Assessing Early Numeracy

Bob Wright

Southern Cross University

New South Wales

Australia

- **Introduction:**

  The compelling case for early numeracy

■ Duncan et al. (2007) estimated the links between three key elements of school readiness,

- school-entry academic
- attention and
- socio-emotional skills

and later school reading and mathematics achievement.

- *Across all 6 studies, the strongest predictors of later achievement are school-entry math, reading, and attention skills. A meta-analysis of the results shows that early math skills have the greatest predictive power, followed by reading and then attention skills. By contrast, measures of socio-emotional behaviors, including internalizing and externalizing problems and social skills, were generally insignificant predictors of later academic performance, even among children with relatively high levels of problem behavior (Duncan et al., 2007).*

- Early mathematics skills are the strongest predictor of later success in mathematics and reading. (Duncan et al., 2007)

Assessing Early Numeracy

- (Tolchinsky, 2003):
- *Numerals seem to be interesting for children to explore.*
- *If written numerals form part of children's constant interactions, then they turn them into a problem space.*
- *They map talk onto the observed graphical shapes and make inferences about functions and functioning.*
- *From the very beginning, a child's notational knowledge should be taken into account and expanded.*
- *Whatever the project in preschool or first grade, written representations should be included without any limit concerning magnitudes.*

- (Mix et al., 2002):
- *Educators should place greater emphasis on the connections between symbols and experiences rather that simply providing experiences. Because the mappings between spoken count words and single-digit numerals are so straightforward, learning them may set the stage for reading in a way that letter-sound pairings cannot, by introducing the idea that a written symbol can stand for a spoken word.*

- **Four books:**

- Wright, R. J., Ellemor-Collins, D., & Tabor, P. (2012). *Developing number knowledge: Assessment, teaching and intervention with 7- to 11-year-olds*. London: Sage Publications.
- Wright, R. J., Martland, J., & Stafford, A. (2006). *Early numeracy: Assessment for teaching and intervention (2nd Ed.)*. London: Sage Publications.
- Wright, R. J., Martland, J., Stafford, A, & Stanger, G. (2006). *Teaching number: Advancing children's skills and strategies, (2nd Ed.)*. London: Sage Publications.
- Wright, R. J., Stanger, G., Stafford, A., & Martland, J. (2006). *Teaching number in the classroom with 4- to 8-year-olds*. London: Sage Publications.

- **Four books:**

- Wright, R. J., Ellemor-Collins, D., & Tabor, P. (2012). *Developing number knowledge: Assessment, teaching and intervention with 7- to 11-year-olds*. London: Sage Publications.

- Wright, R. J., Martland, J., & Stafford, A. (2006). *Early numeracy: Assessment for teaching and intervention (2nd Ed.)*. London: Sage Publications.

- Wright, R. J., Martland, J., Stafford, A, & Stanger, G. (2006). *Teaching number: Advancing children's skills and strategies, (2nd Ed.)*. London: Sage Publications.

- Wright, R. J., Stanger, G., Stafford, A., & Martland, J. (2006). *Teaching number in the classroom with 4- to 8-year-olds*. London: Sage Publications.

■ BIG BIG PICTURE

VIBA — Videotaped Interview-based Assessment

Learning /Assessment framework to profile student knowledge

Elaborated approach to instruction including the use of learning trajectories (learning progressions)

Professional learning to prepare specialist intervention teachers

Professional learning for classroom teachers

Three to five year approach to school change in the approach to teaching early number and whole number arithmetic

■ **Acknowledgements:**

New South Wales Department of Education and Training

United States Mathematics Recovery Council

Catholic Education Office, Melbourne

First People's Center for Education (USA)

Department of Education and Skills – Ireland

Mathematics Recovery Council of UK and Ireland

- **Learning Framework in Number:**

Three Bands:

1. Very early number
2. Early number—three domains
3. Middle number—five domains

- **Early number domains:**

  2A   Early number words & numerals

  2B   Counting

  2C   Early structuring & grouping

■ **Middle number domains:**

3A	Number words & numerals
3B	Structuring to 10 & to 20
3C	Conceptual place value
3D	Addition & subtraction to 100
3E	Multiplication Strategies
3F	Multiplication Basic Facts

- **Focus on four domains:**

  Number words & numerals

  Counting

  Structuring numbers 1 to 20

  Conceptual place value

Four Domains of Early Numeracy

Number Words and Numerals

Counting

Structuring Numbers 1 to 20

Conceptual Place Value

Four Domains of Early Numeracy

Number Words and Numerals

Counting

Structuring Numbers 1 to 20

Conceptual Place Value

Assessing facility with Forward Number Word Sequences (FNWSs):

Saying FNWSs

Start counting from one and I'll tell you when to stop. (Stop at 32.)
Start counting from 47. (Stop at 63.)

Saying the number word after

What comes after 5? What comes after 9? etc.

Difficulties with Forward Number Word Sequences (FNWSs):

Can say from 1 to beyond 10 but can't say number word after in the range 1 to 10.

Can say from 1 to beyond 10 but drops back to say number word after in the range 1 to 10.

Confuses 'ty' and 'teen'.

Difficulties with Forward Number Word Sequences (FNWSs):

Can't go past 29

Proceeds to an incorrect decade

57, 58, 59, 30, 31, 32

Errors after 109: 106, 107, 108, 109, 200

Assessing facility with Backward Number Word Sequences (BNWSs):

Saying BNWSs

Count back from 5. Count back from 10. etc.

Saying the number word before

What comes before 3? What comes before 7? etc.

Difficulties with Backward Number Word Sequences (FNWSs):

45, 44, 43, 42, 41, 30, 39, 38 …(Wrong decuple)

45, 44, 43, 42, 41, 39, 38 …(Omits decuple)

72, 71, 70, 69, 68, 67, 65, 64, …(Omits double digit)

23, 22, 21, 20, 90, 80, 70, 50, …(confuses 'teen' & 'ty')

Decuples: 10, 20, 30, 40, 50 etc.

A **decuple** is a multiple of 10. Versus **decade.**

Assessing facility with numerals:

Numeral Identification

Also known as naming or reading numerals

Could also use tasks involving numeral recognition:

Place the numerals from 11 to 20 on the desk in pseudo-random order:

Which of these is the number 12?

Difficulties with Numerals:

When identifying a numeral greater than 20, students incorrectly apply their knowledge of how to read a teen. So they read right to left and thus indentify 'mn' as 'nm', eg '27' as 'seventy-two'.

When identifying 2-digit numerals, students process one digit at a time.

When writing a teen numeral, students will sometimes write the right-hand digit first.

Difficulties with Numerals:

Identifies '12' as 'twenty' or 'twenty-one'.

Learning to identify '11', is much easier than '12'.

We see a few students in the second year of school who have problems identifying 1-digit numerals.

Four Domains of Early Numeracy

Number Words and Numerals

Counting

Structuring Numbers 1 to 20

Conceptual Place Value

Assessing facility with counting:

"Counting":

involves coordinating a number word with an "item", where "item" refers to a mental construct;

is distinguished from saying a number word sequence.

Assessing facility with counting:

"Counting":

Initial strategies for adding and subtracting arise from counting.

Counting is not regarded as a topic distinct from early addition and subtraction.

Assessing facility with counting:

Consider this task:

Place nine blue counters under a screen.

Under this screen there are nine blue counters.

Place five red counters under a screen.

Under this screen there are five red counters.
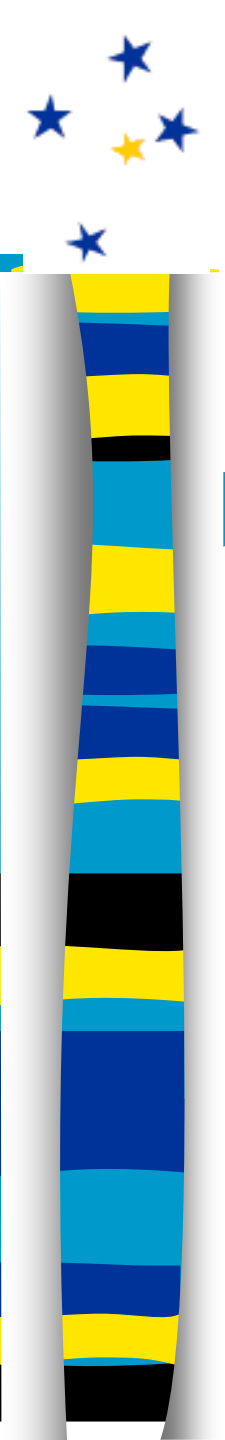
Assessing facility with counting:

Five levels of responding:

0 – Emergent

1 – Perceptual

2 – Figurative

3 – Counting-on

4 – Structuring (Part/part/whole)

Assessing facility with counting:

Level 0 — An emergent counter:

Can't solve when the two collections are screened.

Unscreen the two collections.

Does not count the counters correctly.

Errors in coordinating the number words with the counters or errors with the number word sequence.

Assessing facility with counting:

Level 1 — A perceptual counter:

Can't solve when the two collections are screened.

Unscreen the two collections.

Counts the counters correctly from "one" to "fourteen".

"Perceptual" because, in order to solve the task, the student needs to see the counters.

Assessing facility with counting:

Level 2 — A figurative counter:

Counts from "one" to "nine" to count the first collection (screened) and then continues their count from "ten" to "fourteen" to count the second collection (screened).

This strategy includes activity that, from an adult's perspective, is redundant, that is, the count from one to nine.

From the student's perspective this is necessary in order for "nine" to stand for a count.

The student is mentally visualizing (imaging) the counters.

Assessing facility with counting:

Level 3 — Counting-on:

Counts on from nine: "Nine …, ten, eleven, twelve, thirteen, fourteen!".

Various ways to keep track of the five counts:

Visualize a spatial pattern (e.g. a dice pattern for five)

Use a motor pattern (e.g. raises five fingers or makes five movements)

Use a double count (e.g. "ten is one, eleven is two, twelve is three, thirteen is four, fourteen is five")

Use an auralized pattern for five: bom, bom – bom, bom – bom

Assessing facility with counting:

Level 4 — Structuring or Part/part/whole:

"Nine and one are ten. Ten and four are fourteen!".

Two tasks to assess if a student is at least Level 1 — Perceptual:

Place out 15 counters. *How many counters are there?*

Place out a pile of about 30 counters. *Get me 15 counters from this pile.*

Tasks with screened collections:

Addition task with two screened collections:

Nine red under here, four blue under here. How many altogether?

Subtraction tasks

Missing addend:

Nine under here and some more to make 13. How many more?

Take away:

13 under here. I take away four. How many left?

Missing subtrahend.

13 under here and I take away some. Now there are nine. How many did I take away?

Tasks with screened collections:

Addition task has the purpose of evoking *Count-up-from*.
Missing addend task has the purpose of evoking *Count-up-to*.
Take away has the purpose of evoking *Count-down-from*.
Missing subtrahend has the purpose of evoking *Count-down-to*.

These are the four advanced counting-by-ones
strategies.

Four Domains of Early Numeracy

Number Words and Numerals

Counting

Structuring Numbers 1 to 20

Conceptual Place Value

Structuring numbers 1 to 20:

Learning to add and subtract in the range 1 to 20 with little or no counting by ones. Thus, using five and ten as reference points and using doubles.

Learning to mentally "structure" numbers:

Regard seven as five and two more.

Know seven and three more make ten.

Regard seven as made up of three and three, and one more.

Regard seventeen as made up of ten, five and two.

Know that seventeen is one less than double nine.

.

Assessing Structuring Numbers 1 to 20:

Key combinations and partitions in the range 1 to 10:

Partitions of five — *Two and what make five?*

$$0+5, 1+4, 2+3, 3+2, 4+1, 5+0$$

Small doubles — $1+1, 2+2, \dots 5+5$

Five Pluses — $5+1, 5+2, \dots 5+5$

Partitions of 10 — Eight and what make 10?

$$0+10, 1+9, 2+8, \dots 10+0$$

.

Assessing Structuring Numbers 1 to 20:

Key combinations and partitions in the range 1 to 20:

Ten Pluses — 10+1, 10+2, ... 10+10

Big doubles — 6+6, 7+7, ... 10+10

Partitions of 20 — 12 and what make 20?

0+20, 1+19, ... 20+0

Assessing Structuring Numbers 1 to 20:

All combinations:

Categorize facts into a hierarchy of four levels:

Part + Part = Whole   Whole – Part = Part

Level 1: Parts ≤ 5

Level 2: Whole ≤ 10

Level 3: Parts ≤ 10

Level 4: Whole ≤ 20

Levels

1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

Key

Addends ≤ 5

Totals ≤ 10

Addends ≤ 10

Doubles

Five-plus

Ten-plus

Partitions of 10

Assessing Structuring Numbers 1 to 20:

All combinations:

Level 1: Parts ≤ 5

Examples of additions in Level 1:

4 + 2, 5 + 1, 3 + 3 etc.

Examples of subtractions in Level 1:

7 – 4, 9 – 5, 6 – 3 etc.

Assessing Structuring Numbers 1 to 20:

All combinations:

Level 2: Whole ≤ 10

Level 2 includes all of Level 1

Examples of additions in Level 2 but not in Level 1:
6 + 3, 7 + 2, 2 + 8 etc.

Examples of subtractions in Level 2 but not in Level 1:
9 − 2, 7 − 1, 8 − 2, etc.

Assessing Structuring Numbers 1 to 20:

All combinations:

Level 3: Parts ≤ 10

Level 3 includes all of Levels 1 and 2.

Examples of additions in Level 3 but not in Levels 1 or 2:
6 + 8, 7 + 9, 3 + 8, 9 + 4, etc.

Examples of subtractions in Level 3 but not in Levels 1 or 2:
12–4, 14–6, 15–9, 11–7, etc.
Similarly for Level 4
Levels 1, 2 and 3 constitute the basic facts.

Four Domains of Early Numeracy

Number Words and Numerals

Counting

Structuring Numbers 1 to 20

Conceptual Place Value

Conceptual Place Value:

Learning to increment and decrement numbers flexibly by ones, tens, hundreds, thousands etc.

The traditional instructional topic of *Place Value* was invented in the 1970s to support learning of the vertical algorithms for adding, subtracting etc.

Adoption of *Place Value* as an instructional topic was mainly in Anglophone countries.
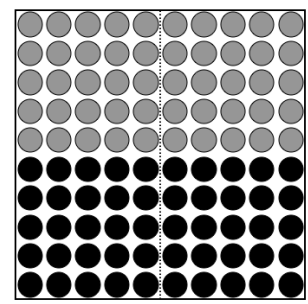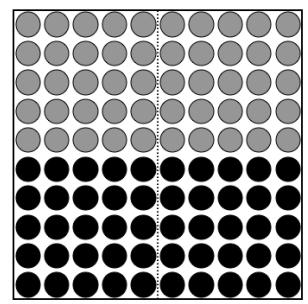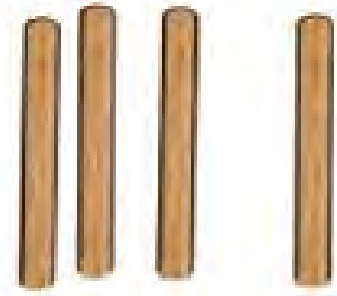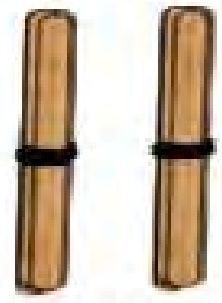
Assessing Conceptual Place Value:

Use base-ten material to constitute quantity such as:

bundling sticks

material consisting of dots organized into ones, tens, 100s etc.
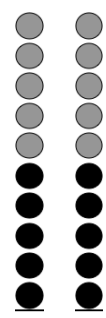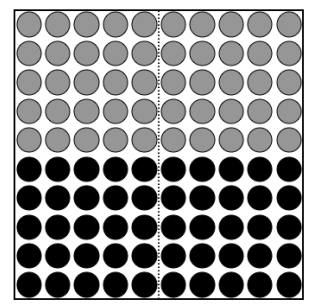
Assessing Conceptual Place Value:

Assessment tasks:

Use base-ten material and a large screen:

- Increment on the decuple
- Increment off the decuple
- Decrement on the decuple
- Decrement off the decuple

A *decuple* is a multiple of ten e.g. 10, 20, 30 etc.

More advanced:

Increment through 100, 200, 1000 etc.

Decrement through 100, 200, 1000 etc.

Four Domains of Early Numeracy

Number Words and Numerals

Counting

Structuring Numbers 1 to 20

Conceptual Place Value

Profiling Students' Knowledge:

Use models that set out levels of students' knowledge on the domains.

Four models for early number learning:

Early arithmetical strategies

Facility with FNWSs

Facility with BNWSs

Numeral Identification

"Facility" is used in the sense of skilfulness or fluency.

Early Arithmetical Strategies:

0. Emergent Counting

1. Perceptual Counting

2. Figurative Counting

3. Counting on and back

4. Structuring (Part/part/whole)

Forward Number Word Sequence (FNWS):

    0. Emergent FNWS

    1. Initial FNWS up to 'ten'.

    2. Intermediate FNWS up to 'ten'.

    3. Facile FNWS up to 'ten'.

    4. Facile FNWS up to 'thirty'.

    5. Facile FNWS up to 'one hundred'.

Backward Number Word Sequence (BNWS):

    0. Emergent BNWS

    1. Initial BNWS up to 'ten'.

    2. Intermediate BNWS up to 'ten'.

    3. Facile BNWS up to 'ten'.

    4. Facile BNWS up to 'thirty'.

    5. Facile BNWS up to 'one hundred'.

Numeral Identification (NID):

0. Emergent numeral identification

1. Facile with numerals to '10'

2. Facile with numerals to '20'

3. Facile with numerals to '100'

4. Facile with numerals to '1 000'

Examples of Students' Profiles

Domains	Students				
	A	**B**	**C**	**D**	**E**
SEAL	0	1	2	3	4
FNWS	1	3	4	5	5
BNWS	0	1	3	4	5
Num. Id.	0	1	2	3	4

Assessment – Student in the first year of school:

1. FNWSs
2. Number Word After
3. Numeral Identification
4. BNWSs
5. Number Word Before

Assessment – Student in the first year of school:

6. Making finger patterns for 2 to 5
7. Ascribing number to flashed dice patterns:

    2 to 6

8. "Get Me" task
9. Additive tasks with two screened collections:

    5 and 3   9 and 4

SHOW VIDEO OF AN ASSESSMENT INTERVIEW