	L	Thirty- nine [*silence flipping fingers like trying to count them. After a long silence]*, Thirty- nine, forty, forty-one, forty-two, forty-three, forty-four.
32.	T	No. Forty-one, forty-two, forty-three [C*ounting on her fingers*] Come on! I want you to practise. [*passing the dice*]
33.	L	[*Plays and get five*] forty-three
34.	T	No, you must not count forty-three. Next number after forty-three?
35.	L	forty-four
36.	T	Good *[passes the dice*]
37.	L	[*Plays and get four*] forty-five, forty-six, forty-seven, forty-eight.
38.	T	Good [*Raising her hand for a 'high five*'] well done! What are we on?
39.	L	48
40.	T	48 [*passing the dice*]
41.	L	[*Plays and get two*] [*silence*] 48 [*counting his fingers*]
42.	T	We don't count forty-eight. Next number after forty-eight?
43.	L	Forty-nine. Forty-nine. [*repeating*]

44.	T	After forty-nine?
45.	L	[*silence*] fifty
46.	T	Good [*passes the dice*]
47.	L	Fifty-one, fifty-two, fifty-three, fifty-four, fifty-five, fifty-six. [passes the dice]
48.	T	Good. We want to hear, we are on fifty-six
49.	L	[*Throws the dice*] fifty-six
50.	T	We finished with fifty-six now you see
51.	LL	fifty-seven, fifty-eight. [*passes the dice*]
52.	L	Why is everybody telling me? [*throwing the dice and gets five*] [*then moving fingers in attempt to count*]
53.	TL	[*Tr flicking his fingers in attempt to help the learner to count*] fifty-nine, sixty, sixty-one, sixty-two, sixty-three, sixty-four.
54.	T	[*Passes the dice to the next learner*]

Extracted from L10, V1

In my communication with Gail she told me that her learners "**really battle to come, out of one ten and get into the next one" (FI2, V1, T116).** In Excerpt 5.2 above, learners struggled to count from thirty-nine to forty, forty-nine to fifty. Gail counted with them through fifty-nine to sixty once she realized that they were having difficulty in bridging the ten. Later in this counting sequence, the learners managed to count from sixty-nine to seventy, but could not count from seventy-nine to eighty. Each time they got to the ninth term of each decade they would get stuck. Gail knew the learners were stuck as a long silence followed while the learner tried to figure out what number should follow. She probed by asking "What number comes next?" The learners managed to bridge the ten once they had been given time to think about it. However, with the transition from seventy-nine to eighty Gail had to tell the learner the answer.

Gail showed her expertise in teaching when counting to her Brown Group by instructing them to play the dice game that would facilitate rehearsal of forward counting in ones (see excerpt 5.2). Gail knew the challenges her learners had with forward counting and provided them with an interesting setting for practicing. Wright (2012) recommends playing games that help learners to rehearse what they have learnt and suggests that rehearsal increases familiarity and leads to automatization.

Counting their throw of dice on from the previous learner's score, learner at 'Turn 43' struggles to count on two from forty-eight. He counts to forty-nine but could not easily proceed to fifty. He had to think seriously before he could figure out that fifty follows forty-nine. Gail gave the learner adequate waiting time to think of the next number. This is crucial because during the process of counting, learners may need time to reorganise their thinking while going through a hurdle (Wright, 2012). Allowing waiting time is advisable to give the learners time to assimilate, thereby enabling counting with understanding. Wright (2012) also suggests that if learners seem not to remember the sequence it is appropriate for the teacher to probe or give clues.

Excerpt 5.3 Showing the challenges learners face bridging eighty

73.	L	[*plays a two*] seventy-six, seventy-seven
74.	T	Well done Dan
75.	L	[*Plays a three*] seventy-eight, seventy-nine, seventy-six.
76.	T	No! What comes after seventy-nine?
77.	L	[Silently pointing at his finger]
78.	T	What comes after seventy-nine?
79.	L	[*Silence*]
80.	T	Eighty.
81.	L	Eighty, eighty-one, eighty-two.
82.	T	Good! Eighty-two

Extracted from L10, V1

Gifford (2005) and Wright (2012) acknowledge that the problem of failing to bridge the decades is common with young learners. Gifford (2005) argues that it is appropriate for the teacher to tell the learners the next number in the counting sequence when they get stuck so that they do not completely lose their confidence. In the above two excerpts, Gail does not immediately tell the learners the numbers as Gifford suggests, rather she asks a question and extends the 'wait time', thus giving the learners time to think about the number that follows.

Gail's MKfT is evidenced in a number of ways in the two excerpts related to the bridging the ten when counting forwards. She is able to anticipate what the learners find difficult when learning to count on, and knows what they are likely to do (i.e. keep silent, have to think for an extended period of time, or substitute an arbitrary number). This knowledge suggests that Gail knows about content and students and is able to combine the two in her teaching. In addition, she has specifically chosen a game to assist the learners in developing their ability to count on and in that process bridge the ten. Gail realises that the game is likely to capture her learners' attention and keep them focused on the purpose of the small group counting session. Her choice of the game also reflects her KCS. However, the use of the game to develop her learners' ability to count on and bridge the ten also indicates that Gail values the use of games as a teaching and learning strategy. Furthermore, the dice provides an opportunity for children to point count if necessary, and to support them in uttering one word for each dot on the dice (i.e. the one-to-one principle). It also provided an opportunity for those children who are able to subitise to do so. As with the previous counting concept, Gail uses careful questioning to encourage her learners to bridge the ten. Gail demonstrated familiarity with the Foundation Phase (FP) mathematics curriculum by planning an activity that require the learners to count on within the number range of zero to one hundred.

The indicators relating to each of the MKfT domains that Gail drew on while developing the concept 'bridging the ten' are summarised in Table 5.3 below.

Table 5.3 Summary of MKfT domains employed relating to the concept of bridging the tens

MKfT Domains	Indicators
SCK	•
KCT	• Remediate counting errors such as counting from seventy-nine to seventy-six • Asks productive questions (e.g. 'what number comes after thirty-nine?) • Recognize what is involved in using a particular representation such as learners playing in turns and counting their throws cumulatively. • Select appropriate dice game to enhance counting • Selects a suitable activity for the learners to count on and bridge the ten • Select representations (i.e. the dice) for a particular purpose
KCS	• Anticipates that the learners will find bridging the ten difficult.

	• Predicts that the learners will find playing a game more interesting and motivating when counting on
KCC	• Demonstrates familiarity with the Foundation Phase (FP) mathematics curriculum by planning an activity that requires the learners to count on within the number range of zero to one hundred • She has an understanding of the competencies related to counting as described in the FP curriculum

Learners did not only have challenges with bridging the decades while counting forward, they also faced the challenge as they counted backward. The next counting session focuses on counting backward.

5.3.2.2 The bridging concept while counting back

Excerpt 5.5 is taken from the middle of the counting session. The learners had been counting as a class in ones from one to eighty while Gail was pulling the beads across the beadstring. She moved away from the beadstring, sat on her chair next to carpet and asked the learners to count back in ones again. This time she does not use the beadstring. Rather she asked them to 'close Mr Eighty's house'. Excerpt 5.5 below highlights what happened during this whole class counting session.

Excerpt 5.5 Showing the challenges learners face bridging the decades while counting back

34.	T	Right we are on eighty now. [*sitting down*] Am I right?
35.	LL	Yes
36.	T	So now we gonna close Mr Eighty's house. Let's close Mr Eighty's house [*using her hands to show the shape of a house starting with the roof]*
37.	TLL	Close Mr Eighty's house
38.	T	Now we gonna fall into Mr who's house?
39.	LL	Mr Seventy's house
40.	T	Mr Seventy's house where? [*Using her hands to show the shape of a house starting with the roof. Her hands are above her head so that everyone can see*]

41.	LL	At the top
42.	T	Right at the top good. Let's go
43.	TLL	Seventy-nine, seventy-eight, seventy-seven, [*staggering their hands as they count down*]
44.	LL	Seventy-six, seventy-five, seventy-four, seventy-three, seventy-two, seventy-one. Close Mr Seventy's house. Sixty-nine, sixty-eight, sixty-seven, sixty-six, sixty-five, sixty-four, sixty-three, sixty-two, sixty-one. Close Mr Sixty's house.
45.	T	Good! We are where?
46.	TLL	Fifty-nine, fifty-eight,
47.	LL	Fifty-seven, fifty-six, fifty-five, fifty-four, fifty-three, fifty-two, fifty-one. Close Mr Fifty's house.
48.	T	Good
49.	LL	Forty-nine, forty-eight, forty-seven, forty-six, forty-five, forty-four, forty-three, forty-two, forty-one. Close Mr Forty's house. Thirty-nine thirty-eight, thirty-seven, thirty-six, thirty-five, thirty-four, thirty-three, thirty-two, thirty-one. Close Mr Thirty's house.
50.	T	Athi, where are we going to go now?
51.	L	[*silent*] twenty-nine
52.	T	Athi, says we fall on twenty-nine. Let's go twenty-nine
53.	LL	twenty-nine, twenty-eight, twenty-seven, twenty-six,
54.	L	[*Tr signals the class to keep quiet and points at one learner who had not been attentive]* twenty-five, twenty-three
55.	T	Use your fingers
56.	TLL	[*Pointing at the fingers as she counts down*] twenty-five, twenty-four, twenty-three, twenty-two, twenty-one. Close Mr Twenty's house
57.	T	Let's go on, nineteen.
58.	LL	Eighteen, seventeen, sixteen, fifteen, fourteen, thirteen, twelve, eleven. Close Mr Ten's house

Extracted from L1, V1

Gail anticipated that her learners would find counting backwards across the decuple a challenge and she devised a strategy that helped her learners to manage and master counting back across the decades. Gail refers to this strategy 'Close Mr Ten's house'.

This Closing Mr Ten's House is a strategy where the learners count down with their hands put together to represent a roof of a house. The numbers are grouped in houses of tens for example numbers between twenty and twenty-nine are called Mr Twenty's house. As they count down they move from one house by staggering down their roof shaped hands from just above their heads until the last number of that decade then they close it saying "Close Mr Eighty's house" and move on to another 'house' They raise their hands again above their heads as they start on the new 'house'

The counting session began with Gail telling her learners to close Mr Eighty's house demonstrating with her hands closing the house as they normally did. The use of the expression prepared learners for counting back. Together with the learners, Gail closed Mr Eighty's house employing the action described above whilst using the expression. Gail asked the learners in which ten's house will they 'fall' into and where exactly will they would 'fall' into. The learners responded that they will 'fall' into Mr Seventy's house. Copying Gail, the learners raised their hands above their heads to indicate that they start counting back from the top of 'Mr Seventy's house' that is at seventy-nine. Together with their teacher they counted back moving their hands down as they counted. Gail's use of the expression and accompanying action assisted the learners in bridging the decade.

The questions Gail asked her learners were used to guide them into being clear about the number before 80. She told me that she liked to do this on the mat with the whole class as there are some learners who are not confident counting backwards. She said that "**weak kids can learn from the stronger children"** (FI2, V2, T39). Gail counted with the learners and then left them to count on their own. She reinforced their success on counting across each decade with a verbal encouragement, a '**good**' and kept on directing them with such questions as **"We are where**?" to help them to keep focused.

Gail checked to see if individual learners were following and counting with the class. For example, she asked one of the learners "**Athi, where are we going to go now?"** When she noticed a learner was not focused on the task, she stopped the other learners and asked that learner to count on his own. This helped all the learners to be attentive and to participate in the counting activity. The learner who had to count on his own, omitted a number as he counted down. Gail asked the learner to count with his fingers. To ensure that the learners knew what to do, she counted with them pointing to her fingers as she counted. The learners also stopped and started counting while pointing to their fingers. When they reached 20 they changed their action to close Mr Twenty's house. Learners were able to count smoothly down until they had closed 'Mr Ten's house'.

Gail used this strategy to alleviate the challenges learners meet when counting backwards and also helped learners to be active participants as they learn. As noted in section 5.2.1 learners face greater challenges when counting backward than when counting forward especially when bridging the decade. Reys et al. (2007) attest that many learners find it difficult to count backward, just as much as many adults find it difficult to recite the alphabet backward. Use of such expressions as 'close Mr Seventy's house' has advantages of helping learners remember how these numbers are grouped into decades.

Gail guided the session through her questions. She asked her learners "After closing Mr Eighty's house whose house do we they fall into?" and "Where do we fall? This helped the learners to bridge the tens accurately. As mentioned earlier bridging the tens is problematic for young learners and it becomes worse when they have to do it backward. Gail's strategy of 'closing' and 'falling' houses seemingly helped her learners to overcome this challenge. Learners could imagine themselves having to go down from a multiple story building with no elevators or stairs where they have to get down by demolishing each floor. Gail commented that 'closing the house' reminds them "**we are now finished with Mr. Seventy's house. Otherwise what they will do is they will go to seventy again or they will go to eighty instead of going to sixty-nine" (FI2, V2, T118).** Gail is echoing Wright's (2016) sentiments that in counting back learners can either fall into the wrong decuple or omit the decuple. Wright (2016) suggests that learners can face a number of challenges as they count backward, such as: a) counting down to the wrong decuple for example, forty-three, forty-two, forty-one, thirty, thirty-nine, thirty-eight…) or b) omitting the decuple (forty-three, forty-two, forty-one thirty-nine, thirty-eight).

I did not witness all these errors in Gail's class as she taught her class in a way that helped her learners to avoid errors and challenges that are common with counting. She would not wait until learners make an error to rectify it but anticipated the errors and taught in a way that helped to avoid them. She taught her learners in a way that was preventive of commonly known errors. This was likely due to her thorough prior knowledge of possible errors learners make during counting.

Gail reflects a deep understanding of the content she is dealing with and the challenges associated with that type of content. She counts with her learners at the problematic area then she leaves them to count on their own. When they manage to correctly bridge the decuple she approves with a "**good**!" Learners get encouraged with this positive reinforcement and they continue to count accurately. At the end of each decuple when they 'close the ten's house' she always asked them a question that would alert the learners to the next number such as "Where are we going to go now?" (L1, V1, T45) or 'Where?'

Gail moves with her learners as they count. She is observant of what learners do as they count. She was able to identify learners that are committed to what they are doing and those who have lost the track. At turn 52, Gail signals the class to keep quiet and points at one learner who had not been attentive. The learner misses the count and counts "25, 23" omitting 24. Gail used this as a way to keep all her learners actively and attentively involved in their learning because no one would like to be put on spot light like that.

Gail realised that the learner mentioned above had lost focus because they had been counting in the same way all the way from eighty. She instructed the learners "**Use your fingers" (L1, V1, T53),** instead of staggering their hands down as they had been doing. Gail avoided being monotonous during her counting session as mentioned earlier "**So instead of just counting and being boring you need to throw in these other things to keep them focused" (FI2, V2, T37).** When learners counted backward to ten, she stopped them then further developed her counting session by connecting counting backward to taking away or subtraction. This is discussed further later in this chapter.

The struggle in backward counting, like in forward counting, was not so evident during the whole class counting as those learners who could not count back just chorused along with those who could count efficiently. However, small group counting revealed the struggling learners as each child would be counting individually. During the whole class counting the evidence of struggle is reflected in L1, V1, T52, where Gail silences the class and asks an individual learner to count down from twenty-six and the learner goes twenty-five, twenty-three. Gail encouraged the learner to be focused and to use his fingers during the counting so as to engage in the one-to-one correspondence. Gail then picked up the counting with the rest of the class and crossed the decade with them from twenty-five to nineteen then left them to count down on their own which they did quite easily. However, small group counting exposed those learners who still had challenges in counting backward. In Excerpt 5.6 extracted from lesson L10, V1 illustrates how the Brown group learners struggled with counting back. They had been playing the dice game counting their throws in ones accumulatively. When they got to eight-three Gail asked them to count backward. The game that was played clockwise is in this extract turned to anti-clockwise to signify counting back is a reverse of counting forward.

Excerpt 5.6 Showing how the Brown group counted backwards.

89.	T	Okay now we will go backwards. We are on eighty-three hey! [*Passing the dice on the anticlockwise direction]*
90.	L	[*plays and gets six but remain silent*]
91.	T	Eighty-three,
92.	L	Eighty-two [*silence again*]
93.	T	Eighty-one
94.	L	Eighty-one [*silence*]
95.	T	Eighty
96.	TLL	Seventy-nine, seventy-eight, seventy-seven
97.	T	[*Passes the dice*]
98.	L	[*plays and gets six*] seventy-seven [*gets stuck*]
99.	T	[helping the boy count his fingers] seventy-six
100.	L	Seventy-six [*silence*] seventy-five [*silence*]
101.	T	Seventy-four [*counting on the boy's fingers and the boy counting after the teacher*]
102.	L	Seventy-four
103.	T	Seventy-three
104.	L	Seventy-three
105.	T	Seventy-two
106.	L	Seventy-two
107.	T	[*Pointing at the boys next finger*]
108.	L	Seventy-one
109.	T	[*Passes the dice*]
110.	L	[*Plays then looks up*]
111.	T	Seventy-one
112.	L	[*Silence*]

113.	T	Seventy-one take away one
114.	L	Sixty
115.	T	Ehhh! Seventy-one, sixty! Thus, a common error. Wait now I'm going to show you [*looking for some flard cards besides her*] let me find my seventies here. [*She put the flard card with seventy in front of the learners then looks for one. Puts 1 on top of 0 of 70 so that it reads 71*] ok! Seventy-one, look here now! Can you see 71?
116.	LL	Yes
117.	T	Is it a seven and a one
118.	LL	Silence
119.	T	[*Tr pulls off 1 from 70*] it is seventy and one am I right?
120.	LL	Yes
121.	T	[*Teacher puts back the 1 on top of the 0 for 70] If I take away 1 [taking away the flard card with 1*]
122.	LL	Seventy
123.7	T	Seventy, take away one

Extracted from L10, V1

The first learner to count backwards struggled to count down six from eighty-three. He literally counts each number after the teacher. He struggles to count down six from eighty-three. The next learner also struggles to count down six from seventy-seven. He could not count even one number down from seventy-seven. The teacher helps by leading the count. When the learner gets to seventy-one, he is stuck once more. Gail asked him seventy-one take away one and the learners gave sixty as an answer. Gail jumps in **"Ehhh! Seventy-one, sixty. Thus a common error" (L10, V1, T115).** She reflects the knowledge of the pattern of errors proposed as common counting back huddles by (Wright, 2016), discussed earlier in this chapter. The teacher is expected to know and understand the learners' misconceptions as this enables the teacher to respond flexibly and appropriately to the challenges experienced by learners (Cotton, 2010). Cotton (2010) asserts that focusing on the common misconceptions within each topic allows the teacher to develop an understanding of how each individual learner comes to understand mathematics. Because Gail has knowledge of what errors to expect she has means of helping learners to avoid them or readily overcome them when they surface during teaching and learning. In this case, she used the flard cards to demonstrate to the learner how to count down one from seventy-one as described in the excerpt 5.6 above and the learner manages to get the answer right. Gail displays knowledge that learners learn better when they use their senses. In this case, she engaged the sense of sight and hearing as she used the flard cards while she was speaking. She then encourages the learner to use the ‘Close Mr Ten’s house’ strategy. The learner manages to count down as demonstrated in the next excerpt 5.7 below although he omitted sixty-seven and got corrected.

The above excerpts demonstrate Gail’s employment of MKfT in her teaching. She is able to evaluate strategies that will work for different concepts she teaches. For example, she devised the ‘Close Mr Ten’s house’ strategy and noted that it can help learners to master not only crossing the decades but can be applied for crossing the hundreds and thousands as well. The same way it is used for the tens can apply to other levels of counting back. Her ability to evaluate procedures that work and are generalizable reflect her SCK and putting these into practice requires her KCT. She employed her KCS to anticipate that her learners would find it quite challenging to count down across the decades and also that they would find it interesting and motivating to use the ‘Close Mr Ten’s house’ strategy.

Gail is able to sequence her lesson from the known to the unknown, from the simple to complex. Learners start by counting in ones from one going up. All the learners were familiar with counting in ones especially between one and twenty because they had been doing that since Grade R. After counting forward, she instructs them to count back which is more complex. Gail demonstrates her KCC by keeping the counting within the range for Grade 2 counting numbers and facilitating both the forward counting and the backward. The DBE (2011) indicates that by the end of Grade 2 learners should be able to count forward and backward within the range of two hundred. Gail kept her range within one hundred which is the minimum for Grade 1. Gail also demonstrates her KCC through her knowledge of what resources learners can use for counting. She requested that her learners use their fingers in their counting. She stated that: “**Well our fingers are part of a resource Yah I mean your fingers can be ones, they can be twos, they can be fives, they can be one hundreds, anything (FI2, V2, TT 153, 155).** Gail encourages learners to use their fingers because fingers are **“part of them” (FI 2, V2, T157).** Her emphasis on using fingers is influenced by her understanding that “**Children need to learn, they need concrete objects in order to do maths. You can’t just tell them something. Thus, the basis as well is you got to discover things using concrete apparatus. (***Wiggling her fingers***) this is concrete” (FI 2, V2, T159).** She goes on to justify the use of fingers **“Mmhmm its much better for them to use their own fingers than to get**

up and walk around to go use your number grid, they can, but you're going to have lots of children walking around whereas fingers are on your hands the whole time" (FI2, V2, T159). This also reflects that Gail is concerned about order in the learning environment. Table 5. 5 below summarises the aspects of each domain that were prevalent during the development of the concept of bridging the tens while counting back.

Table 5.4 Summary of the MKfT domains prevalent in development of bridging the tens concept while counting back

MKfT Domains	Indicators
SCK	• Evaluate that the 'Close Mr. Ten's house' strategy' works for counting back even across the hundreds or thousands and they will always fall at the top of each hundred or thousand
KCT	• Remediate errors (e.g. when a learner counts back from seventy-one to sixty she uses the flash cards to help the learner to realize that seventy-one count back one is seventy not sixty • Asking productive questions. The questions she asks learners about which decade do they fall to at the end of each decade and where they exactly do they fall into help learners to be prepared to count back • Recognize what is involved in using a particular representation • Select appropriate representations to illustrate content. • Sequence mathematical content and instruction. For example, she instructs her learners to count back after they have counted forward • Select representations for a particular purpose. The Close Mr. Ten's house was selected to aid the learners count back across the decades
KCS	• Anticipate what learners will find difficult or easy when counting back especially across the decades. She counts with them on areas she anticipates to be challenging and leave them to count on their own where she anticipates they will find easy. • Predict what learners will find interesting and motivating when choosing a representation. The 'Close Mr. Ten's house' strategy proved interesting and motivating to learners

	• Identify counting from seventy-one to sixty as a common errors • Anticipate that learners may think that when they fall back into the previous decade they start counting from where that house begins and start counting forward, for example when they finish the seventies they fall into the sixties and may count from sixty upwards
KCC	• Articulate the expectations of the Foundation Phase mathematics curriculum (Grade 2 learners are expected to be able count in ones forward and backward between one and one hundred. • Articulate familiarity with the FP mathematics curriculum (learners at Grade 2 are expected to do a lot of counting • Demonstrate knowledge of what resources learners can use to count (e.g. their fingers) • Gail demonstrated knowledge of the topics in the grade 2 curriculum. (e.g. counting and subtraction)
HCK	• Makes connections between counting back and subtraction. E.g. when a learner is challenged to count back from seventy-one she asks the learner to take away one from seventy-one to get the number the learner needs to count back to

Counting backwards did not only challenge learners in crossing the decades but learners also struggled to follow the downward sequence of numbers. The next counting concept focuses on the struggle learners had with counting backward besides bridging the tens.

5.3.3 The subtraction concept

It is almost ten minutes into the first whole class counting session (L1, V1) described in Vignette 5.1. The learners had counted forwards in ones from one to eighty, then backward from eighty to ten using the 'Close Mr Ten's house' described in section 5.2.3 above. The learners were starting to lose focus whilst counting backwards. Gail stopped them at number ten and asked them what they were doing. Excerpt 5.7 below projects how Gail linked counting back and subtraction

Excerpt 5.7 Connecting counting and subtraction

56.	LL	Eighteen, seventeen, sixteen, fifteen, fourteen, thirteen, twelve, eleven, close Mr Ten's house
57.	T	Ok you guys you know what? [*Leaning forward and balancing her hands on her knees*] What are we actually doing?
58.	LL	Mumbling
59.	T	What does counting backwards mean?
60.	L	Mmm, it's like forward but you are doing it upside down.
61.	T	Do you know what! What does counting backwards mean? Look at me [*picking up a stick and pointing to the numbers on the chart*] Twenty-nine, twenty-eight, twenty-seven.
62.	L	It's like the opposite.
63.	T	What am I doing all the time? You can tell me, what I am doing?
64.	L	You are counting backwards.
65.	T	I am counting backwards! But What does counting backwards mean?
66.	L	It's like counting in ones but doing it the opposite way.
67.	T	What does that mean? [*pauses*] What does that mean?
68.	LL	[*silence*]
69.	T	[*Tr standing*] I am here. I am on twenty-nine [*Taking one step back at a time*] twenty-eight, twenty-seven. What am I doing? I am taking a step…
70.	TLL	Backward.
71.	T	Taking one away each time. Doesn't it? Am I not taking one away?
72.	LL	Yes!
73.	T	So, what does counting backward mean?
74.	LL	Taking away.
75.	L	You are taking away [*Siting and pointing to the numbers on the chart*] Thirty take away one?
76.	LL	twenty-nine
77.	T	Take away one?
78.	LL	Twenty-eight
79.	T	Take away one?
80.	LL	Twenty-seven

Extracted from L1, V1

Gail began this excerpt with a simple question: "What are we actually doing?" It seemed obvious to the learners that they had been counting backwards and therefore they got confused by the question. They had no clear response. Gail rephrased the question in a way that was clearer to learners suggesting she realised they did not understand the first question. Now she asked the question in a more direct way, "What does counting backwards mean?" A couple of learners interpreted the question literally and tried to explain counting backward in relation to counting forward. Gail repeated the question a few more times when she did not get the response she expected she illustrated counting back using a number chart. She tried to guide learners into understanding what her question intended to ascertain from them. When learners did not follow, she stood up and started taking steps backwards. Each time she took a step, she counted backwards. She then asked the learners what she was doing as she was walking backwards. The learners did not make the link with subtraction until she highlighted to them that she was taking one away each time she took a step back and counted back. Having established that counting backwards was the same as subtraction, she asked them a few subtraction oral questions (e.g. twenty-nine take away one). The learners responded accurately to all the problems posed. Gail asked them what counting back meant. The learners responded accurately. Finally, Gail linked skip counting to subtraction.

Gail was able to determine what learners found confusing in the first question and rephrased her question in order to engage the learners about the meaning of counting back is and why they were doing it. Gail reinforced Gifford's (2005) and Wright's (2008) notions that counting should always be done for a purpose and that learners should be able to determine the purpose. She employed various strategies to help her learners understand the concept she wanted them to learn. She used questioning, illustrated counting backward on a number chart and actively demonstrated it by physically taking steps backwards. All these strategies reflected Gail's MKfT. Like Cotton (2010), Gail demonstrated that a teacher should use various ways of presenting mathematical content and should understand which of the available strategies is most appropriate to engage learners in meaningful learning. The demonstrations of counting back that Gail made with the number chart and taking steps back helped the learners to realise what happens when they are counting backward.

According to Cotton (2010), "at foundation stage we are encouraging learners to notice what happens when we add or subtract one from a number" (p. 86). Gail's actions helped her learners to associate counting back with moving back and counting on as moving forward. This in a way, lays foundations for use of a number line. By taking steps back as she counted backwards, Gail introduced learners to the reduction structure of subtraction which in turn introduced learners to the number line (Haylock & Cockburn, 2008). The image Gail created in the minds of her learners of moving one digit/count to the right/forward to add and one digit/count to the left/back to subtract is essential to support them later when they explore more complex ideas (Cotton, 2010). Learners should be able to keep in mind that addition and subtraction involve moving up and down the number line respectively.

Gail demonstrated her ability to sequence the content she was teaching in ways that are recommended by the literature reviewed earlier for teaching. The counting session started off with counting forward, which represents addition. This was followed by counting backward as indicated in Excerpt 5.7. Counting backwards was used to introduce the learners to the meaning of subtraction. This illustrated Gail's ability to sequence the mathematics content she is teaching and the ability to connect the content she is teaching to previously taught content with view to future topics. She took the learners from what they knew (counting) and guided them towards the new knowledge (counting backward as a strategy for subtraction). The way Gail taught backward counting reflected her knowledge of the four structures of subtraction: the partitioning structure (take away); the comparison structure (find the difference between two or more quantities); the reduction structure (counting back) and the inverse of addition structure (Haylock & Cockburn, 2008). Gail's awareness that subtraction is more than just partitioning or taking away is evidenced by her introducing her learners to an important and fundamental structure of subtraction, the reduction structure. Learners were familiar with the partitioning structure of subtraction where they had been using objects to partition or to take away. Gail introduced to learners that subtraction is not all about taking away or partitioning but there are other structures (such as the comparison, the difference and the reduction structure) which she introduced one at a time. She then introduced them to another subtraction structure, the reduction structure.

Gail was cautious of the language she used when teaching mathematics to her Grade 2 learners. She explained to learners that counting backward is taking away. She chose to use the term 'take away' rather than subtraction or minus because the learners had used this term in Grade 1 and even out of school. The learners are familiar with 'taking away' because they have been doing 'taking away' problems in previous grades. Gail once more was seen teaching from the known to the unknown, from the simple to the complex. In one of the PGCE FP lectures, Gail discouraged the pre-service teachers from using the terms ‘subtraction’ and ‘minus’. She told them that the learners do not understand these terms and they have nothing to associate the words with. She proposed that the learners understand the meaning of ‘take away’ because they have been counting objects in groups and taking away some of the objects in Grade 1. Furthermore, the term ‘take away’ is familiar with the learners’ life experiences where they can for example ‘take away’ an apple from the fruit basket and count how many fruits are left (Personal journal, 27 July 2016).

Haylock and Cockburn (2008) argue that language plays an important role in teaching the different structures of subtraction and advise that “teachers need to be aware of the range of situations, language and pictures that have to be connected in developing an understanding of this (subtraction) concept” (pp. 63-64). Gail used the term ‘take away’ which is associated with the partitioning structure, to help learners to understand counting back which is the reduction structure of subtraction. The staff of Motion Math (2012) explain that using the term ‘take away’ “helps learners connect the concept of subtraction to their real life experiences” while the counting back “accounts for both size and direction (vector) of a subtraction problem, enabling students to subtract a larger number from a smaller number (for example, 5 – 7 = -2)” (p.1). Gail connected counting to the learners’ life experiences and at the same time prepared them for the mathematical calculations they will meet as they progress in mathematics education.

Gail facilitated the concept development with a series of questions. Her questions were productive (e.g. what are we actually doing? What does counting backward mean?). She gave her learners ample time to think and figure out the answers to these questions without simply telling them. She guided the learners through questions to construct their own knowledge as she claimed that a teacher is “**a facilitator, you facilitate learning, you facilitate learning that is what maths is all about. […] You don’t show methods! (FI2, V2, T14)**. Gail believes in facilitating learning through questioning and providing appropriate activities for the learners to construct their own knowledge.

Gail’s way of teaching demonstrates her understanding of what the work of teaching entails. She unpacked the mathematical ideas and linked them one to another. She demonstrated her SCK through her knowledge of the subtraction structures. She understands what her learners are trying to put across when trying to answer her question on what counting back is (KCS). In demonstrating counting backwards, Gail used various representations, such as the number chart

(KCT). Her KCT is also evidenced by her ability to ask questions that enhance teaching and learning. Gail is able to connect counting back and subtraction (HCK) and demonstrates knowledge of the subtraction structures as highlighted above. The knowledge of these structures reflects her SCK while her ability to teach each of these structures to her learners reflects her KCT. This kind of knowledge is only relevant to teaching. It is the kind of knowledge that a teacher does not have to teach to her learners but employs it in the development of concepts. Table 5.6 below gives a summary of the MKfT domains that are evident in the excerpt where Gail developed in learners an understanding of counting backward as a strategy of subtraction.

Table 5.5 Summary of the MKfT domains prevalent in development counting back as a strategy of subtraction

MKfT Domains	**Indicators**
SCK	• Knowledge of the subtraction structures. • Knowledge of the relationship between counting backwards and subtraction.
KCS	• Hear and interpret learners' emerging and incomplete ideas. She understands the learners' explanation of what counting back is although it is not what her question desires to get from learners. • Predicts what learners may find interesting and motivating when choosing an example and multiple forms of representation (e.g. number grid, walking back etc.).
KCT	• Sequences mathematical content from forward counting to backward counting then using counting back as a method of subtraction. • Selects appropriate representations to illustrate what counting back means. • Selects examples to take students deeper into the mathematical content. • Promotes meaningful learning by questioning learners to encourage them to develop an understanding of the relationship between counting back and subtraction.

	• Asks questions that facilitate the learners thinking and questions that lead learners to discover how counting back is related to subtraction.
HCK	• Makes connections across the topics in mathematics such as linking counting back with subtraction. • Articulates how the reduction structure introduces the number line.
KCC	• Demonstrate familiarity with the structure of the mathematics curriculum. Learners need to count back and forward and be able to relate counting to the basic operations addition (counting on) subtraction (counting back) multiplication (skip counting).

In the next section I will discuss how place value was developed during the counting session.

5.3.4. The place value concept

The development of the place value concept occurred at the beginning of the whole class counting session outlined in Vignette 5.1. The learners were counting in 1s while Gail was pulling beads across the beadstring with each count. She stopped the counting first at thirty-six and then at forty to ask the learners about the composition of these two numbers. She stopped the counting for a third time to ask learners how fifty-five is composed. Excerpt 5.8 begins when she asked the learners what fifty-five is made of.

Excerpt 5.8 Showing how place value was developed

56.	T	(The learners have been counting forwards in ones. They reach fifty-five.) Stop! What numbers do I need to make fifty-five? Listen you can hear it, fifty-five [*Tr emphasising fifty with her voice*].
57.	L	A fifty and a five.
58.	T	Who is shouting?
59.	LL	Trevor
60.	T	Trevor next time you do that there will be a consequence. Okay! This is very rude! What do I need Morgan?
61.	L	A fifty and a five.
62.	T	A fifty and a five. Listen! If I put that number in my mouth and pull it out like a piece of chewing gum. Listen! Fifty-five. I will show you how it

		actually works [*Tr taking the number cards from the box*]. Look [*Showing the learners 55 using the flard cards cards*] Can you see?
63.	LL	Yes
64.	T	I want you to watch this. Here is fifty-five [*showing them the card number once more*]. I want you to watch [*putting the cards in her mouth then pulling them out one at a time starting with 50*] fifty [*then pulling out 5 and putting it on 50 so that it reads 55*] five
65.	LL	Ha! Five.
66.	T	What two numbers?
67.	LL	Fifty and five
68.	T	Right! We are on fifty-five [*going back to the beadstring and pulling the beads across the string once more*]
69.	TLL	Fifty-six, fifty-seven.
70.	LL	Fifty-eight, fifty-nine, sixty, sixty-one, sixty-two, sixty-three sixty-four, sixty-five, sixty-six, sixty-seven.,
71.	T	Stop! What two numbers do we need to make sixty-seven? If you can't remember pull it out of your mouth [*demonstrating pulling it out*] as you say it.
72.	TLL	Sixty-seven
73.	T	Dan, can you try! Sixty-seven.
74.	L	Sixty and seven.
75.	T	Well done! Ok we were at sixty-seven. Let's go.

Extracted from L1V1

Gail stopped the learners at various times as they counted forwards in ones. She asked them: "What numbers do I need to make fifty-five?" She repeated and emphasised the pronunciation of ***fifty-five*** and once more requested the learners to listen carefully so that they could hear the numbers that make fifty-five. One learner shouted out the answer. Gail acknowledged the accuracy of the given answer but reached for her flard cards and demonstrated how fifty-five is made of a fifty and a five. She emphasised to the learners that they should watch and listen carefully as she demonstrated fifty-five using the flard cards. She placed the '5' onto the '50' so that the learners could read the number aloud '55'. Gail placed the cards in her mouth and pulled the cards out one at a time. She did this slowly as if she had gripped a piece of 'chewing gum' (T62) and was pulling it out of her mouth. As she did this she called out the names of the numbers that composed fifty-five. She started with fifty then five. She put the cards together again so that the learners could read fifty-five. This demonstration was followed with a loud chorus from learners "Ha five" which reflected an 'aha' moment; a sudden discovery of new knowledge or understanding. Gail asked the learners once more what two numbers make fifty-five. There was a loud response from learners that it is made out of a fifty and a five. The children resumed counting on in ones from fifty-five. Gail stopped them at sixty-seven and asked what sixty-seven is made of. She encouraged learners to pull out the imaginary cards if they were not sure of the numbers. The learners were able to give the correct answer.

The episode began with a question: "What numbers do I need to make fifty-five?", showing the value Gail places on questioning as a teaching strategy. The whole excerpt hinges on this question as she used the counting session to develop the learners understanding of place value.

Choosing to demonstrate how fifty-five is composed reflected Gail's understanding of how the learners take time to understand the place value system and to realise that in two digit numbers the left refers to the tens and the right the units (Gifford, 2005). Through using the number fifty-five (55), Gail made the learners realise that though the digits may be the same ('5'), the position of the digits determined their values. In this way, she highlighted the significance of place value.

In this excerpt, Gail demonstrated her ability to choose appropriate representations that successfully illustrated the concept she was teaching. Gifford (2005) asserts that reading and writing figures above ten is confusing to young counters and may take the learners a long time to learn. She argues that the use of flard cards helps learners to decode the numerals. Gifford (2005) claims that some learners confuse left and right and to them it is not obvious that in two digit numbers, the number on the left is tens and the one on the right is units, she thus recommends the use of flard cards suggesting it makes it easier to understand how the first 'five' is fifty and the second, 'five'. By using the flard cards Gail helped her learners relate the sound of the number as it was being said to how it should be written. She praised the use of flard cards in teaching saying, **"We use flard cards. Fantastic! You can't actually do addition of two digits' numbers or any kind of addition or subtraction without them" (FI2, V2, T83).** Flard cards make it possible for learners to break up numbers so that they can easily add or subtract two or more digit numbers.

Gail evaluated the learners' responses during her teaching. She knew which responses to accept, which to praise, which to follow up on, and which to ignore. She ignored the correct response given by the learners when she asked them what two numbers are needed to make fifty-five (e.g. fifty-five is made out of a fifty and a five) so as to create an opportunity to ensure everyone understood. She anticipated that not all her learners could understand the composition of two digit numbers hence she demonstrated this using the flard cards. She claimed that she purposefully chose to extract the cards from her mouth like a piece of chewing gum **"for a bottom group to realize, and for other children who haven't got that" (FI 2, V2, T93).** She acknowledged, **"so you're actually showing them using the flard cards" (FI 2, V2, T95).** This suggests that when Gail planned her counting sessions she had all her learners in mind. While this was a whole class counting session, Gail ensured that she reached all the learners at their different levels of understanding. Gail appeared to know what would appeal to the more competent learners and what would make learning meaningful and understandable to the learners deemed less competent. She argued that a teacher's "**teaching ability lays with the bottom group, that is where your teaching ability lays [nodding] it's very easy to teach a top child" (FI2, V2, T103).** This is evident in Gail's teaching as she made every effort to ensure that her learners deemed less competent were engaged with the learning process. The representation and examples used are carefully selected to ensure her learners develop a solid grounding in understanding place value.

Gail used examples of things learners are familiar with and that make her explanation easier to understand. A chewing gum stretches when pulling it out and so is the sound of ***fif-ty*.** The illustration given does not only create a mental image in learners that they can refer to each time they deal with two digit numbers but also help them to relate mathematics and their life experiences. In turn 28 she asks them once more **"What 2 numbers do we need to make 67? If you can't remember pull it out of your mouth (demonstrating pulling it out) as you say it" (L1, V1, T28).** Gifford (2005) propose that learners often have challenges writing figures above twenty. She argues that if they had to write '67' for example they would write '607' and therefore suggest that overlapping flard cards can help them to see how the number is made up.

Gail was able to articulate how understanding the composition of numbers fits into the mathematical concepts which learners will meet later in mathematics. She used counting as an opportunity to develop learners' understanding of place value, which learners need to do addition and subtraction of two digit numbers. Understanding place value is helpful when learners have to break and build numbers, a strategy learners will use to solve addition and subtraction problems.

Gail demonstrated a variety of MKfT domains during the place value excerpt in her counting session. She exhibited her KCT through her use of questioning during the session to lead learners to the construction of an understanding of place value. In addition, her choice of fifty-five as her initial example for demonstrating how two digit figures are composed was an interesting selection. This choice also reflects her KCS in that she was able to anticipate that learners may confuse which five represents the tens and which one represents the units. Gail evaluated the learners' responses and reacted to them accordingly. She knew which responses to take, which to respond to, and which to ignore. All these aspects of her lesson point to both her KCS and KCT. Gail revealed her knowledge of the connection between content and curriculum through her decision to use the flard cards as a representation to teach place value and her knowledge of the expectations of the Grade 2 mathematics curriculum. Flard cards are a recommended resource in the CAPS for the teaching of place value. Evidence of Gail's horizontal knowledge is based on her understanding that place value is an important concept when learners start operating with two-digit numbers. The table below reflects the MKfT domains that were employed during the excerpt where Gail facilitated the development of place value during the whole class counting session.

Table 5.6 Showing the MKfT domains in action during the place value excerpt

MKfT Domains	Indicators
KCT	• present mathematical ideas that two digit numbers are made out of tens and units. • sequence the mathematical content from simple counting to development of place value. • selects and uses flard cards to demonstrate how fifty-five is composed. • uses an example of pulling out a piece of chewing gum to illustrate how the number should be pronounced in a way that make it possible to determine its components. • asks productive questions such as what two number are needed to make fifty-five to lead learners to understand place value.

	• recognises what is involved in using a particular representation such as pulling out the flard cards like a chewing gum from one's mouth demonstrate the composition of numbers.
KCS	• recognises and articulates misconceptions learners carry about a particular mathematics concept for example that fifty-five is five and five. • demonstrate knowledge of how Grade 2 learners construct knowledge about the composition of numbers through the use of flard cards. • Anticipates that the weak learners may find it difficult to understand how two digit numbers are composed. • Knows which of the learners' responses should be pursued, ignored or put on hold. Gail ignored initial response that fifty-five is made out of a fifty and a five.
KCC	• demonstrates a familiarity with the structure of the curriculum. Grade 2 learners should develop some of the difficult concepts during the whole class counting sessions. • know what instructional materials (flard cards) are available, what their purpose is and how effective they are for teaching place value.
HCK	• articulates how place value fits into breaking and building up of numbers and addition and subtraction that that comes later.

This chapter analysed how Gail employed her MKfT in developing number sense through counting. The MKfT domains indicators summarised in Table 5.1 were used during the analysis. These indicators were useful when Gail was focusing on number operations or place value, but not necessarily for counting, which led to me having to develop my own. Different concepts that were developed during the counting sessions were identified and analysed. These were the *teen* and *ty,* the *bridging of the decades* during forward counting and during the backward counting, the *counting back* outside bridging the decades, counting backwards as a strategy for *subtraction* and the *place value.*

During the analysis of these counting sessions I found that Gail reflected knowledge that there is more to mathematical ideas and concepts than what is obvious to those who use mathematics outside the field of teaching. The SCK that Gail demonstrated included an understanding of the counting principles, the significance of developing rote counting, rational counting and advanced counting. She knew that mathematics is about patterns and relationships and that learners learn mathematics best when they are able to identify patterns. In addition, she had knowledge of the errors common to counting. She had a deep understanding of different ways to represent concepts and the mathematics inherent in her representations. As such, she was able to evaluate the representations and examples that would be beneficial during the learning process. Such knowledge is only useful to the work of teaching. This kind of knowledge is what Ball et al. (2008) identified as the SCK.

Gail's KCT was evidenced through her teaching. She demonstrated her knowledge through: identifying and remediating learners' errors; asking productive questions, to which she gave the learners adequate time to think and respond; selecting appropriate representations and examples to illustrate the various concepts and successfully utilising those representations; and sequencing the mathematics content in a way that facilitated meaningful understanding.

Gail was able to anticipate what the learners would find challenging, interesting and motivating. In the choice of activities, representations and examples she would keep that in mind. She also made use of a variety of exercises, examples and representations in order to avoid any monotony. She listened carefully to her learners, interpreted their emerging and incomplete ideas and respond appropriately. This mirrored her KCS.

Gail demonstrated the knowledge of the topics that are covered in each grade at FP and how these topics can be connected. She drew on a wide range of mathematical ideas to articulate and often made links for problem solving. Her lessons were built on the work that learners had already covered hence she was familiar with the FP curriculum and how it links with the next phase. She did not focus on one topic during her teaching but used each counting session as an opportunity to lay foundations for the other mathematical concepts that would be taught later. She knew what resources were available for her to use during her planning and teaching. She used a variety of representations suggested in the FP mathematics curriculum in her teaching. Thus, one can conclude Gail also had KCC and HCK.

Although Gail's teaching was analysed using the different MKfT domains, during her teaching there was no significant indication that she is now moving from one domain to another. There was a seamless flow between these domains which signified that Gail did not take a moment's break to think of what she was to do next but taught like an experienced driver who when driving a car does not stop to thick which gear to change to but automatically changes the gears when the need arises without having to think about it. This seamless flow between the concepts indicates expertise in the profession of teaching.

5.4 CONCLUSION

This chapter has established what Subject Matter Knowledge and Pedagogical Content Knowledge Gail employed in conducting her counting sessions. However, the aim of this research was to establish MKfT in relation to the development of numbers sense through counting. As it was established in Chapter One, one of the reasons for the continued poor performance in mathematics in South Africa is that learners exit FP without number sense (Graven et al., 2013). While my emphasis in this chapter has been on the MKfT that Gail draws on when teaching counting, and related concepts, I provided a narrative to make an explicit link to the development of the learners' number sense. I suggest that learners' number sense emerges from their experience with counting and related concepts. Counting, in and of itself, is not number sense, but it is key in the development of number sense.

As discussed in Chapter Three, number sense is a well organised conceptual framework of number information that enables learners to understand numbers and their relationships and to solve mathematical problems that are not bound by traditional algorithms (Bobis, 1996). To develop this framework, as suggested in Chapter Three, learners need to develop (1) an awareness of the relationship between number and quantity; (2) an understanding of number symbols, vocabulary and meaning; (3) an understanding of different representations of number; (4) an ability to compose and recompose numbers; (5) an awareness of number patterns including recognising missing numbers; (6) an awareness of magnitude and comparisons between different magnitudes; (7) competence with simple mathematical operations and (8) an ability to engage in systematic counting, including developing notions of cardinality and ordinality (Back, 2014). Gail's counting sessions as analysed in this chapter involved her learners in counting activities that addressed the above-mentioned requirements for developing number sense. I mention a few examples below of how she used her counting sessions to develop learners' number sense.

Gail employed the counting principles in her counting lessons and engaged her learners in systematic counting and always had representations to facilitate one-to-one correspondence counting, which helped her learners to develop understanding of the muchness of numbers, their stable order and cardinality. All the counting sessions provided learners with an opportunity to count systematically by employing one-to-one correspondence. For example, in a lesson described in Vignette 5.1, Gail tagged the beads from the beadstring while the learners counted in ones systematically. In Vignette 5.4, learners played the dice game and counted out blocks equivalent to their throws. This, aside from enhancing understanding of the muchness of numbers and mastering their sequence, provided learners with an opportunity to work and play with concrete materials, to count for a purpose rather than just for the sake of counting, and to explore number patterns and their relationships.

During the counting session learners were introduced to different problem solving strategies such as relating counting forward to addition, counting backward to subtraction and skip counting to multiplication. Gail made sure her learners understood the composition of numbers so that they would be able to compose and decompose numbers - a skill they will need for addition and subtraction of two or more digit numbers. During such moments, Gail gave her learners oral problems where they would use each strategy to solve mathematical problems. She always insisted on learners explaining how they got their answers and this developed their conceptual understanding and competence with simple mathematical operations.

Gail emphasised the point of learners identifying patterns as they did their counting activities for she believed that mathematics **"is all about patterns"**. When a learner had identified a pattern, she encouraged them to share with their peers then apply the pattern in their own learning. For example, in the lesson described in Vignette 5.3, when a learner discovered that they could add or subtract tens by moving up or down the number grid Gail asked the learner to demonstrate how it works and encouraged the other learners to use the skill to solve the problems she gave them orally during that group activity. From the analysis of data presented in this chapter, accompanied by this overview link of the counting sessions and what is expected of teachers to develop number sense, one can conclude that Gail's MKfT enabled her to effectively develop number sense in her learners.

Having presented and analysed my data in this chapter, in the next chapter I conclude this study by discussing my concluding remarks, key findings, some insights emerging from my research and opportunities for further research.

CHAPTER SIX

CONCLUDING REMARKS

The research process and the presentation of this thesis was guided by the question:

> *What MKfT in relation to development of number sense through counting does an expert Grade 2 teacher have and use in her teaching?*

The question enabled me to uncover both the content knowledge and the pedagogical knowledge Foundation Phase (FP) teachers need to develop FP learners' number sense through counting. In Chapter One it was established that both national and international studies have shown that learner performance in mathematics in South African is below the expected standards. Learners in South Africa are performing poorly in mathematics and this is a cause for concern for the South African government, policy makers, researchers, teachers and the South African nation as a whole (Kazima et al., 2008; Henning, 2004; Graven et al., 2013; Spaull & Kotze, 2014).

Previous research relating to mathematics teaching and learning established a number of factors contributing to poor learner performance. Some of the major factors are that the many mathematics teachers in South Africa lack sufficient content and pedagogical knowledge to teach mathematics effectively. This has led to learners' inadequate knowledge and skills acquisition in their early years of learning subsequently resulting in poor learner performance in mathematics (Fleisch, 2008; NEEDU, 2013; Spaull, 2013).

Research that seeks to understand this dilemma in the Foundation Phase has established that learners exit FP without developing strong number sense to calculate and solve mathematical problems efficiently and effectively (Schollar, 2008; Hoadley, 2012; Askew, 2012; Graven et al., 2013). Number sense as shown in Chapter Three, develops in learners the flexibility and fluidity in working with numbers. It equips them with knowledge and skills they need to meet the demands of mathematics learning as they go up the grades. According to Graven et al. (2013), "number sense and mental agility are critical for the development and understanding of algorithms and algebraic thinking introduced in the intermediate phase" (p. 131). They suggest that if a learner has not developed strong number sense in the FP they will have challenges in learning mathematics, not only in the Intermediate Phase, but throughout their life in the mathematics classroom.

The goal of my research was to learn and establish from the real classroom situation of an expert FP teacher, the knowledge and skills FP teachers require to develop learners' number sense. I wanted to learn from practice in order to inform pre-service and in-service teacher education programs of what Mathematics Knowledge for Teaching (MKfT) FP teachers need to develop learners' number sense successfully enough to equip them for learning mathematics with understanding. In my research I chose to investigate how counting sessions can be used to effectively develop number sense, since counting is the first mathematical concept learners are formally introduced to at school (Naudé & Meier , 2014). I made this decision in order to establish how teachers can start developing number sense from the very beginner concept of their learning career. In this research I was guided by Ball et al.'s (2008) MKfT framework to investigate the knowledge Gail has and uses to develop number sense in her Grade 2 learners.

6.1 MY THEORETICAL FRAMEWORK: MKFT

As mentioned earlier in this research I used Ball et al.'s (2008) MKfT framework to analyse the collected data so as to determine the knowledge that Gail employed to develop number sense through counting. This framework is the most ideal framework as it focuses specifically on the knowledge required for teaching mathematics effectively. It seemed to fit perfectly with what the mathematics education community is looking to provide: an informed intervention to the current predicament of poor learner performance in mathematics in South Africa. This MKfT framework, as discussed in Chapter Two, has six knowledge domains.

Having established that the South African teachers lack both the mathematics content knowledge and the pedagogical knowledge, this framework is a suitable framework to use as it specifically meets the criteria of what I am investigating. The framework provides a basis for analysing Gail's teaching to find out the knowledge she employed during her counting sessions to develop the learners' number sense. I compared Gail's knowledge and teaching skills demonstrated during teaching to the indicators of each of the five domains of the MKfT (that is, SCK, HCK, KCT, KCS and KCC) in order to establish what knowledge is necessary for developing number sense. As discussed in Chapter Two, I did not focus on Gail's CCK on the assumption that Gail's positioning as an expert teacher and a part time lecturer on mathematics methodology assumed her possession of CCK.

6.1.1 Limitations of using MKfT theoretical framework

I found working with Ball et al.'s (2008) MKfT framework rather complex. It was not easy to establish the various domains of MKfT in practice in the classroom. It was difficult to distinguish the knowledge domain behind each teaching action and interaction because of the 'thin line' between these knowledge domains. The knowledge domains do not have clear definitions and boundaries. Indeed Ball et al. (2008) acknowledge that "it is not always easy to discern where one of our categories divides from the next, and this affects the precision (or lack thereof) of our definitions" (p. 402). Ball and her team made use of examples of mathematical calculations and learner errors to communicate their ideas about each domain. Pinsky (2013) argues that "it is preposterous to imagine, for example, a conference presentation in which the presenters do not justify their mathematical ideas, use mathematical representations effectively, or make clear the language they are using" (p. 40). Given the above, it was difficult to determine what knowledge was used where and when since any given task could easily require knowledge from many or all of the other domains.

Ball et al.'s (2008) MKfT framework diagram (Chapter Two), when considered with the definition given to SCK, gives an impression that SCK is dominant in the work of teaching. I expected to find Gail relying strongly on this knowledge domain, however it was difficult to establish the difference between SCK and CCK. While I did not focus on CCK, it seemed that Gail relied mostly on her CCK to inform her teaching. The definitions, provided by Ball et al. (2008), of these two domains do not draw an explicit line between the two. Markworth, Goodwin and Glisson (2009) define SCK as "content knowledge needed for the teaching of mathematics, *beyond* the common content knowledge needed by other professions" (p. 69) while CCK is defined as the knowledge required in order to solve such tasks as are given to pupils (Wilkie, 2015). It then becomes difficult to determine the content knowledge needed beyond other professions because we do not know what mathematics content is needed by other professions. Flores et al. (2012) concur that none of SCK definitions in literature "specifies the nature of the knowledge itself, but they all evoke external agencies" (p. 2). In other words, they suggest that SCK seems to be embedded in pedagogy (e.g. identifying student errors) rather than specifying the mathematical misconception behind the error.

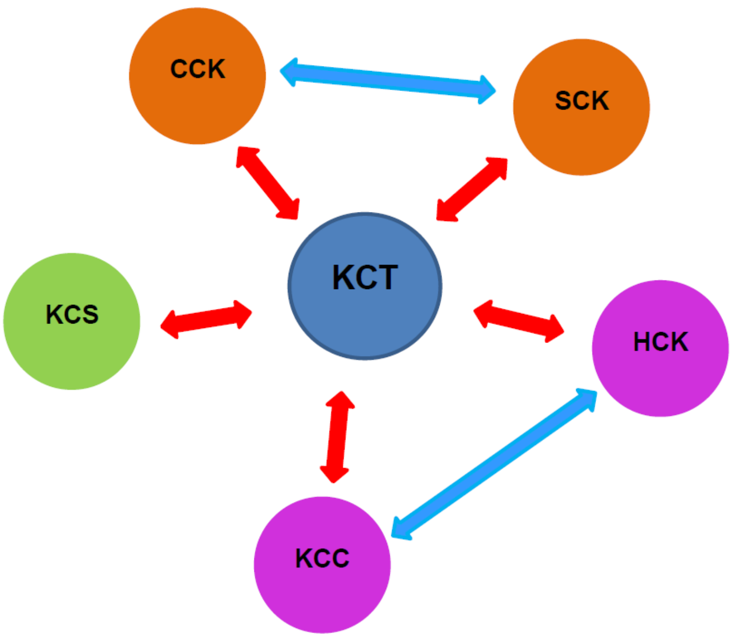
6.1.2 Key findings from my research

I found that the indicators that Ball et al. (2008) has established for each domain were not particularly useful in relation to the teaching of counting. I surmise that they may be better suited to studying the MKfT teachers draw on in teaching number operations and other mathematical concepts. Given this, I had to establish my own indictors by drawing on Ball et al.'s (2008) definitions of each of the domains. Most notably was the challenge in identifying Gail's SCK in relation to counting.

In this study, I found that KCT seemed to be the centre of MKfT. My study found KCT directly linking the teacher and the learner. Gail expressed KCC, HCK KCS, SCK through her teaching. The knowledge of the other five domains only become beneficial to the learner through KCT where the Gail actually interacted with the learner and employed the knowledge of the other five domains to improve teaching and learning. For example, Gail's knowledge about the misconceptions and errors made in their counting (KCS) was crucial to her teaching. She taught in a way that addressed possible errors prior to the learners even making the errors. For example, she knew that learners confused the ***teen*** and the ***ty*** numbers. She thus taught them in such a way that that this possible error would be addressed. She emphasised the ***teen*** sound on the teen numbers like thir***teen***, four***teen*** and the ***ty*** sound with numbers like thir***ty*** and fort***y.*** She also asked her learners what the difference was between four***teen*** and for***ty***. I thus suggest that the five domains (CCK, SCK, KCC, HK and KCS) influence her KCT.

At the same time, the other knowledge domains CCK, KCC, SCK, HCK and KCS would not be very useful if the teacher cannot actually present or carry out the actual work of teaching by presenting the content sequentially, making good use of the appropriately selected representations and examples, asking productive questions and so on. Thus, the other knowledge domains facilitated better teaching by informing the teacher what content to teach (KCC); how it related to the topics that have already been taught or still to come (HCK); what misconceptions and errors are associated with the chosen content (KCS), what principles underlay the teaching of the content (SCK). The teacher must then have a deep understanding of that content area so that she can accurately present it and evaluate the work she assigns to the learners (CCK). I present my understanding of how MKfT domains relate in Figure 6.1. However, it is important to note that I am not underestimating the value of the other domains in the work of teaching by foregrounding KCT.

Figure 6.1 How the MKfT domains relate to each other



I found that there is a strong interdependence between these MKfT domains and thus a very ‘thin line’ between them. Imagine a teacher employing good teaching strategies to teach blind learners how to write the alphabet, with the assumption that they would learn in the same way as a learner who was able to see. While the teacher may have adequate KCT, their lack of KCS may make their teaching in vain because no matter how good the lesson would be the blind learners will not be able to write the alphabets in a normal way and therefore one cannot get the expected results.

All the domains work together to enable the teacher to effectively present mathematical ideas in ways that learners will find meaningful. As reflected in the MKfT diagram in Figure 6.1 above, all the other five domains inform and influence how Gail taught, and in a similar way Gail’s KCT, for example, is enriched by the experience Gail gains as she teaches. For example, for Gail to know that learners have challenges in *bridging the tens* she would probably have experienced it a number of times during the counting lessons or learned it from someone or from a text. That knowledge would enable her to design teaching strategies and representations that would address the learners’ problems. The learners would only benefit if Gail uses the strategies informed by her knowledge in her teaching.

Although it may be difficult to determine where one knowledge domain begins and ends, Pinsky (2013) argues that “the importance of Ball et al.’s work resides not in clearly drawing the boundaries between them, but rather in establishing their existence" (p. 40). Gail's teaching established the existence of these domains. Through my research, I realised that Gail was not always conscious of her MKfT. Her counting sessions were seamless and in many respects her MKfT was automated. Like an experienced driver, Gail did not stop to think about what MKfT she was drawing on at any particular moment in time.

In my research, I found that Gail used all six knowledge domains during the process of teaching counting and developing children's number sense through counting. She had deep understanding of the knowledge that underpins the content she was teaching, such as the knowledge of counting principles, and applied this knowledge in her teaching. I learnt that Gail's MKfT was influenced by what she reads. For example, Gail talked of the five principles of learning, implying that she had read about them and therefore employs them in her teaching. This suggests the need for FP teachers to be wide readers and researchers.

In this study, I found that there is a close link between the KCC and HCK where HCK is dependent on how much knowledge of the curriculum one has. Understanding the link between these domains assists the teacher to appropriately sequence both the lessons and the content they are teaching. The ability to connect and sequence the lessons and the content accurately helps the teacher to teach from the known to the unknown which helps the learners to construct their new knowledge from the knowledge they already have. This brings about conceptual understanding (Griffin, 2004). Gail promoted connectionist teaching. As elaborated in Chapter Five, her counting sessions, and the content explored in those counting sessions, were presented in an interconnected manner. She did not focus on one topic at a time but would always: (1) connect her current lesson with previous and the future lessons; and (2) explore other concepts (e.g. place value) through counting. Cotton (2010) assert that this kind of teaching make learners grasp concepts quicker than when one uses the transmission[11] or discovery[12] methods of teaching.

In relation to Gail's KCT, which I found to be the dominant domain, Gail employed a variety of strategies in her teaching, such as sequencing her mathematical content, asking productive questions, selecting appropriate representations to illustrate the content she was teaching and selecting examples that took her learners into deeper understanding of the content she was

[11] Teaching method where learners are expected to perform the standard procedures and routines as taught. It is a teacher-centered approach in which the teacher is the dispenser of knowledge, the arbitrator of truth, and the final evaluator of learning.
[12] Discovery learning refers to various teaching methods that engages learners in learning through discovery

teaching. I experienced the significance of employing effective teaching strategies was evident, where the teacher did not teach learners methods but encouraged them to come up with their own strategies. The teacher also had to build on learners' strategies as ways of developing more general ideas and systematic approaches. For example, Gail exposed her learners to a mathematics calculation where they were using number grids to count on (add) or count back (subtract). One learner discovered that they can add or subtract multiples of ten by moving up and down the number grid. Gail asked the learner to explain to the other learners how his 'newly discovered' method works then encouraged the other learners to try it as they solved the problems that she gave them. This strategy brought excitement among learners as they discovered how the method simplified their calculations. Teachers of FP learners should take advantage of such occurrences as it is important in the development of number sense to honour individual learners' thinking and reasoning.

The use of questioning as an effective way of teaching was found to be a useful strategy at FP level. Questions that lead to explanations, extensions and the development of new understandings were used by Gail in this study. The teacher in this observed class facilitated the learning of mathematics with understanding. In the analysis of all the excerpts that were selected to illuminate development of concepts during counting, Gail facilitated learning through questions. Questions helped her learners not only to engage with the content they are learning but to also keep focussed.

Gail also ensured there were interesting learning activities, problems and stimulating mathematics-based conversations during each mathematics lesson. This motivated her learners to have a positive attitude towards mathematics and to always looking forward to the next mathematics lesson. Finally, this study confirmed that the way one teaches has an impact on how learners learn and their attitude towards the subject. If the teacher has adequate knowledge and skills to teach individual mathematics concepts the learners will always understand and enjoy the subject, leading to positive learning outcomes.

Through this research I found that competent teaching is not dependent on one's qualifications. I found that having many academic qualifications does not necessarily make one an effective teacher. Gail has never formally upgraded her qualification. She has a three-year teaching diploma. Despite this, she has been: positioned as an expert teacher; requested to lecture at a university to students who hold degrees; elected as a cluster leader; and appointed by her the Association for Mathematics Education in South Africa to facilitate various mathematics workshops in Eastern Cape and in a number of other provinces. In my interaction with her as a teacher and a lecturer I learnt that Gail values updating her knowledge. She reads extensively about teaching and learning, and participates in a number of learning fora (e.g. the Professional Teacher Development programme at the university). This is an interesting finding that would be worth future research.

6.2 SOME INSIGHTS EMERGING FROM MY RESEARCH

My research showed that an expert FP teacher possesses the knowledge of both the content and the pedagogy necessary to teach mathematics at FP. This calls for the teacher educator programs to ensure that their courses are rich in both content and pedagogy and that these are presented in a way that correlate particular content with its pedagogy. Both pre-service and in-service teachers need to be made aware of the extent of the knowledge required to teach FP mathematics. Opportunities need to be created within Higher Education Institutions and in schools for teachers (pre-service and in-service) to reflect on their teaching in a way that enables them to identify and critique their practices by exploring the MKfT they draw on as they teach.

Competent and experienced FP teachers of mathematics should be elected to mentor younger and less experienced colleagues in teaching FP mathematics. Platforms where FP teachers meet and share information regularly need to be encouraged. The teachers could also be encouraged to form online communities of practice where they can share the best ways of teaching particular concepts.

6.3 RECOMMENDATIONS FOR FURTHER RESEARCH

More research on intervention strategies to improve the MKfT of FP teachers, both in-service and pre-service, is crucial. This needs to be done by focusing on ‘expert’ FP mathematics teachers in a wide range of South African schools with different contexts. This research has contributed one case study in the particular context of an ex model C school. It would be important to draw on a wide range of teachers and contexts to create a fuller picture of the MKfT that expert South African FP teachers draw on in achieving successful mathematical learning. A focus on such expertise is important, especially given the dominance of deficit discourse in research that has focused on teacher who do not have the required MKfT and produce weak mathematical learners and performance results

Research that focuses on the MKfT drawn on by teachers who are working in more challenging environments, and in less functional schools, may be worthwhile so as to ascertain what MKfT teachers have. This knowledge can be used to inform the development of teacher education programs.

6.4 A FINAL WORD

This research study has, in many respects, changed me. Not only has it advanced my research capabilities, but it has taught me about learning and teaching in the Foundation Phase. My own teaching experience was as a high school mathematics and accountancy teacher. When I joined Rhodes University, I found myself being drawn into the Post Graduate Certificate in Education (Foundation Phase) (PGCE (FP) program. The experience of working with students registered for the PGCE (FP) led me to combine my interest in mathematics, albeit at high school level, with learning and teaching in the Foundation Phase. I have had the privilege to not only observe Gail in her Grade 2 classroom, but also to attend her mathematics method lectures in the PGCE (FP) program. It is this experience that has given me not only enormous insight, but also huge respect for FP teachers. I look forward to sharing my experiences and knowledge gained from this research process with future PGCE (FP) students.

REFERENCES

Adler, J. (2002). Global and local challenges of teacher development. In J. Adler, & Y. Reed (Eds.), *Challenges of Teacher Development: An Investigation of Take-up in South Africa* (pp. 1-16). Pretoria: Van Schaik.

Adler, J. (2005). Mathematics for teaching: What is it and why is it important that we talk about it? *Pythagoras, 62*, 2-11.

Adler, J., & Davis, Z. (2006). Opening Another Black Box: Researching Mathematics for Teaching in Mathematics Teacher Education. *Journal for Research in Mathematics Education, 37*(4), 270-296.

Adler, J., & Pillay, V. (2017). Mathematics Education in South Africa. In J. Adler, & A. Sfard, *Research for Educational Change:Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 9-24). New York: Routledge.

Adler, J., & Venkat, H. (2013). Mathematical knowledge for teaching and subject matter knowledge. In S. Lerman, & etal (Eds.), *The Encyclopaedia of Mathematics Education.* Springer.

Alvior, M. (2014, December 13). *The Meaning and Importance of Curriculum Development*. Retrieved October 17, 2016, from simplyeducate.me: http://simplyeducate.me/2014/12/13/the-meaning-and-importance-of-curriculum-development/

Ary, D., Jacobs, L. C., & Razavieh, A. (2006). *Introduction to Research in Education. (7th, Ed.).* USA, Belmont: Wadsworth, Cengage Learning.

Askew, M. (2008). Mathematical discipline knowledge requirements for prospective primary teachers, and the structure and teaching approaches of programs disigned to develop that knowledge. In P. Sullivan, & T. Wood (Eds.), *Knowledge and Beliefs in Mathematics Teaching and Teaching Development* (pp. 13-35). Rotterdam, NL: Sense Publishers.

Askew, M. (2012). *Transforming primary mathematics.* New York: Routledge.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective Teachers of Numeracy.* London: School of Education, King's College.

Back, J., Sayers, J., & Andrews, P. R. (2013). The development of foundational number sense in England and Hungary: a case study comparison. *In Conference Proceedings: Comparative Studies in Mathematics Education* (pp. 1835-1844). Antalya, Turkey: CERME 8 (ERME).

Ball, D. L., & Bass, H. (2009). With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures. *Paper presented at the 43rd Jahrestagung für Didaktik der Mathematik, March 1 – 4, 2009.* Oldenburg, Germany .

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, Fall 2005, 29*(1), 14-22, 43-46.

Ball, D. L., Thames, M. H., & Phelps, G. (2005). Knowledge of mathematics for teaching: What makes it special? *Paper presented at the AERA Annual Conference* (pp. 1-13). Canada: Montreal. Retrieved February 19, 2016, from https://pdfs.semanticscholar.org/2daa/67d3bbde0d751eab7302c6704f41f9f31f97.pdf

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes it Special? *Journal of Teacher Education, 59*(5), 389-407.

Banegas, D. L., & Villacañas de Castro, L. S. (2015). A look at ethical issues in action research in education. *Argentinian Journal of Applied Linguistics, 3*(1), 58-67.

Bansilal, S., Brijlall, D., & Mkhwanazi, T. W. (2014). An exploration of the common content knowledge of high school mathematics teachers. *Perspectives in Education, 32*(1), 34-50.

Baroody, A. J. (1987). *Children's Mathematical Thinking: A Development Framework for Preschool, Primary, and Special Education Teachers.* New York: Teachers College Press.

Baroody, L. M., & Wilkins, A. J. (1999). The Development of Informal Counting, Number, and Arithmetic Skills and Concepts. In J. Copley (Ed.), *Mathematics in the early years, birth to five* (pp. 49-64). National Council of Teachers of Mathematics (NCTM).

Bernard, H. R. (2002). *Research Methods in Anthropology: Qualitative and quantitative methods. 3rd Edition.* Walnut Creek, California: AltaMira Press.

Bertram, C., & Christiansen, I. (2014). *Understanding research: An introduction to reading research.* Pretoria: Van Schaik.

Bevir, M., & Kedar, A. (2008). Concept formation in political science: An anti-naturalist critique of qualitative methodology. *Perspectives on Politics, 6*(3), 503-17.

Bilbao, P. P., Lucido, P. I., Iringan, T. C., & Javier, R. B. (2008). *Curriculum Development.* Philippines: Lorimar Publishing, Inc.

Bloomberg, L. D., & Volpe, M. (2008). *Completing your qualitative dissertation: a road map from beginning to end.* Los Angeles: Sage.

Bobis, J. (1996). Visualisation and the development of number sense with kindergarten children. In J. Mulligan, & M. Mitchelmore (Eds.), *Children's Number Learning : A Research Monograph of the Mathematics Education Group of Australasia and the Australian Association of Mathematics Teachers* (pp. 17-34). Adelaide: Australian Association of Mathematics Teachers (AAMT).

Bogdan, R. C., & Biklen, S. K. (1998). *Qualitative research for education: An introduction to theory and methods. 3rd Edition.* Boston: Allyn and Bacon.

Bolowana, A. (2014, November 30). *Ineffective teaching behind poor performance in schools*. Retrieved from Motshekga: http://www.sabc.co.za/news/a/2badc50046713e3793fdbfb6166ecff3/Ineffective-teaching-behind-poor-performance-in-schools:

Bolowana, A. (2014, December Thursday 4). Ineffective teaching behind poor performance in schools: Motshekga. *SABC News.*

Bowen, G. (2009). Documentary analysis as a qualitative research method. *Qualitative Research Journal, 9*(2), 27-40.

Bruce, B., & Threlfall, J. (2004). One, two, three and counting. *Educational studies in Mathematics, 55*, 3-26.

Burgess, R. G. (Ed.). (1989). *The Ethics of Educational Research.* Philadelphia, PA: Falmer Press.

Burns, M. (1997). "How I Boost My Students' Number Sense.". *Instructor Magazine*, 49-54.

Burns, M. (2007). *About Teaching Mathematics: A K-8 Resource* (3rd ed.). Sausalito, CA: Math Solutions.

Carlyle, A., & Mercado., B. (2012). *Teaching Preschool and Kindergarten Math: More than 175 Ideas, Lessons, and Videos for Building Foundations in Math,a Multimedia Professional Learning Resource.* Sausalito, CA: Math Solutions.

Carnoy, M., & Arends, F. (2012). Explaining mathematics achievement gains in Botswana and South Africa. *PROSPECTS, 42*(4), 453–468.

Carnoy, M; Chisholm, L; Addy, N; Arends, F; Baloyi, H; Irving, M; Raab, E; et al. (2011). *The process of learning in South Africa: The quality of mathematics teaching in North West Province. Technical report, 11 June 2011.* Stanford University School of Education & HSRC.

Carnoy, M; Chisholm, L; et al. (2008). *Torwards Understanding Student Academic Performance in South Africa: A Pilot Study of Grade 6 Mathematics Lessons in South Africa.* Pretoria: HSRC Press.

Carrillo, J., Climent, N., Contreras, L. C., & Muñoz-Catalán, M. C. (2012). Determining Specialised Knowledge for Mathematics Teaching. In B. Ubuz, C. Haser, & M. A. Mariotti (Ed.), *Proceedings of CERME 8* (pp. 2985- 2994). Antalya, Turquía: ERME.

Center for Development and Enterprise (CDE). (2014). *What does research tell us about teachers, teaching and learner perfomance in mathematics.* Pretoria: CDE.

Center for Innovation in Education. (2011). *Attaching quantitaive meaning to the counting sequence.* California: Saratoga.

Charmaz, K. (2006). *Constructing Grounded Theory: A Practical Guide through qualitative analysis.* London: Sage.

Chrossely, N. J. (2007). *Growing Ideas of Numbers.* Camberwell Victoria: ACER Press.

Clements, D. H., & Sarama, J. (2014). The importance of the early years. In R. E. Slavin (Ed.), *Science, technology & mathematics (STEM)* (pp. 5-9). Thousand Oaks, CA: Corwin.

Clements, D., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach.* New York: Routledge.

Clerkin, A., Perkins, R., & Cunningham, R. (2016). *TIMSS in Ireland: Mathematics and Science in primary and post primary school.* Dublin: Educational Research Centre.

Cohen, D., & Crabtree, B. (2006). *Semi-structured Interviews.* Retrieved from Qualitative research guidelines project: http://www.qualres.org/HomeSemi-3629.html

Cohen, L., Manion, L., & Morrision, K. (2000). *Research methods in education, 5th Edition.* London: Routledge Falmer.

Cohen, L., Manion, L., & Morrison, K. (2005). *Research methods in education.* London: Routledge.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education, 7th Edition.* London: Routledge.

Connel, M. (2012, November 27). *Number Sense: What it is, why it's important, and how it develops.* Retrieved September 09, 2016, from Native Brain: http://www.nativebrain.com/2012/11/number-sense-what-it-is-why-its-important-and-how-it-develops/

Cooper, D., & Schindler, P. (2001). *Business Research Methods, 7th Edition.* New York, NY: McGraw-Hill.

Corry, M. (1996). Jean Piaget's Genetic Epistemology. Retrieved June 24, 2016, from http://home.gwu.edu/~mcorry/corry2.htm

Cotton, T. (2010). *Understanding and Teaching Primary School Mathematics.* London: Pearson Education Limited.

Cousins, L. (2010, December 28). *Three Kinds of Knowledge.* Retrieved 2016, from Psych Central: http://blogs.psychcentral.com/always-learning/2010/01/three-kinds-of-knowledge/

Coyne, I. T. (1997). Sampling in qualitative research. Purposeful and theoretical sampling: Merging or clear boundaries? *Journal of Advanced Nursing, 26*, 623-630.

Creswell, J. M. (2009). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches. (3rd ed.).* Los Angeles: Sage.

Creswell, J. W. (2007). *Qualitative Inquiry & Research Design: Choosing Among Five Approaches, 2nd Edition.* Thousand Oaks, CA: Sage.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches (3rd ed.).* Thousand Oaks, CA: Sage.

Denzin, N. K., & Lincoln, Y. S. (2000). *Handbook of Qualitative Research* (2nd ed.). London: Sage.

Denzin, N. K., & Lincoln, Y. S. (2005). Introduction: The discipline and practice of qualitative research. In N. K. Denzin, & Y. S. Lincoln (Eds.), *The sage handbook of qualitative research, 2nd Edition.* Thousand Oaks, CA: Sage.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education 34, 34*, 12-25.

Department of Basic Education (DBE). (2009). *National Examinations and Assessment: Report on the National Senior Certificate Examination Results, Part 2.* Pretoria: Department of Basic Education (DBE).

Department of Basic Education (DBE). (2013). *Macro Indicator Report.* Pretoria: Department of Basic Education.

Department of Basic Education (DBE). (2015). *National Senior Certificate Examination Report 2015.* Pretoria: Department of Basic Education.

Department of Basic Education (DBE). (2016). *Education Statistics in South Africa 2014.* Pretoria: Department of Basic Education.

Department of Basic Education (DBE); Department of Higher Education and Training (DHET). (2011). *Integrated strategic planning framework for teacher education and development in South Africa 2011- 2025.* Pretoria: Department of Basic Education & Department of Higher Education and Training.

Department of Basic Education, (DBE). (2012). *Numaracy Handbook for Foundation Phase Teachers: Grade R-3.* Pretoria: Department of Basic Education.

Department of Education (DoE). (2004). *Qualitative overview of the Further Educationa and Training Sector: A Sector in Transition.* Pretoria: Department of Education.

Doyle, A. (2016, November 30). How to Handle an Informal Interview with Prospective C-workers. Doyle & Doyle Communications Inc. Retrieved from http://www.careertoolbelt.com/how-to-handle-an-interview-with-prospective-co-workers/

Education Development Center (EDC). (2015, November 9). *Mathematics in the Early Grades: Counting & Cardinality.* Retrieved December 28, 2016, from Interactive STEM: http://interactivestem.org/wp-content/uploads/2015/09/Interactive-STEM-Brief-Counting-and-Cardinality-Sept-16-Final-File.pdf

Edward, S., Ebert, I. I., Christine, E., & Bentley, M. L. (2013, July 19). *The Educator's Field Guide: From Organization to Assessment (and Everything in between).* New York: Corwin Press. doi:http://dx.doi.org/10.4135/9781452275154

Emanuel, E. J., Wendler, D., & Grady, C. (2000). What Makes Clinical Research in Developing Countries Ethical? The Benchmarks of Ethical Research. *The Journal of the American Medical Association, JAMA, 283*(20), 2701-2711.

Ensor, P., Hoadley, U., Jacklin, H., Kuhne, C., Schmitt, E., & Lombard, A. (2009). Specialising pedagogic time and text in Foundation Phase mathematics classrooms. *Journal of Education, 47*, 4-30.

Fleisch, B. (2008). *Primary Education in Crisis: Why South African School-children Underachieve in Reading and Mathematics.* Johannesburg, South Africa: Juta & Co.

Flick, E. (2011). *Introducing Research Methodology.* London: Sage.

Flores, E., Escudero, D., & Carrillo, J. (2013). A theoretical review of Knowledge. In B. Ubuz, C. Haser, & M. A. Mariotti (Ed.), *Proceedings of the Eighteenth Congress of the European Society for Research in Mathematics Education* (pp. 3055-3064). Ankara, Turkey: European Society for Research in Mathematics Education.

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing nimber sense, addition and subtraction.* Portsmouth: Heinemann.

Fries-Gaither, J. (2016, October 11). *Beyond Penguins and Polar Bears*. Retrieved from http://beyondpenguins.ehe.osu.edu.

Fuson K.C. & Hall. J. (1992). The acquisition of early number word meanings; a conceptual analysis and review. . *In H.P Ginsburg (Ed) children's mathematical thinking*, 49-107.

Fuson, K. C. (1988). *Chiltlren's Counting and Concepts of Number.* New York: Springer-Verla.

Gelman, R., & Gallistel, C. (1978). *The Child's Understanding of Number.* Cambridge, MA: Harvard University Press.

Gervasoni, A. M. (2003). Difficulties children face when learning to count. In L. Bragg, C. Campbell, & G. M. Herbert (Ed.), *Mathematics education research: Innovation; networking; opportunity. Proceedings of the 26th annual conference* (pp. 388-395). Melbourne, Australia: Mathematics Education Research Group of Australasi.

Ginsburg, H. P. (1977). *Children's Arithmetic.* New York: D. Van Nostrand Co.

Graeber, A., & Tirosh, D. (2008). Pedagogical content knowledge: A useful or an elusive notion? In P. Sullivan (Ed.), *Knowledge and Beliefs in Mathematics Teaching and Teaching Development .* Amsterdam, The Netherlands: Sense Publishers.

Graven, M. (2002). Coping with new mathematics teacher roles in a contradictory context of the curriculum change. *The Mathematics Educator, 12*(2), 21-27.

Graven, M. (2016). When Systemic Interventions Get in the Way of Localized Mathematics Reform. *For the Learning of Mathematics, 36*(1), 08-13.

Graven, M. H. (2013). Poverty, inequality and mathematics performance: the case of South Africa's post-apartheid context. *ZDM Mathematics Education, 46*(7), 1039-1049.

Graven, M., & Heyd-Metzuyanim, E. (2014). 'Exploring the limitations and possibilities of researching mathematical dispositions of learners with low literacy levels'. *Scientia in Educatione, 5*(1), 1-16.

Graven, M., Venkat, H., Westeway, L., & Tshesane, H. (2013). Place value without number sense: Exploring the need for mental mathematical skills assessment within the Annual National Assessments. *South African Journal of Childhood Education, 3*(2), 131-143.

Green, W., Parker, D., Deacon, R., & Hall, G. (2011). Foundation phase teacher provision by public higher education institutions in South Africa. *South African Journal of Childhood Education, 1*(1), 109-122. Retrieved July 14, 2016, from http://www.sajce.co.za/index.php/sajce/article/viewFile/80/81

Griffin, S. (2004). Teching Number Sense. *Educational Leadership, 61*(5), 39-42.

Halmos, P. R. (1985). *I want to be a Mathematician: An Automathography.* New York: Springer-Verlag.

Hasting, S. (2003, july 4). Questioning. *TES Newspaper*. Retrieved November 18, 2015, from teacher tools: http://www.tes.co.uk/article.aspx?storycode=381755

Haylock, D., & Cockburn, A. (2008). *Undestanding mathemaics for young children; A guide for Foundation Stage & Lower Primary teachers.* London: SAGE Publications.

Henning, E. (2004). *Finding your way in qualitative research.* Pretoria: van Schaik.

Henning, E. (2005). *Finding your way in qualitative research.* Pretoria: Van Schaik.

Hidden curriculum. (2014, August 26). *The glossary of education reform*. (S. Abbott, Editor) Retrieved October 17, 2016, from http://edglossary.org/hidden-curriculum.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. *American Educational Research Journal, 42*(2), 371-406.

Hoadley, U. (2012). What do we know about teaching and learning in South African Primary schools? *Education as Change, 16*(2), 187-202.

Hopkins, D. (2008). *A teacher's guide to classroom research. 4th Edition.* New York, NY: McGraw- Hill.

Hugo, W; Jack, M; Wedekind, V; Wilson, D; et al. (2010). The state of education in KwaZulu-Natal: A report to the Provincial Treasury. *Pietermaritzburg: KZN Provincial Treasury*.

Human, A., van der Walt, M., & Posthuma, B. (2015). International comparisons of Foundation Phase number domain mathematics knowledge and practice standards. *South African Journal of Education, 35*(1), 01-13.

Hurrell, D. P. (2013). What teachers need to know to teach mathematics: An arguement for a reconceptualised model. *Australian Journal of teacher education, 38*(11), 54-64. doi:http://dx.doi.org/10.14221/ajte.2013v38n11.3

Jakobsen, A., Thames, M. H., & Ribeiro, C. M. (2013). Delineating issues related to Horizon Content Knowledge for mathematics teaching. In B. Ubuz, C. Haser, & M. A. Mariotti (Ed.), *Proceedings of CERME 8, the Eighth Congress of the European Society for Research in Mathematics Education* (pp. 3125–3134). Antalya: Middle East Technical University, Antalya.

Jansen, J. (2011, October 13). No hope for teachers. *Times LIVE*. Retrieved June 21, 2016, from http://www.timeslive.co.za/opinion/columnists/2011/10/13/No-hope-for-teachers1

Johansson, R. (2004). *Theory of science and research methodology.* Stockholm: Royal Institute of Technology. Department of Infrastructures.

Jordan, N. C., & Montani, T. (1997). Cogntive arithmethic and problem solving. A comaprison of children with specific and general mathematics learning difficulties. *Journal of learning disabilities, 30*(6), 624-634.

Kamii, C. (1997). *Number in Preschool and Kindergarten: Educational Implications of Piaget's Theory.* Washington, D C: National Association for the Education of the Young Child.

Kawulich, B. B. (2005). Participant observation as a data collection method. *Forum Qualitative Sozialforschung / Forum: Qualitative Social Research, 6*(2).

Kazima, M., Pillay, V., & Adler, J. (2008). Mathematics for teaching: observations from two case studies. *South African Journal of Education, 28*, 283-299.

Kim, H. (2013). Towards teaching for developing students' mathematical reasoning: A case study on teaching formative assessment lessons. *Paper presentation at the 2013 Joint International Conference on Mathematics Education.* Seoul, Korea.

Klenke, K. (2008). *Qualitative research in the study of leadership.* Bingley, UK: Emerald Group.

Koehler, M. S., & Grouws, D. A. (1992). *Mathematics Teaching Practices and Their Effects." In Handbook of Research on Mathematics Teaching and Learning.* New York: Macmillan.

L, T. Y., & Lin, Y. C. (2012). Elementary School Teachers' Understanding: Towards the Related Knowledge of Number Sense. *US – China Education Review B1.*, 17 – 30.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry.* Beverly Hills, CA: Sage.

Lyon, L. M., & Hardesty, L. H. (2005). Traditional healing in the contemporary life of the Antanosy people of Madagascar. *Ethnobotany Research and Applications, 3*, 287-294.

Mabogoane, T., & Pereira, C. (2008). Using trends to hold schools accountable. *Presentation to the Association for Evaluation and Assessment in South Africa Conference.* Pretoria.

Mack, L. (2010). The Philosophical Underpinnings of Educational Research. *Polyglossia, 19*. Retrieved from http://r-cube.ritsumei.ac.jp/bitstream/10367/1887/1/1-Polyglossia19_The%20Philosophical%20Underpinnings%20of%20Educational%20Research.pdf

Maclellan, E. (1997). The importance of counting. In I. Thompson (Ed.), *Teaching and Learning Early Number.* Buckingham: Open University Press.

Marmasse, N., Bletsas, A., & Marti, S. (2000, May 11). *Numerical Mechanisms and Children's Concept of Numbers.* Retrieved from The Media Laboratory: Available at: http://web.media.mit.edu/

Marshall, C. (2006). *Data Collection Methods.* Retrieved from www.sagepub.com/upm-data/10985_Chapter_4.pdf

Marshall, C., & Rosmann, G. B. (2006). *Designing qualitative research.* London: Sage.

Marshall, C., & Rossman, G. B. (1989). *Designing Qualitative Research.* Newbury Park, CA: Sage.

Marzano, R. J., Pickering, D. J., & Pollock, J. (2001). *Classroom instruction that works: Research-based strategies for increasing student achievement.* Alexandria, VA: Association for Supervision and Curriculum Development.

Mason, J. (2006). "Mixing Methods in a Qualitatively Driven Way". *Qualitative Research, 6*(1), 9-25.

Maxwell, J. A. (2013). *Qualitative Research Design: An interactive approach.* London: SAGE.

Maxwell, J., & Loomis, D. (2003). Mixed method design: an alternative approach. In A. Tashakkori, & C. Teddlie (Eds.), *Handbook of mixed methods in social and behavioral research* (pp. 241-271). Thousand Oaks, CA: Sage.

McAuliffe, S., & Lubben, F. (2013). Perspectives on Pre-Service Teacher Knowledge for Teaching Early Algebra. *Perspectives in Education, 31*(3), 155-169.

McMillan, J. H., & Schumacher, S. (2010). *Research in Education: Evidence-Based Inquiry, 7th Edition.* Boston, MA: Pearson.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation.* San Francisco, CA: Jossey-Bass.

Moloi M, Q., & Chetty, M. (2010). *The SACMEQ Ill Project in South Africa: A Study of the Conditions of Schooling and the Quality of Education South Africa: Country report.* Pretoria, South Africa: Department of Basic Education. Retrieved August 12, 2016, from http://www.education.gov.za/Portals/0/Documents/Reports/SACMEQ%20Country%20Report.pdf?ver=2015-03-20-090641-533

Moloi, M., & Chetty, M. (2011). *SACMEQ Policy Brief: Trends in Achievement Levels of Grade 6 Pupils in South Africa.* Southern and Eastern Africa Consortium for Monitoring Educational Quality. Retrieved May 19, 2016, from http://www.education.gov.za/LinkClick.aspx?fileticket=v34F8luZQGQ%3D&tabid=358&mid=2596&forcedownload=true.

Morris, M. W., Leung, M., Ames, D., & Lickel, B. (1999). Views from Inside and Outside: Integrating Emic and Etic Insights about culture and Justice Judgement. *Academy of Management Review, 24*(4), 781-796.

Morse, J. M. (2000). Determining sample size. *Qualitative Health Research, 10*(1), 3-5.

Mosvold, R., & Fauskanger, J. (2014, September 25). Teachers' Beliefs about Mathematical Horizon Content Knowledge. *International Journal for Mathematics Teaching and Learning, 9*(3), 311-327. Retrieved November 22, 2015, from https://brage.bibsys.no/xmlui/bitstream/handle/11250/280221/Article_Mosvold_Fauskanger.pdf?sequence=1&isAllowed=y

Motion Maths Staff. (2012, May 28). *How to Understand Place Value: 3 Fun Family Math Activities.* Retrieved from Motion Math Blog: http://motionmathgames.com/how-to-understand-place-value-3-fun-family-math-activities/

Mourshed, M., & Barber, M. (2007). *How the world's best performing school systems come out on top.* New York: McKinsey & Company.

Mouton, J. (2001). *How to Succeed in Your Master's and Doctorial Study.* Hatfield, Pretoria: Van Schaik.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, D C: U.S. Department of Education. Retrieved from https://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Education Evaluation and Development Unit (NEEDU). (2013). *NATIONAL REPORT 2012: The State of Literacy Teaching and Learning in the Foundation Phase, April 2013.* Pretoria: National Education Evaluation and Development Unit.

National Education Evaluation and Development Unit (NEEDU). (2013). *Teaching and Learning in Rural Primary schools: National Report 2013.* Pretoria: National Education Evaluation and Development Unit (NEEDU).

Naudé, M., Meier, C., & Bosman, L. (2014). *Teaching foundation phase mathematics : a guide for South African students and teachers.* Hatfield, Pretoria: Van Schaik Publishers.

Newman, H. B., Evans, D., & Wong, M. (2012). The Teacher Subject Matter Knowledge of Number Sense. In J. Dindyal, L. P. Cheng, & S. F. Ng (Ed.), *Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 130-137). Singapore: Mathematics Education Research Group of Australasia (MERGA).

Nolan, B., Dempsey, M., Lovatt, J., & O'Shea, A. (2015). Developing Mathematical Knowledge for Teaching (MKT) for pre-service teachers: a study of students' developing thinking in relation to the teaching of mathematics. In A. G (Ed.), *Proceedings of the British Society for Research into Learning Mathematics, February 2015*, *35 (1)*, pp. 35-54. Dublin.

Noor, N. M. (2008). Method of study. In N. M. Noor (Ed.), *An insight into post-independent Malaysia women's well being* (pp. 43-52). Kuala-Lumpur: IIUM press.

Oduol, T. (2014). *The challenges of ethical leadership: a case study of secondary school leader's experiences in Kenya. Unpublished PhD thesis.* Wellington, New Zealand: Victoria University of Wellington. Retrieved from http://researcharchive.vuw.ac.nz/xml

Ono, Y., & Ferreira, J. (2010). A case study of continuing teacher professional development through lesson study in South Africa. *South African Journal of Education, 30*, 59-74.

Organisation for Economic Co-operation and Development (OECD). (2008). *Review of National Policies for Education: South Africa, Policies for Education.* Pretoria: South Africa: Organisation for Economic Co-operation and Development.

Park, S., & Oliver, J. S. (2008). Revisiting the Conceptualisation of Pedagogical Content Knowledge (PCK): PCK as a Conceptual Tool to Understand Teachers as Professionals. *Research in Science Education, 38*, 261-284.

Patton, M. (2002). *Qualitative research and evaluation methods.* Thousand Oaks, CA: Sage.

Patton, M. Q. (1990). *Qualitative evaluation and research methods, 2nd edition.* Newbury Park, CA: Sage.

Petrou, M., & Goulding, M. (2011). Conceptualising teachers' mathematical knowledge in teaching. In G. A, & P. Boer (Eds.), *Handbook of Research on the Psychology Mathematics Education: Past, Present and Future* (pp. 429-459). Rotterdam: Sense Publishers.

Piaget, J. (1965). *The Child's Conception of Number.* New York: Norton.

Pope, C., & Myers, N. (2001). *Qualitative research in health care.* London: BMJ Books.

Pournara, C., Hodgen, J., & Adler, J. &. (2015). Can improving teachers' knowledge of mathematics lead to gains in learners' attainment in mathematics? *South African Journal of Education, 35*(3), 1-10.

Randolph, J. J. (2007). What's the difference, still? A follow-up critique of the contemporary quantitative research methodology in distance learning. *Informatics in Education, 6*(1), 179-188.

Reddy, V. (2006). *Mathematics and science achievement at South African schools in TIMSS .* Pretoria: HSRC Press.

Reddy, V. (2005). Cross-national achievement study: Learning from South Africa's participation in the Trends in International Mathematics and Science Study (TIMMS). *Compare, 35*, 63-77.

Reddy, V., Zuze, T. L., Visser, M., Winnaar, L., Juan, A., Prinsloo, C. H., . . . Rogers, S. (2015). *Beyond Benchmarks: What twenty years of TIMSS data tells us about South African education.* Cape Town: HSRC Press.

Reddy, V., Zuze, T. L., Visser, M., Winnaar, L., Juan, A., Prinsloo, C. H., . . . Rogers, S. (2015). *Beyond Benchmarks; What twenty years of TIMSS data tell us about South African education.* Cape Town: HSRC Press.

Reys, R. E., Lindquist, M. M., & Smith, N. L. (2007). *Helping children learn mathematics.* Hoboken: John Wiley & sons.

Robertson, S.-A., & Graven, M. (2015). Exploring South African mathematics teachers' experiences of learner migration. *Intercultural Education, 26*(4), 278-295.

Robson, C. (2002). *Real world research (2nd ed.).* Malden: Blackwell.

Rockoff, J. E., Jacob, B. A., Kane, T. J., & Staiger, D. O. (2011, January). "Can You Recognize an Effective Teacher When You Recruit One?". *Education Finance and Policy, MIT Press, 6*(1), 43-74.

Saddler, F. (2009). Help! They Still Don't Understand Counting. *Teaching Exceptional Children Plus, 6*(1), 1-12. Retrieved September 22, 2016, from http://files.eric.ed.gov/fulltext/EJ875422.pdf

Sandelowski, M. (1995). Sample size in qualitative research. *Research in Nursing & Health, 18*(2), 179-183.

Schollar, E. (2008). *Final Report: The Primary Mathematics Research Project: 2004-2007. Towards evidence-based educational development in South Africa .* Johannesburg: Eric Schollar and Associates c.c.

Schwartz-Shea, P., & Yanow, D. (2012). *Interpretive Research Design.* New York: Routledge.

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. S. (1987). "Knowledge and teaching: Foundations of the new reform". *Harvard Educational Review, 57*(1), 1-23.

Shulman, L., & Grossman, P. (1988). *The intern teacher casebook.* San Francisco, CA: Far West Laboratory for Educational Research and Development.

Simpson, M., & Tuson, J. (2003). *Using observations in small-scale research: a beginner's guide.* Glasgow: The Scottish Council for Research in Education.

Sood, S., & Jitendra, A. K. (2007). A Comparative Analysis of Number Sense Instruction in Reform-Based and Traditional Mathematics Textbooks. *Journal of Special Education, 41*(3), 145-157.

Spaull, N. (2013). *South Africa's education crisis.* Johannesburg: Centre for Development and Enterprise (CDE).

Spaull, N. (2013). *South Africa's Education Crisis: The quality of education in South Africa 1994-2011.* Johannesburg: Centre for Development and Enterprise. Retrieved August 15, 2016, from http://www.section27.org.za/wp-content/uploads/2013/10/Spaull-2013-CDE-report-South-Africas-Education-Crisis.pdf

Spaull, N. (2013, October 30). *Teachers can't teach what they don't know*. Retrieved from Independent Education: http://www.ieducation.co.za/teachers-cant-teach-what-they-dont-know/

Spaull, N., & Kotze, J. (2014). "Starting Behind and Staying Behind in South Africa: The case of insurmountable learning deficits in mathematics". *Working Papers 27/2014*. Cape Town: Stellenbosch University, Department of Economics.

Spaull, N., & Kotze, J. (2015). Starting behind and staying behind in South Africa. *International Journal of Educational Development, 41*, 13–24.

Taylor, N. (2008, February 28-29). *What's wrong with South African schools? Presentation to the Conference What's Working in School Development. JET Education Services, 28-29 February 2008*. Retrieved from www.jet.org.za.

Taylor, N. (2009). Standards-based accountability in South Africa. *School Effectiveness and School Improvement, 20*(3), 341-356.

Taylor, N. C., & Moyane, J. (2005). *Khanyisa Education Support Programme: baseline study part I.* Johannesburg: ET Education Services.

Taylor, N., & Taylor, S. (2013). Teacher knowledge and professional habitus. In N. Taylor, S. van der Berg, & T. Mabogoane (Eds.), *Creating Effective Schools.* Johannesburg: Pearson.

Thanheiser, E., Browning, C. A., Moss, M., Watanabe, T., & Garza-Kling, G. (2010, December). Developing Mathematical Content Knowledge for Teaching Elementary School Mathematics. *IUMPST: The Journal (Content Knowledge), 1*, 1-13.

Thompson, P. W. (1994). Concrete materials and teaching for mathematical understanding. *Arithmetic Teacher, 41*(9), 556-558.

Thorndike, E. L. (1922). *The Psychology of Arithmetic.* New York: Macmillan.

Tongco, M. D. (2007). Purposive sampling as a tool for informant selection. *Ethnobotany Research & Applications, 5*, 147-158.

Toombs, W. E., & Tierney, W. G. (1993). Curriculum Definitions and Reference Points. *Journal of Curriculum and Supervision, 8*(3), 175-195.

Ursula, H. (2010). *What do we know about teaching and learning in primary schools in South Africa? A review of the classroom-based research literature.* Cape Town.

Van der Berg, S., & Louw, M. (2006). Unravelling The Mystery: Understanding South African Schooling Outcomes in Regional Context. *Centre for the Study of African Economies.* Oxford: Oxford University Press.

Venkat, H. (2013). Curriculum Development Minus Teacher Development ≠ Mathematics Education. In Z. J. Davis (Ed.), *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa. 1*, pp. 4- 16. Cape Town: AMESA. Retrieved from http://www.amesa.org.za/AMESA2013/Files/P02.pdf

Venkat, H. (2015). Mathematical practices and mathematical modes of enquiry: same or different? *International Journal of STEM Education, 2*(6), 1-12. Retrieved September 15, 2016, from http://link.springer.com/article/10.1186/s40594-015-0018-8

Venkat, H., & Spaull, N. (2014). *What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. Working Papers 13/2014.* Cape Town: Stellenbosch University, Department of Economics, revised 2014.

Vinjevold, P., & Crouch, L. (2001). *Literacy, Numeracy and Policy: Towards Assessing Learning in Grade 3.* Pretoria: Research Triangle Institute.

Wilkie, K. J. (2015). Learning to teach upper primary school algebra: changes to teachers' mathematical knowledge for teaching functional thinking. *Mathematics Education Research Journal, 28*(2), 245–275. doi:10.1007/s13394-015-0151-1

Woolfolk, A. (2010). *Educational Psychology. 11 Edition.* Upper Saddle River, NJ: Merrill.

Wright, & J, R. (1991). What number knowledge is possesed by children beginning the kindergarten year of school? *Mathematics Education Research Journal, 3*(1), 1-16.

Wright, A. (2012). Tablets over textbooks. *Communications of the ACM, 55*.

Wright, B. (2016). Assessing Early Numeracy. Southern Cross University, New South Wales, Australia: SANC, Rhodes University. Retrieved from https://www.ru.ac.za/media/rhodesuniversity/content/sanc/documents/Bob%20Wright%20Assessing%20Early%20Numeracy%20presentation%20Sept%202016.ppt

Wright, J. R. (2008). Interview bassed assessment of early number knowldge. In I. Thompson (Ed.), *Teaching and Learning early number* (2nd ed., pp. 193-204). Maidenhead: Open University Press.

Wright, R. (2003). A Mathematics Recovery: Program of intervention in early number learning. *Australian Journal of Learning Disabilities, 8*(4), 6-11.

Wright, R. J., Martland, J., Stafford, A. K., & Stanger, G. (2006). *Teaching Number: Advancing children's skills and strategies (2nd Edition).* London: Paul Chapman Publishing Ltd.

Wynn, K. (1990). Children's understanding of counting. *Cognition*, 155-193.

Yanow, D., & Schwartz-Shea, P. (Eds.). (2006). *Interpretation and Method: Empirical Research Methods and the Interpretive Turn.* Armonk, NY: M.E. Sharpe.

Yin, R. K. (2003). *Case study Research: Design and methods. 3rd edition, volume 5.* Thousand Oaks, CA: Sage.

Yin, R. K. (2009). *Case study research: Design and methods. 4th edition.* Thousand Oaks, California: Sage.

Yin, R. K. (2011). *Qualitative Research from Start to Finish.* New York: Guilford Press.

APPENDICES

APPENDIX 1 – PRINCIPAL CONSENT FORM

RHODES UNIVERSITY

Grahamstown • 6140 • South Africa

Dear _______________

Re: Permission to conduct research in the classroom of Ms G's class

My name is Samukeliso Chikiwa, a Masters in Education student in the South African Numeracy Chair at Rhodes University. I am currently working on a research project that explores what Mathematics Knowledge for Teaching teachers in the Foundation Phase require in order to teach mathematics competently and confidently. Given the nationwide concern about learner performance in mathematics in South Africa, this research has the potential to make a valuable contribution to the education of teachers in South Africa. My research is being supervised by Lise Westaway (Senior Lecturer in the Education Department at Rhodes University) and Professor Mellony Graven (Numeracy Chair in the Education Department at Rhodes University).

The research I wish to undertake seeks to understand the mathematical content and pedagogical knowledge that a *leader teacher* in the Foundation Phase has. Gail is regarded in the Eastern Cape as a leader teacher in mathematics. She runs workshops on mathematics teaching and learning for the District Office, the Eastern Cape Department of Education and the National Association for Professional Teachers of South Africa (NAPTOSA). In addition, she is responsible for the mathematics education component of the Post Graduate Certificate in Education (Foundation Phase) qualification at Rhodes University.

The focus of the research is the teacher. However, I have to research Gail in context, which means that the Grade 2 learners at your school will be in the classroom and interacting with their teacher while I am conducting my research. As a research scholar in the Education Department, I am bound to the ethical principles of the department and Education Faculty's Higher Degrees Committee. This has implications for how I carry out my research.

I wish to observe Gail teach mathematics on daily basis for a maximum of five weeks. These lessons will be video recorded. While the video camera will be focused on Gail, it is likely that images of children working on the mat with her, will be captured on the video. However, my interest is not in the grade 2 learners, but their teacher. In compliance with the ethical standards alluded to above, the video data will be treated with the utmost confidentiality. The only people who will be able to view the videos will be Gail, myself and, if necessary, my two supervisors. Viewing of the video material will only take place in collaboration with me. No video data will be available for any other purposes other than my research.

I am an experienced teacher and researcher who has previously conducted research in classrooms with young learners and realise the importance of confidentiality and anonymity in the research process. In the thesis there will be no mention of the name of the school or the teacher. Pseudonyms will be used throughout in order to respect the confidentiality of the school and the teacher. My research proposal has been reviewed by the Education Faculty's High Education Degrees Committee to ensure that it meets ethical guidelines.

I hereby request permission from you, as principal to conduct research in Ms Gail's class. I desire to commence the research on 1 February 2016 but I will visit the class for familiarization as from Monday 25 January 2016

Thank you for your co-operation.

S. Chikiwa
MEd Scholar and Part-time Lecturer
Rhodes University
Grahamstown
6139

I, the undersigned__________________________, consent to the above research being conducted by Samukeliso Chikiwa from Rhodes University in Ms Gail's Grade 2 class.

Signature--------------------------

Date ---------------------------

APPENDIX 2 – TEACHER CONSENT FORM

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

Dear Ms G

Re: Permission to conduct research your Mathematics Knowledge for Teaching

My name is Samukeliso Chikiwa, a Master's in Education student in the South African Numeracy Chair at Rhodes University. I am currently working on a research project that explores what Mathematics Knowledge for Teaching teachers in the Foundation Phase require in order to teach mathematics competently and confidently. Given the nationwide concern about learner performance in mathematics in South Africa, this research has the potential to make a valuable contribution to the education of teachers in South Africa. My research is being supervised by Lise Westaway (Senior Lecturer in the Education Department at Rhodes University) and Professor Mellony Graven (Numeracy Chair in the Education Department at Rhodes University).

The research I wish to undertake seeks to understand the mathematical content and pedagogical knowledge that a *leader teacher* in the Foundation Phase has. You have been regarded in the Eastern Cape as a leader teacher in mathematics at Foundation Phase level, evidenced by the responsibilities you have been given by the Department of Education and your involvement with teacher education at Rhodes University. I wish to investigate what knowledge of teaching you employ in your teaching of mathematics at grade 2 level.

I wish to observe you teach mathematics on daily basis for a maximum of five weeks. May it be clear to you that I am in your class to learn from you NOT to judge your teaching style. These lessons will be video recorded. In compliance with the ethical standards the video data will be treated with the utmost confidentiality. The only people who will be able to view the videos will be you as the teacher, myself and, if necessary, my two supervisors. Viewing of the video material will only take place in collaboration with me. No video data will be available for any other purposes other than my research. I will also request to carry out a few formal and non-formal interviews. I will be considerate of your time

I am an experienced teacher and researcher who has previously conducted research in classrooms with young learners and realise the importance of confidentiality and anonymity in the research process. In the thesis there will be no mention of the name of the school or the teacher. Pseudonyms will be used throughout in order to respect the confidentiality of the school and the teacher. My research proposal has been reviewed by the Education Faculty's High Education Degrees Committee to ensure that it meets ethical guidelines.

I hereby request permission from you, as the leader teacher, to conduct research with you.
Thank you for your co-operation.

S. Chikiwa
MEd Scholar and Part-time Lecturer
Rhodes University
Grahamstown

6139

I, the undersigned___________________________, consent to take part in the above research conducted by Samukeliso Chikiwa from Rhodes University

Signature----------------------------

Date ---------------------

APPENDIX 3 – INFORMATION SHEET AND PARENT CONSENT FORM

RHODES UNIVERSITY

Grahamstown • 6140 • South Africa

INFORMATION SHEET FOR PARENTS OF LEARNERS IN MS GAIL'S CLASS.

My name is Samukeliso Chikiwa, a Masters in Education student in the South African Numeracy Chair at Rhodes University. I am currently working on a research project that explores what Mathematics Knowledge for Teaching teachers in the Foundation Phase require in order to teach mathematics competently and confidently. Given the nationwide concern about learner performance in mathematics in South Africa, this research has the potential to make a valuable contribution to the education of teachers in South Africa. My research is being supervised by Lise Westaway (Senior Lecturer in the Education Department at Rhodes University) and Professor Mellony Graven (Numeracy Chair in the Education Department at Rhodes University).

The research I wish to undertake seeks to understand the mathematical content and pedagogical knowledge that a *leader teacher* in the Foundation Phase has. Gail[13] is regarded in the Eastern Cape as a leader teacher in mathematics. She runs workshops on mathematics teaching and learning for the District Office, the Eastern Cape Department of Education and the National Association for Professional Teachers of South Africa (NAPTOSA). In addition, she is responsible for the mathematics education component of the Post Graduate Certificate in Education (Foundation Phase) qualification at Rhodes University.

The focus of the research is the teacher. However, I have to research Gail in context, which means that your child will be in the class when I conduct my research. As a research scholar in the Education Department, I am bound to the ethical principles of the department and Education Faculty's Higher Degrees Committee. This has implications for how I carry out my research.

I wish to observe Gail teach mathematics on daily basis for a maximum of five weeks. These lessons will be video recorded. While the video camera will be focused on Gail, it is likely that images of children working on the mat with her, will be captured on the video. However, my interest is not your child nor any of the other Grade 2 learners. My focus is their teacher. In compliance with the ethical standards alluded to above, the video data will be treated with the utmost confidentiality. The only people who will be able to view the videos will be Gail, myself and, if necessary, my two supervisors. Viewing of the video material will only take place in collaboration with me. No video data will be available for any other purposes other than my research.

I am an experienced teacher and researcher who has previously conducted research in classrooms with young learners and realise the importance of confidentiality and anonymity in the research process. My research proposal has been reviewed by the Education Faculty's High Education Degrees Committee to ensure that it meets ethical guidelines. Although the principal, and Ms Gail have agreed to my research, I do require permission from all the parents in Gail's class. **Please return a signed copy of the consent form attached by 27 January 2016…….**

Thank you for your co-operation.

[13] I have used pseudonyms in this document to protect the identity of the school, principal and teachers.

S. Chikiwa
MEd Scholar and Part-time Lecturer
Rhodes University
Grahamstown
6139

RHODES UNIVERSITY
Grahamstown • 6140 • South Africa

CONSENT FORM FOR GRADE 2 PARENTS

I, the undersigned__________________________ give permission for _____________________to participate in a research project conducted by Samukeliso Chikiwa from Rhodes University. I understand that the project is designed to gather what Mathemetics Knowledge for teaching is employed by the Grade 2 teacher to be recognised as an expert teacher so as to inform teacher education programs.

1. I am aware that participation in this project is voluntary. I understand that no payment will be paid for participation in this study. My child may withdraw and discontinue participation at any time without penalty.

2. Participation involves being interviewed, audio and video recorded during each learning session by the researcher from Rhodes University.

3. I understand that the researcher will not identify my child by name in any reports using information obtained from each session, and that his/her confidentiality as a participant in this study will remain secure. Subsequent uses of records and data will be subject to standard data use policies which protect the anonymity of individuals and institutions.

4. I understand that this research study has been reviewed and approved by the Higher Degrees Committee of the Faculty of Education at Rhodes University. The institution may be contacted for research problems and questions regarding the research.

5. I have read and understand the explanation provided to me. I have had all my questions answered to my satisfaction, and I voluntarily agree that my child participate in this study.

6. The study will begin on Monday 25 January 2016 so please confirm your permission before then. Otherwise your child will <u>not</u> be included.

Signature of parent / guardian…………………………………………………………..

Signature of the Researcher……………………………………………………………………..

For further information, please contact:

Researcher: Ms S.Chikiwa 078 385 6781
Supervisors: Ms L. Westaway (046 603 8774) and Prof M. Graven (046 603 7227)

APPENDIX 4 – EXAMPLE OF AN INTERVIEW

Turn	Speaker	Information
	R	**Thank you for allowing me to carry out this interview with you. We are going to view together one of your lessons I captured. I wish to understand better what you were doing and why you were doing it.**
	G	**No problem**
	RG	[*Plays the video and together they watch and listen*]
	R	**Why is it important for them to know the composition of numbers, like you want them to break up 36, you wanted them to understand the make, what 36 is made of. Why is it important that they should have that understanding?**
	G	**It is very important because otherwise they see 36 as a three and a six, and 36 actually is not a three and a six but a thirty and a six.**
	R	**Okay. When you… When they are learning about numbers, like when they learning to identify numbers say at Grade 1, are they…., before they get into realization that 36 is a 30 and a 6 are they allowed all to like…..do you… Can you allow them to like identify 36 as a three and six so that they can be able to pick up 36 number from a number line, a three and six?**
	G	**They are not going to see it as a three and a six. Three and a six means they don't actually know what that number is actually made out of. They just gonna see it as 3 otherwise if they see it as 3 tens there is a difference but a lot of children just see it as 3 and a 6 which makes nine. They don't refer to it as tens.**
	R	**Okay.**
	G	**You see what you are actually doing is part of strategies that children need in building up and breaking down numbers so now you are actually breaking down the numbers to see what they actually composed of.**
	R	**[***Nods her head***]**
	G	**It is very important. Another thing that we are actually doing when we are breaking down those numbers and building up numbers is that you can actually say to them put your finger under 3. What is that? They have got to be able to identify that 3 as either three tens or it's a 30. It's not just the three. A 3 is 3 ones or 3 units.**
	R	**Okay. One more thing! And I asked this before and I will ask it again. I realize that all your lessons you start them with counting, they count on, they count backwards, why are you putting so much emphasis on counting?**
	G	**Because counting is the basis of mathematics [***silence]***
	R	**Explain that further**
	G	**Well the thing is you want children to……. Counting is not just a component of just rote count, spit out of your mouth and it means absolutely nothing, which is what lot of teachers actually do they think it's just count, count, count. They don't even know if children are actually counting the right number on the right word or whatever, or if you didn't give the counting apparatus on the floor or things like that. What you want to do is to use the counting skills in your calculations.**
	R	**Okay…. So why?**
	G	**…and also with counting, what comes before, what comes after, if you look at the CAPS document, everything that I am actually doing I'm taking, I build all those things, reading their numbers, counting, breaking, building**

		up and breaking down numbers, doubling and halving. All those problem-solving they're all built into my lessons. I don't just say "right! Today I am doing problem-solving, tomorrow I am just going to count". All these build in. I am giving them what we call conceptual knowledge, basis of mathematical knowledge. I am building up their network in logical mathematical knowledge.
	R	**Okay.**
	G	**Yeah.**
	R	**Why do you use counters? You see, you don't just do counting like "okay [***clapping hands like to put order in class***] okay boys sit down boys lets count! One, two, three….." You actually go to your….**[*Pointing to the beadstring]*
	G	**Yes, the abacus …**
	R	**… abacus and you will be pulling out…**
	G	**It must be one to one, it must be a uttering the correct word on…**
	G R	**[***In unison***] the correct number.**
	G	**Yes.**
	R	**So what is the, what is the importance of using the abacus?**
	G	**Like I can use the numbers set, I can use the number grid, I can use anything as long as they are actually uttering the word to the correct number.**
	R	**Okay, so the abacus is helping them to realize that this number means this quantity?**
	G	**And also if you look at the abacus right now** *[pointing to the abacus]* **I can already see we are on 32** *[silence]***. How can I see we are on 32? Because it's 10, 20, 30 and 2 [***pointing to the beads on the abacus***]**
	R	**[***Looking at the abacus and smiling***] yes, so it makes it easy for them to identify numbers. Okay** *[playing the video, watching and listening***].**
	G	**The reason why you also break is because you want to keep them focussed.**
	S	**Okay.**
	G	**You are working with the whole group …**
	R	**Um**
	G	**… and you're going to get day dreamers.**
	R	**Okay.**
	G	**So instead of just counting and being boring you need to throw in these other things to keep them focused.**
	R	**[***Nodding***] Okay.**
	G	**And the other reason why you deal with the whole group is that weak kids can learn from the stronger children.**
	R	**So important that sometimes put them together and they …..**
	G	**…. I always starts with what I call mental maths at the beginning of my lesson.**
	R	**Okay.**
	G	**The whole class.**
	R	**[***Nodding***]**
	G	**At the moment it takes the round-about 10 to 15 minutes. At the moment it takes a little bit longer but there is quite a lot of conceptual knowledge that I need to get through to them to discover through my mental mathematics and learning from each other.**
	R	**Okay** *[playing the video once more***] it's not very loud.**
	G	[*Leaning forward to listen*] **Yeah!**
	S	**When do they learn about tens and units?**
	G	**Coming up now.**
	S	**Coming up now!**

	G	**We are doing revision up to nine……. And then of course I am building in all my things then we start breaking up tens, but they have already done up to 19. The minimum requirements in Grade one is the work up to 19. So they know that's if 15 is a 10 and a 5, 19 is 10 and 9. They have already picked that up. It is 10 and 9 ones.**
	S	**Ok, so it's like when you stopped them at 40 you asked them, "Ah how many tens in 40?"**
	G	**They have been counting in tens in Grade one.**
	S	**Okay, so you expect them to know?**
	G	**Well! That's what I am looking for. I am still doing a lot of…..**
	S	**Assessment**
	G	**Not assessment but baseline, to see actually what knowledge they have.**
	S	**Okay, so it's like as you do your counting, it's just having them understand counting but you also get to understand them better is you ask them questions.**
	G	**[***Nodding]***]**
	S	**Okay.**
	G	**I mean you could actually see that with the bottom group, the week ones, some of those children they can't count.**
	S	**Yeah, they can't!**
	G	**They can't [***shaking her head***]**
	S	**So is it ideal for you to say you have realized, like this….. What's the name of the group?**
	G	**My Brown Group**
	S	**Your Brown Group, they still need an lot of learning to count, can it be a deal for you to sit them on the mat and you start counting using that the number line with them counting**
	G	**[***Nodding***] Yeah! This is why I do use the number grid.**
	S	**Okay.**
	G	**Each child has a number grid.**
	S	**[***Nodding***] Okay.**
	G	**Yeah, and I can actually do what they call point counting. There are lots of different ways of actually counting.**
	S	**Um**
	G	**There are five principles of counting that govern meaningful counting.**
	S	**Okay?**
	G	**I am not even going to those, but then it's a lecture that I will do. You can go and study that the five principles of counting. And this one is… It is not one of the principles but you get rote counting which is just spitting out, it's just rote, they don't sometimes even know. You may think the child, your child is being clever, a four year old child or three year old can be counting up to 19 or 20 all whatever but the thing is if you give them counters, the go "one, two, three, four, five, six" [***pulling imaginary counters in a way that is not corresponding with the counting of numbers***] but they have lots only five counters or whatever. Where they point count they literally point, this is what we call one to one correspondence. Say the word and move it, say the number and move it, say the number, move it** *[moving the imaginary counters]*
	S	**Okay [***playing the video***]**
	G	**You see that is one to one correspondence, [***Pointing to the screen***] there, so they are saying as I am moving [***moving the imaginary bids with his fingers***]. They are saying the number I can stop them and say "what have we counted to?" 10, 20, 30, 40, 50 whatever [***pointing to the beadstring on the screen***] You can see it, 10, 20,30, 40, 50, 55.**

	GS	**[***watching the video***]**
	S	**Hmm I enjoyed this part of your lesson. [***Smiling***]**
		[*silence as the two watch the video***]**
	G	**You can hear it, we also want them to hear the number.**
	S	**Hmm. Can you just tell me about the illustration, Why did you do it? [***Pointing the imaginary flood cards from the mouth as shown in the video***] Why use those cards? How does it help them?**
	G	**Break up numbers we use flood cards. Fantastic! You can't actually do edition of two digits numbers any kind of edition or subtraction without them.**
	S	**Okay.**
	G	**Yeah you know what? When a child does addition of numbers two digits and one digit numbers they actually pull them apart horizontally.**
	S	**Hmm.**
	G	**There is no vertical maths. We don't do vertical maths. We do everything horizontally.**
	S	**Okay.**
	G	**So if I have 36+42, they pull the number of parts [***pulling her hands apart***] and then the social knowledge that I give them, okay say go from the biggest to the smallest. So they go a 40 and a 30 whatever whatever whatever. 40 count on 30 makes 70. How do you know? 10, 20, 30, 40, 50, 60, 70. You see how counting is coming in? 40 and 30 makes 70….. Plus a 2 plus a 4 whatever.**
	S	*[Silence]*
	G	**So everything is horizontal.**
	S	**Okay, like when you did this, this illustration. Before you did the illustration you asked what is, what is 55 made of? Or something almost like that and one learner gave you the answer a 50 and a 5 but you went on to illustrate how it becomes a 50 and a 5 why did you do that?**
	G	**For a bottom group to realize, for your other children who haven't got that.**
	S	**Okay.**
	G	**So you actually showing them using the flood cards.**
		Okay.
	G	**So they can actually see it, pulling the numbers apart** [*pulling apart imaginary flood cards]*
	S	**So you are trying to go with the whole class.**
	G	**[***Nodding***]**
	S	**You realize you have the, the top ones are with you but you also keep in mind you…**
	G	**You have to!**
	S	**Okay [***noddin***g]**
	G	**It is very easy to teach a clever child, your teaching ability lays with the bottom group, that is where your teaching ability lays [***nodding***] it's very easy to teach a top child**
	S	**Okay [***nodding***] but to pull along the bottom ones is what teaching is all about.**
	G	*[Addresses the children who were coming back to the classroom]*
	S	*[Playing the video]*
	G	**What am I actually doing here is the revision of grade one work because they should be able to count up to 100**
	S	**Okay.**
	G	**[***Nodding***] Hmm.**

	S	**In grade 1?**
	G	*[Nodding*] **Hmm.**
	S	**And in grade 2?**
	G	**200.**
	GS	**[***Watching the video***]**
	S	**What does that mean, "close Mr.80's house"?**
	G	**You see the thing is like Samu they battle. They really battle to come, to come out of one ten and go into the next one if the accounting backward.**
	S	**Okay?**
	G	**So now what you are wanting them to do, what I have discovered that over the years is you start at the top [raising her hands above her head] so can 79, 78, 77, 76, 75,74, 73, 72, 71 [***staggering her hands downwards on each count***] Close Mr 70's house. [***Pulling hands apart in a way of closing and speaking softly***] we are now finished with Mr. 70s house. Otherwise what they will do is they will go to 70 again or they will go to 80 instead of going to 69.**
	S	**Okay, so that 'house' help them realize now we have nothing to do with the 70s, we going to the …**
	G	**We are finished with the 70s we are now falling into the 60s house and you fall in at the top [***raising her hands above her head***] then you count backwards [***staggering her hands downwards***] it's something I have learnt over the years.**
	S	**And it's quite interesting.**
	G	**It works! It's definitely does help them actually realizing we have finished the 70s**
	S	**And it also helps all of them to be with you because all of them have to be working [***raising hands above the head and staggering them down like they do when counting backward***].**
	G	**Oh yes, [***also raising hands***] you see that is actually getting the children, with the boys the boys have to move to learn.**
	S	**Okay.**
	G	**That is why you often see me doing that counting in 2s, get up, get down, it's to get them focused again. It's to get them moving then back to your next bit of work.**
	S	**Okay** *[nodding and playing the video again*].
	GS	*[Listening and watching]*
	S	**So is there any possibility that they can, let me use the word sing, sing this close Mr whatever the bottom group without understanding what is happening.**
	G	**Hopefully they do understand because I have also shown them. We did a house and we had all those numbers that are part of seventy. So now, I mean there are only nine numbers that are part of seventy and only nine numbers that are part of eighty, see what I mean, so when u are finished with those where do u go from there?**
	S	**What do u mean there are only nine numbers that are part of eighty?**
	G	**81, 82, 83, 84, 85, 86, 87, 88, 89 you then go to 90.**
	S	**Okay.**
	G	**And all the numbers in ninety are 91 to 99 basically there are only nine ones then you go to tens then you get ten and nine ones then you go to twenty.**
	S	**So basically does it mean if a child as well understood counting back from 10 to 0 it becomes easy for them to count from any number coming down?**
	G	**Not necessarily coming from nine to zero, not at all, not necessarily from nine, um, it's very easy and also parrot fashion when they do it that way, 9,**

		8, 7, 6, 5, 4, 3, 2, 1 then they blast off. I don't know, get all these weird things coming in, um, no it's when you get to the bigger numbers that it becomes difficult for them to actually finish. They will go like 71, 60 or 71, 80 they won't say 70 finish that ten.
	S	**Okay [***nodding***] Close Mr seventy's house.**
	G	**Yah but now you will actually see initially we were closing but now I just do [***shows hand signal]* **they don't do close Mr seventy's house anymore they're now counting just backwards 79-70 we're not saying "Close Mr seventy's house" anymore.**
	S	**Now they understand that sign [***showing sign***] we are closing, okay.**
	G	**Yah.**
	S	**Okay so with time will you still have to do the actions or you can get to a stage were you just go down, go down, go down with having to…**
	G	**Yah eventually you will actually see by the end of the year I am not doing half as much counting.**
	S	**[***nodding***]**
	G	**It's only at the beginning of the year that I do an enormous amount of counting.**
	S	**Okay.**
	G	**Yah towards the end of the year I a much further they know how to use their counting skills in their calculations so I am not doing half of this much counting.**
	S	**Okay.**
	GS	**[***Watch video***]**
	S	**There is that boy [***pointing at video***] when we started just now who was using his fingers to [***showing what boy was doing***] what's is important for them to be able to use their fingers in counting. I have realised even when you are doing counting with them at times you say ''use your fingers, use your fingers'' what's….**
	G	**… well our fingers are part of a resource.**
	S	**Mmm.**
	G	**Yah I mean your fingers can be ones, they can be twos, they can be fives, they can be one hundreds, anything …**
	S	**Mmm.**
	G	**Children need to learn, they need concrete in order to do maths, you can't just tell them something. That's the basis as well is u got to discover things using concrete apparatus. [***wiggling her fingers***] this is concrete**
	S	**Mmm. Okay but why don't u ask them to use the number line? Like they look at the number line or they look at the fingers on the board but they use their own fingers. Is there any special thing about their own fingers?**
	G	**It's part of them.**
	S	**It's part of them.**
	G	**Mmm. It's much better for them to use their own finger that to get up and walk around to go use your number grid, they can, but you' re going to have lots of children walking around whereas fingers are on your hands the whole time.**
	S	**Okay.**
	GS	**[***Watch video]*
	G	**How much more I have to go?**
	S	**Ok maybe I wanted to understand about this part [**pointing at the video**]**
	G	**Quickly let's do.**
	S	**Okay.**
	GS	*[Watch video]*

	G	**They need to realise that counting backwards you are taking away one every time.**
	S	**Okay.**
	G	**So you are trying to, through counting, you are trying to let them realise that counting is a strategy. Forward, addition and backwards is taking away.**
	S	**Ok so that's what you wanted them to realise to…?**
	G	**So I am taking away one each time…**
	S	**Each time you counting back using one you are taking away one!**
	G	**And if I am counting back in twos I am removing two.**
	S	*[Silence]*
	G	**So u see how counting helps you with addition and subtraction.**
	S	**Okay.**
	G	**Yah.**
	S	**So your counting, you use it for building skills for addition and subtraction?**
	G	**[***Nodding in agreement***]**
	S	**Okay.**
	G	**There you see taking away one. [***referring to the video***]**
	SG	**[***Watch the video]*
	G	**There we go! [***referring to the video***]**
	S	**[***Silence]*
	G	**You see 10, 20 is 2 tens and 30 is 3.**
	SG	*[Watch video]*
	G	**Look at this one! [***Pointing to screen***] He's not listening at all!**
	S	**[***Laughing***]**
	G	**Okay now you are going ask me why that …**
	S	**Mmm** *[smiling]*
	G	**Alright can I write here?** *[pointing at the paper in front of the researcher***]**
	S	**Ya you can write at the back. [***giving her a sheet of paper***]**
	G	**Patterns, maths is also about patterns, okay, so I went, I have, [***addresses a staff member]*
	S	*[Looks at the page]*
	G	**It's a pattern** *[continues to write on page the multiples of two up to ten in a row, then up to twenty underneath such that the 2s are in the same column and the 4s,6setc form a pattern)* **all the two, here [***pointing at her thumb]***.**
	S	**Okay.**
	G	**It doesn't matter I can start 22, 24, 26, 28, [***pointing at all her fingers starting with thumb***] 30, 32. This is a 2, a 4, a 6, a 8. I only need five fingers to count in twos.**
	S	**Okay.**
	G	**Ya go it's all about patterns. [***handing paper back to Samu***]**
	S	**So you make them realise the patterns?**
	G	**I make them aware of patterns all the time. The same thing if you see with my counting in 10s, I said is we went down what could we actually see, the 6 was all the same what was changing was the 10 was going to 20 because from 16 I added on 10 to get 26**
	S	**Okay.**
	G	**That's all about patterns and if a child can see patterns they can do maths.**
	S	**Okay [***nodding head***]**
	G	**Maths is a pattern. It's the same thing over and over.**
	S	**So they just need to pick the pattern?**

	G	**It takes a long time for your weak group to see patterns though, long time, sometimes they don't even see it.**
	S	**Oh that's why you were saying, you may not have to use that break down you used there with your top group, with your bottom group.**
	G	**Oh! They are a sweet.**
	S	**No I will see then by the end of the year they will be, I think they will be up there.**
	G	**Oh they will be able to do something but…**
	S	**They will be up there.**
	G	**They won't be anywhere near my top group.**
	S	**Okay, now because you want to go?**
	G	**Yah I have to go.**
	S	**I will stop here. I guess some of the things I could have asked about in this lesson some of them I will still find them in another lesson and some of them I will just…**
	G	**I am sure you will** *[nodding*] **Yah.**
	S	**I will ask you some other time but thank you so much for your time and thank you for answering all my questions [***shaking hands***].**

APPENDIX 5 – WHOLE CLASS COUNTING SESSION LESSON 1

APPENDIX 6 – WHOLE CLASS COUNTING SESSION LESSON 2

APPENDIX 7 – WHOLE CLASS COUNTING SESSION LESSON 4

APPENDIX 8 – WHOLE CLASS COUNTING SESSION LESSON 10

GLOSSARY OF ABBREVIATIONS AND ACRONYMS

ANA	Annual National Assessment
BEd	Bachelor of Education
CAPS	Curriculum and Assessment Policy Statement
CCK	Common Content Knowledge
CDE	Centre for Development and Enterprise
DBE	Department of Basic Education
FP	Foundation Phase
HCK	Horizon Content Knowledge
HoD	Head of Department
JPTD	Junior Primary Teachers' Diploma
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
MKfT	Mathematics Knowledge for Teaching
NCTM	National Council of Teachers of Mathematics
NEEDU	National Education Evaluation and Development Unit
NSC	National Senior Certificate
PCK	Pedagogical Content Knowledge
PGCE-FP	Post Graduate in Education (Foundation Phase)
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SANCP	South African Numeracy Chair Projects
SCK	Specialized Content Knowledge
SMK	Subject Matter Knowledge
TIMSS	Trends in International Mathematics and Science Study
Tr	Teacher