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INSTRUCTIONAL DESIGN IN PURSUING EQUITY: THE CASE OF THE 'FRACTION AS MEASURE' SEQUENCE

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ABSTRACT

The aim of this paper is to discuss an approach to instructional design in which we intertwine mathematics education research with development, draw insights from collaborations across international contexts, and place educational equity firmly at the centre of our work. Using an example of the instructional sequence on 'Fraction as Measure,' we illustrate how the two key aspects of the design process allow for developing instructional resources with equitable learning opportunities in mind. The two aspects entail specifying (a) the prospective endpoints of an instructional sequence (i.e., forms of students' reasoning to be developed), and (b) how the learning process might be realized in the classroom, so that students would come to develop the specified forms of reasoning. We discuss how considerations of teacher learning, and processes of travel and scaling up of designed instructional innovations, need to be considered by designers aiming to advance educational equity.

Data from international comparisons suggest that both within and across countries, varying access to high quality educational opportunities accounts for much of the difference in resulting achievement. They also indicate that most of the world's children receive mathematics education in classrooms that do not resemble those in which instructional innovations are typically developed. The instructional design work on which we report here explores the very question of supporting equitable mathematics learning in and beyond under-resourced classrooms.

The instructional design in which we engage typically involves conducting classroom design experiments, where a research team engages with a group of students for a period that can last between a few weeks and several months (Cobb, 2000). Once initially developed, the instructional resource is re-tested and modified in numerous classrooms, until it can be seen to reliably support the targeted forms of mathematical reasoning, and until the instructional conditions under which it does so are well understood. At first glance, our work can be regarded as unsuitable for the pursuit of equity in mathematics education, since the resulting instructional resources cannot be readily and effectively implemented by all teachers. This is because the instructional conditions under which the resources contribute to successful, equitable learning (including a kind of teaching required) are complex and take time and assistance to develop. Public education systems are rarely organized to proactively support long-term development of this kind of teaching, and conditions tend to be far less favourable in schools that serve the children of marginalized communities.

Despite the challenges our resources present for teachers' learning, equity is a driving concern in our research. We use this paper as an opportunity to explain how we conceptualize and pursue equity in our design work. We address some of the contributions that this approach to instructional design can make to increasing the opportunities of children from historically excluded communities to make sense of key mathematical ideas.

We first situate our work methodologically and discuss the history of developing and adapting an instructional sequence on fraction as measure. We then discuss how we use the two key aspects of the design process when developing instructional resources with equitable learning opportunities in mind. These aspects entail specifying (a) the prospective endpoints of an instructional sequence, and (b) how the learning process might be realized in the classroom. While our major focus in this paper is the design and adaptations of classroom resources for equitable teaching and learning, we touch upon broader issues that must be considered in instructional design for equity. These include teachers' learning, and travel and scaling up of designed instructional innovations. We illustrate how these broader issues shape our collaborations and design decisions.

CLASSROOM DESIGN EXPERIMENTS

Classroom design experiments are a research methodology used to study the process of students' mathematical learning, as it occurs in the social context of the classroom, and the means by which that learning is organized and supported. The resource developed is often referred to as an *instructional sequence*, recognizing the sequential nature of increasingly sophisticated forms of reasoning in the classroom that require teacher's proactive support.

Central to the instructional sequence is a hypothetical learning trajectory (HLT) consisting of a series of conjectures about how the mathematical development of a classroom community is expected to unfold in response to instructional intervention. The HLT specifies the prospective learning endpoints, the learning starting points, the general path by which students' mathematical reasoning will develop, and the means with which teachers would support this development (Cobb, 2000). The HLT provides the underlying *rationale* of an instructional sequence. It is explicitly formulated so as to allow (and, ideally, support) teachers to make reasoned adaptations of the resource, to address the specific circumstances of their classrooms.

The 'Fraction as Measure' instructional sequence includes a series of tasks, in which students are expected to use a variety of measuring tools, graphs, written symbols and oral expressions. However, the means of support specified by the HLT extend beyond these. They include guidance for the teacher on how to organize classroom activities in which students engage with the tasks and tools, so that this engagement cultivates students' mathematical interests, provides cognitive challenge for individual students, and maintains opportunities for classroom discussions. Most importantly, it is intended that a particular kind of microculture will be constituted in the classrooms, so as to orchestrate rich whole-class discussions that can give rise to the expected forms of mathematical reasoning. Amongst other things, students are expected to listen to each other, explain and justify their solutions, attempt to make sense of the explanations of others, and indicate agreement and non-understanding.

‘FRACTION AS MEASURE’ COLLABORATIONS

For over a decade, the first two authors have been developing and improving the ‘Fraction as Measure’ instructional sequence. The first of our formal classroom design experiments took place in 2007. Since then, we have engaged in a process of revising and improving the sequence. We have conducted other classroom design experiments in Mexico, including one that was implemented by a Mexican teacher, Guadalupe, with her regular fifth-grade classroom, where the first author served as a university mentor, never entering the school.

Most recently, the latter two authors incorporated the instructional sequence into their research and development work in the South African Numeracy Chair Project (SANCP) in the Eastern Cape. They first trialled the sequence in an after-school club and then conducted a four-lesson implementation study, where the sequence was taught by the researcher, in three grade 3 classrooms (108 students), where the language of instruction was not the home language of the overwhelming majority of the learners[6].

Across the studies, the ways in which students came to reason about the inverse order relation of unit fractions (grades 3 and 2) and about the relative size of any two fractions (grades 4 and above) are both consistent and impressive. For example, Vale and Graven (2018) documented the changes in grade 3 students’ participation and unit fraction understanding after the brief 4-lesson implementation of the sequence in three classrooms. In addition to increase in students’ contributions during classroom discussions, change was visible in the increase in the numbers of children who could correctly compare three pairs of unit fractions (1/2 and 1/4; 1/5 and 1/3; and 1/4 and 1/8). The comparison comes from 83 learners who took both tests and attended all the 4 lessons. While only 7 out of the 83 students compared all three fraction pairs correctly on the pre-test, 59 did so on the post-test. While only four of these students *also* indicated correctly that 3/4 is less than 3/3 on the pre-test, 57 of the 59 students who successfully compared all unit fraction pairs also compared 3/4 and 3/3 correctly on the post test. However, since there were still some learners who were not able to consistently make fraction comparisons correctly in the post test begs the question: can the sequence be used to support *all the children* in a classroom to learn fractions?

Guadalupe’s fifth-grade classroom design experiment might be indicative here. She taught the sequence in her regular classroom during 18 dedicated weekly sessions, approximately 35 minutes each. Only 4 of the 20 students could reliably compare three pairs of unit fractions on the pre-test[7]. In contrast, *all* could do so on the exit written test, comparing nine unit-fraction pairs including 1/4 > 1/456. In addition, 18 students correctly placed three fractions (16/11, 10/3, and 15/5) on a number line, while the remaining 2 students placed correctly two of those and made a numerical error in estimating the position of the third[8].

[6] It is worth clarifying that we view implementation as a process of conjecture-driven adaptation and adaptations made to the sequence in this context were expected and were the focus of our learning from this study.

[7] These were the same 3 pairs as used in SA on pre- and post- tests.

[8] This task was not pre-tested.

Granted, the institutional conditions for the implementation of the sequence were quite different in these two cases. The first study aimed to assess the viability of the sequence adaptation in a new setting, across 3 different classrooms, in a way that would be convincing to the local teachers and school leaders. Fitting the implementation study into one week of lessons was important since the pace of teaching was dictated by a departmental push for curriculum coverage. The latter is a result of the work of one teacher who had experienced the sequence as worthwhile for her students' learning during the initial lessons, had decided that all of her students' learning with understanding was a reasonable goal with this tool, and dedicated more time in her classroom for such learning (Visnovska & Cortina, 2017).

These and many more of our classrooms design experiments were conducted in under-resourced schools that serve children living in poverty. However, as we explain next, the orientation towards equity in our research agenda goes beyond the selection of collaborating schools.

SPECIFYING INSTRUCTIONAL ENDPOINTS

Delineating the endpoints of an instructional sequence entails specifying how learning a mathematical idea could have enduring value for learners. This value must be judged in terms of what it represents for students' future mathematics performance *and* for their learning in other fields. In addition to addressing the goals related to what Biesta (2010) refers to as *qualification* purpose of education, we find it important to judge the extent of the broader social significance of these goals, or how they address what Biesta refers to as the *socialization* and *subjectification* purposes of education. For instance, the contribution of the envisioned goals to both students' access to and participation in the current social order, and their formation as informed, critical, and unique citizens, able of shaping their society, are taken into consideration.

We regard this aspect of the instructional design process to be of particular importance for the mathematics education of marginalized groups. The resources that are made available to these learners are often limited. Before entering school, children from these communities often have few opportunities to develop mathematical ways of engaging that are valued and expected at school reception, such as counting in the language of instruction. The schools that are available to them are often under-resourced. The teachers who work there are frequently less qualified. Student absenteeism due to illness, economic, and family circumstances is then more common.

There is certainly much that societies and governments can and need to do to improve the learning opportunities of children of historically marginalized communities. From an instructional design perspective, clarifying what is most critical or beneficial when supporting these students in developing specific mathematical ideas could allow for better use of the relatively limited instructional time and resources currently available to them.

Fractions are widely regarded as both mathematically and culturally indispensable concept, the first extension of our numeration system beyond whole numbers. They occupy an important place in mathematics curricula of countries all over the world. The vast body of research literature concerned with teaching and learning fractions goes back many decades. Recognized as a complex, multifaceted idea involving a variety of meanings that relate to each other in an intricate manner, fractions provide a conceptual entry point to proportional (multiplicative) reasoning (Thompson & Saldanha, 2003), a type of reasoning that is indispensable for both individual wellbeing, and participatory democracy.

At the same time, fractions continue to be widely regarded as a difficult topic for teachers to teach and for students to learn. Alarmingly, the mathematics instruction typically made available to students from low income communities does not allow the vast majority of them to adequately make sense of this concept (Graven, Venkat, Westaway, & Tshesane, 2013). Given the substantial time that teaching and re-teaching fractions takes across the years of instruction, mostly to no avail, one has to ask whether this time could be better used. Can we steer away from the early introduction of formulaic procedures and symbol manipulation and focus on sense-making as the learning goal? Could the learning goal, and the learning pathway be expressed in terms of the specific meanings, images, and forms of reasoning that students should develop?

The endpoint of the 'Fraction as Measure' instructional sequence entails an image of fractions as numbers that account for the size of a measured length. In this image, it is expected that the denominator of a fraction will be construed by students as a number that accounts for the size of a subunit of measure, and the numerator as one that accounts for the iteration of that subunit, a certain number of times. For instance, a fraction 7/6 would be interpreted as a length that corresponds to seven iterations of a subunit that is one sixth as long as a reference unit (see Figure 1).

It is worth clarifying that, in formulating this image of a fraction, we have been influenced by the insights and considerations of several authors, including Thompson and Saldanha's (2003) ideas about understanding fractions as expressing reciprocal relations of relative size, Freudenthal's (1983) concept of fraction as comparer, and Davydov's (1969/1991) work on teaching fractions as length measures (Cortina, Visnovska, & Zúñiga, 2015). The endpoint of our instructional sequence is consistent with how Steffe and his colleagues (Steffe & Olive, 2010) expect students to reason when having developed relatively sophisticated understandings of fractions.

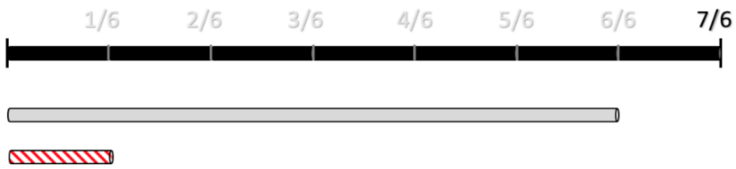

Figure 1: Fraction 7/6 (black) as a length that corresponds to seven iterations of a subunit (striped) that is 1/6 as long as a reference unit (grey).

We consider this image of fractions to have enduring value for students' mathematical development in three ways. First, it has a strong *quantitative* component, which allows associating fractions to the measurement of continuous magnitudes. For instance, this image is consistent with construing a measure such as 1375 mg as 1375 times a mass that is 1/1000 as much as a gram. Second, the image is *multiplicative*: it can serve as a basis for carrying out proportional comparisons, and for reasoning about relative size. As an example, the image is consistent with judging that if the value-added tax is 20%, then the price that a buyer always pays for a product is 120 times a one one-hundredth of its original value (or 6/5 of the original value). This type of reasoning is both difficult and unusual, especially in English or Spanish language environments, where relationships involving percentages are almost always expressed in an additive language, that is, “20% *more* than original value” or “25% *off*” (as opposed to multiplicative expressions such as “120% of the original value”, or “75% of the original price”, which are typical in some southeast Asian language environments). Third, since length is a unidimensional magnitude, this image is consistent with regarding fractions as *numbers* that occupy a specific place on the number line, an important sense-making tool.

Most importantly, this endpoint image of fractions is consistent with introducing students to *mathematics as a human activity* (Freudenthal, 1973), in which individual and collective sense-making are the primary tools for establishing the value of mathematical tools and ideas. We conjecture, and our experiences in classrooms to this point confirm this, that prioritizing the meaning-based goals for learning fractions, and the classroom microculture that can support such goals, would be beneficial for students’ mathematical development. This would be particularly so for those pupils whose opportunities to engage with and talk about mathematics are relatively limited.

A PATHWAY FOR LEARNING

As previously mentioned, central to the instructional design approach that we follow is the formulation of a HLT that specifies how students’ mathematical reasoning is expected to develop, and the means that would support this development. Underpinning this formulation is a view of mathematical learning as socially situated. Consequently, the role of instructional design is not limited to developing resources that can support what is regarded as rather stable and autonomous processes of conceptual development (Cobb, 2000). Instead, it is assumed that students’ participation in classroom cultural practices strongly influences their development of mathematical notions and ways of reasoning.

It is interesting to note that research literature on fractions is riddled with reports on the conceptual challenges experienced by learners, including *misconceptions*, *hurdles*, and *interferences,* and that these are positioned as an inevitable part of fraction learning. From a situated perspective on learning, however, these conceptual challenges are understood as a product of the social and cultural practices in which students engage with the idea. We argue elsewhere (Cortina, Visnovska, & Zúñiga, 2014) that these challenges are productively understood as *didactical obstacles* (ones resulting from teaching; Brousseau, 1997), rather than ontogenetic (ones resulting from learners’ cognitive development) or epistemological (ones resulting from the nature of mathematics).

From this theoretical perspective, the conceptual challenges in learning fractions are largely a function of the types of instructional activities, metaphors, representations, and tools that students typically encounter during instruction. This opens the possibility that a thoughtful change in the instructional resources that perpetuate current classroom cultural practices will make it possible to overcome, or even bypass altogether, the currently identified and prevalent learning challenges. Discovering the alternative paths and the resources to support them, should be, from an equity perspective, an important instructional design task. It would certainly be of much relevance for the many children of marginalized communities, who are seldom provided with the necessary resources to overcome the challenges that emerge as a result of instruction.

The HLT that substantiates our instructional sequence on fraction as measure serves as an example of what different instructional practices and meanings, to those that are typically employed, might look like. We describe it in detail so as to clarify both what those differences are, and why they make a difference to students' learning.

The HLT aims at guiding students in *reinventing length measurement* that requires the use of a reference unit, and subunits. It encompasses the consecutive emergence of three classroom mathematical practices. In the first one, the collective activity of the classroom community centers on measuring lengths with a standardized unit, and on reasoning about the advantages and insufficiencies of using a single reference unit to measure various objects. In the second practice, the activity shifts to producing additional measuring resources, namely, subunits of measurement the size of unit fractions; and reasoning about their relative size. Finally, in the third mathematical practice, the activity centers on reasoning about the relative size of lengths produced by iterating the subunits.

The first classroom mathematical practice

For this mathematical practice to emerge, it is expected that students will first become mindful of the limitation of using non-standard units—such as hands—when measuring. One of the means we developed to support the emergence of the first mathematical practice is a narrative that includes a series of stories about the trials and tribulations of an ancient community, who sought to find better ways of measuring. These stories are a resource to orient the students to view themselves as investigators of how ancient people developed the ability to measure with accuracy and innovation.

The first mathematical practice emerges once the students recognize the need for a standardized unit if people are to make consistent measures. Prior to arriving at such recognition, students explore measuring with parts of their bodies, such as their hands and feet. The teacher points to the inconsistencies of measures carried out by different students and makes these an issue for discussion in the classroom. For instance, the teacher might make it noticeable to the class that the number of handspans counted when different people measure the chalkboard is not always the same. Once the need for a standardized unit of measure is established, wooden sticks of the same length (about the width of this page) are introduced as a tool that fulfilled this need in the measurement narrative.

It is worth pointing here to one of the differences between the means of support involved in our instructional sequence and those that are commonly brought to bear in initial fraction instruction. In the 'Fraction as Measure' instructional sequence, the value of 1 is embodied by the stick[9]. The stick is expected to be iterated, and each iteration is understood to be of the same length. The unrestricted availability of the equal-sized units introduced through this process makes it relatively easy for students to construe the stick's length as a reference unit for measurement. They also construe the iteration of a measurement unit as a process by which different lengths can be created and compared.

In comparison, students are typically expected to first associate the value of 1 with a *unit-whole*, usually portrayed as a food item that can be easily divided (e.g., a cake). While this unit-whole is available as a source for the creation of sub-units, it often appears unique and not easily reproducible. In this process, comparison among different unit wholes is not established (Freudenthal, 1983), which can later challenge construing fraction quantities bigger than 1.

The second classroom mathematical practice

The emergence of the second mathematical practice in the classroom is supported by activities, in which students produce subunits of the reference unit. For these activities to become meaningful, students first need to become mindful of the insufficiencies of measuring with this single unit (the stick). As students engage in measuring the length of different things, the teacher makes it noticeable that, frequently, there is a remainder that the measuring stick does not cover exactly. For instance, the teacher's desk can be more than three sticks tall, but not as tall as four sticks (see Figure 3).

Figure 2: A desk that is taller than three iterations of the stick, but not as tall of four iterations.

The making of the subunits of the stick (reference unit) is introduced as a solution to the problem of accounting for the lengths of the remainders, in a systematic way. Building on the narrative of how ancient people measured, students are asked to make subunit rods by cutting straws, so that their lengths satisfy specific conditions. For instance, they can be asked to make a rod that would require five iterations of its length to cover the length of the stick, exactly (see Figure 3). If a straw is too long, given that five iterations of its length surpass the length of the stick, a student will have to make it

[9] It is, in fact, embodied by any of the many sticks that, while owned by different people, are all understood to be of equal length.

shorter, using a pair of scissors. If a straw is cut too short, they will have to try again with a longer straw.

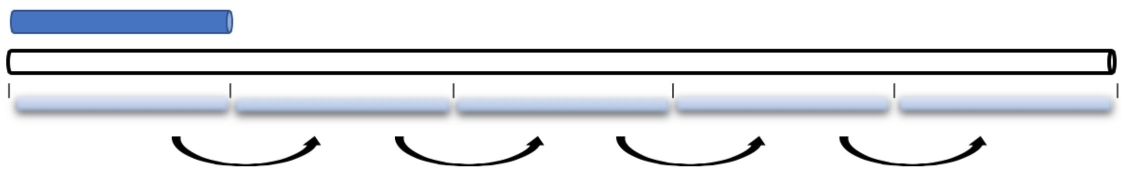

Figure 3: A rod of such length that five of its iterations cover the length of the reference unit.

Usually, students are asked to make at least five rods whose lengths are unit fractions of the length of the measuring stick (see Figure 4).

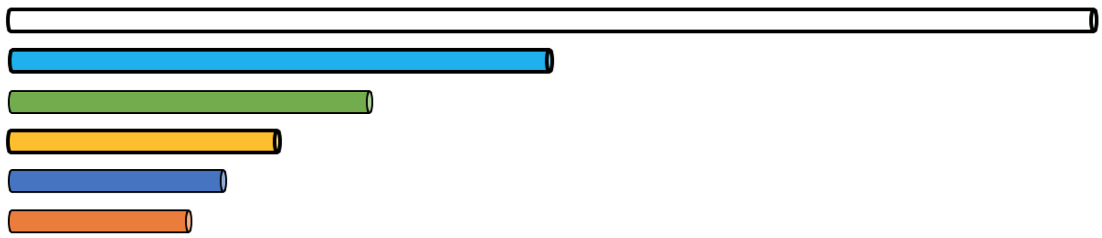

Figure 4: The measuring stick (reference unit) and measuring rods (subunits) of lengths 1/2, 1/3, 1/4, 1/5 and 1/6.

This second mathematical practice is considered to have been constituted once students can readily reason about the relative size of different rods, without having to physically produce them. For instance, they would be able to readily judge that a rod whose length is 1/11 of the length of the reference unit would be longer than one whose length is 1/15. Their reasoning would be based on the consideration that the length of the latter rod would have to be iterated more times to coincide with the length of the reference unit, so it would have to be shorter.

Here too, the differences between the means of support involved in our instructional sequence and those that are commonly brought to bear in initial fraction instruction are important. First, most commonly, unit fractions are construed as a quantity formed by one part, when a whole is partitioned into a certain number of equal parts. This process experientially only gives rise to the creation of a limited number of the entities that are expected to embody unit fractions (e.g., only 5 fifths are available), which are understood as being contained within a unit-whole. As a consequence, only some common fractions can be created (1/5, 2/5,... , 5/5). According to Steffe and Olive (2010) this is consequential for many novice fraction learners, for whom regarding the unit fraction quantities created in this way presents a major challenge to understanding fractions bigger than 1 later on.

The 'Fraction as Measure' instructional sequence addresses this challenge in that unit fractions are expected to be construed as the lengths of rods that, when iterated a certain number of times, fulfil a specific condition: producing a length equal to that of the measuring stick (see Figure 4). This condition does not limit the number of times that each resulting sub-unit of measure can be subsequently iterated (see Figure 1 for an example).

The second important difference rests on the observation that once unit-fraction pieces have been created in a part-whole model, students' access to the object that represented the unit-whole is no longer experientially supported. In contrast, in the 'Fraction as Measure' sequence, the entities that students are expected to regard as embodiments of unit fractions—namely, the rods—are always apart from the reference unit (see Figure 4). This facilitates establishing unit fraction as a *relationship of relative size* between *two* quantities that are both accessible to students.

Finally, when the part-whole scenarios of cutting food items are used, students can only construct *some* of the unit fractions independently with reasonable accuracy (e.g., 1/2, 1/4, 1/8). This is problematic, as fraction learners invariably rely on the process by which unit fractions are produced when making the initial comparisons of their relative sizes. For instance, reasoning about the different sizes between the fractions 1/5 and 1/7, using a pizza or a chocolate bar as a reference, is only experientially accessible to some learners when pre-made manipulatives representing these quantities are readily available, which is less likely to be the case in under-resourced classrooms. In contrast, in the 'Fraction as Measure' sequence, all fractions are equally experientially accessible to students, and the process of their physical creation acts as a means of supporting students' initial reasoning about their relative size.

The third classroom mathematical practice

The third mathematical practice entails using the subunits as units of measure, in their own right. In the instructional activities that are used to support the emergence of this practice, students are asked to use the rods to make paper strips of specific lengths. For instance, they might be asked to make a strip the length of which corresponds to three iterations of the 1/2 rod.

As these activities are implemented, the meaning of fraction inscriptions becomes established, where it is negotiated that the denominator will indicate the subunit that was used in making a measurement, and the numerator, the number of times that the subunit was iterated. This way of interpreting fraction inscriptions coincides with the endpoint of the instructional sequence, described in the previous section.

This mathematical practice becomes established once students develop the ability to readily gauge a fraction inscription as quantifying a length, so that it can be judged to be shorter than, longer than, or as long as the length of one or several iterations of the reference unit. Hence, students will be able to judge the fraction 5/6 as being shorter than one iteration of the reference unit, the fraction 4/3 as being longer, and the fraction 20/10 as being as long as two iterations of the reference unit.

The emergence of this practice is contingent on the specific meanings about length measurement that become established in the prior two practices, including the understandings that any subunits can be created from a reference unit, and that their resulting lengths can be iterated without restriction. Practices that emerge from scenarios that rely on construing fractions as being formed by equal sized parts of a unit whole thus seem lacking. This might be why an additive interpretation of mixed fractions is used extensively in initial fraction instruction for handling quantities that are bigger than 1, thus contradicting the proclaimed aim of fractions becoming an entry into proportional reasoning.

CAPITALIZING ON THE INSTRUCTIONAL DESIGN INNOVATIONS

We conclude this paper addressing how the instructional innovations developed through the described instructional design approach can be recognized as an answer to a problem, by initiatives such as the SANCP, which are concerned with improving the quality of teaching and learning in school systems that predominantly serve low-income children. We use this as an opportunity to address how teachers' learning, innovation travel, and scaling up also shape our design decisions.

In the SANCP the latter two authors work with so-called previously disadvantaged primary schools in the Eastern Cape. In terms of physical and pedagogical resources, these schools still face serious challenges, which shape learner access to mathematical sense making and reasoning. In this context, the third author as the incumbent Chair actively searches for research-informed resources that enable an equity focused agenda of increasing learner agency and participation in mathematics sense making. A range of such resources have been investigated, trialled, and implemented in various professional development projects and the after-school clubs.

In the case of fractions, a wide range of literature points to mathematics teachers and teacher educators grappling with the teaching for decades. The grade 3-6 teachers in SANCP faced such challenges as well. In particular, fraction teaching in SANCP tended to focus almost solely on part-whole models. This limited focus tended to restrict exploring generalized understanding and talk of principles for key concepts such as comparing the relative size of unit fractions and non-unit fractions. Furthermore, because teaching and learning occurs in a language that is not the home language of the majority of SANCP learners, teachers found the adoption of more exploratory discussions of key concepts challenging (see Graven & Robertson, in press, for an example).

The 'Fraction as Measure' instructional sequence was seen as an instructional design innovation that could potentially enable a way forward to the challenges experienced in teaching fractions in our schools. One of its favourable aspects was its focus on iteration and measure versus part-whole. The inclusion of a story with a driving problem needing solving (enabling experientially real engagement) particularly appealed to the South African context where increasingly mathematics educators are using stories to enable talk, discussion, and informal exploratory engagement including in SANCP professional development and parent projects (Graven & Coles, 2017; Graven & Jorgensen, 2018). While teachers often find it hard to shift from more traditional pedagogies where talk is teacher dominated and student talk is limited to providing brief answers to teacher questions, the introduction of a story and a series of activities that learners engage with experientially was seen to provide a promising way forward. It was equally important that the sequence supports teachers with key prompts for the kinds of discussion to be promoted at key learning points (discussed in the three practices above).

Thus, the instructional sequence was seen to provide both key conceptual resources for the development of fraction sense making as well as resources for enabling increased student agency, participation, and discussion in the learning process. As many SANCP learners are learning in English (not their home language), the opportunity—built into the sequence—to talk through one's thinking and reasoning in informal language was particularly important. Having trialled and adapted the sequence to local circumstances, we now look to supporting teachers in understanding and using the sequence in our PD programs (with departmental representation) and in our pre-service teacher courses at our university. This work is but the start of the process of addressing issues of scalability in making this instructional sequence available to others. The sharing of resources captured within the HLT, and the teaching experiences in different classroom settings were important in guiding the local adaptation and trials of 'Fraction as Measure' sequence. They were also essential in providing more equitable access to quality opportunities for addressing seemingly intractable didactical obstacles.

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