ABSTRACT
Local, regional and international research consistently points to a crisis in primary mathematics learning whereby grade 4 learners are already almost two grades behind grade level expectations. Research across local contexts points to a failure of progress for the majority of learners beyond concrete one-to-one counting methods, even well into the intermediate phase. Yet systemic teacher support largely ignores this learning gap and the fact that since mathematics is a highly hierarchical subject learner progress is constrained if earlier grade concepts are not established. This paper shares the empirical findings of a 5 year research and development project that focused on supporting grade 3 and 4 numeracy and mathematics teachers in developing foundational knowledge of learners (not achieved in earlier grades), through focusing active participation, connection of concepts and sense making, in order to support learner progress. The paper points to both successes and challenges of improving numeracy performance of learners across a wide range of schools.

Introduction and context
South Africa is a country of extremes with one of the largest gaps in the world between rich and poor (Wilkinson & Picket, 2009) and related to this has one of the largest gaps in mathematical performance between learners in wealthier and poorer quintiles of schools (Reddy et al., 2015). Redressing inequality has been a post apartheid priority, emphasized in the rhetoric of educational policy. Yet implementing a national curriculum, without taking into account the extreme inequality of the contexts in which implementation occurs, will not redress inequality and as South African learner performance across the Trends in International Mathematics and Science Study (TIMSS) 1999 and TIMSS 2003 shows it may even exacerbate inequality in performance of wealthier and poorer learners (Reddy, 2006).

Redressing this inequality and the crisis of extreme poor performance of learners previously disadvantaged under apartheid is a central aim of the South African Numeracy Chair Project (SANCP), which began in 2011 at Rhodes University. The project merges research with development work and is responsible for both improving the results of learner performance in the schools it partners with as well as searching for sustainable ways to address the challenges of primary mathematics education. Through partnering with local schools in the broader Grahamstown area the project runs a teacher development program, after school mathematics clubs, a homework drive, and a range of family mathematics events. All work is aimed at addressing the gaps in foundational knowledge of learners in these schools as well as creating strengthened mathematics learning dispositions that foreground sense making, steady effort, resilience, confidence and a love of mathematical activity and engagement. The SANCP, for which I am the Chair, is currently in its 5th year of work.

The teacher development program is called the Numeracy Inquiry Community of Leader Educators (NICLE). The program focuses on grade 3 numeracy and grade 4 mathematics teachers as these grades involve the critical transition from Foundation Phase to Intermediate Phase where increasingly abstract thinking and ways of working are required. Additionally since research has shown that by Grade 4 the majority of our learners are already two grades behind (Spaull & Kotze, 2015) and that progression from 1-1 concrete methods of working is a key challenge (e.g. Schollar, 2008) early intervention was considered a priority. Each year since 2011 at least 39 teachers from twelve schools have participated in NICLE which is functions as a community of practice meeting regularly over a long period of time to participate in and engage with a range of learning activities focused on improving mathematics teaching. The SANCP team is responsible for coordinating NICLE activities and ensuring teachers have access to
high quality resources that stimulate learning. These resources include for example: opportunities for active mathematical participation and engagement about general and local pedagogical challenges within a supportive community; access to a range of local and international mathematics education ‘experts’ (or in Lave and Wenger’s (1991) term ‘masters’) who bring high quality learning activities to stimulate engagement, as well as research informed teaching resources for use in class. Several NICLE teachers have presented sessions or parts of sessions and teachers contribute the critical resource of grounded local experience essential for engagement. While the vast majority of teachers teach grade 3 and 4 there are also several teachers of other grades ranging from grade 0 – 6. Most NICLE teachers have been participating since 2011 although each year some new teachers join and some leave due to changes in schools, grades, retirement or other factors.

Over the past five years our project team has gathered learner performance data across grade 3 and 4 learners in ten core participating schools twice a year in the form of two data gathering instruments adapted for use from other international projects. Additionally the project has gathered a range of qualitative data from teacher questionnaires, interviews, video recordings of lessons, learner books, learner task based interviews, learner dispositional interviews and so forth. In this paper I report on the annual performance data of over a thousand grade 4 learners from two ‘learner performance’ instruments across participating SANCP schools. The modest average improvements in learner results over time, with strong improvements in some schools, point to relative success in the strategy of focusing on redressing foundational knowledge of earlier grades in the Intermediate Phase. However, several factors challenge the opportunity for greater impact on learner performance across schools. These challenges are discussed in the concluding section with discussion of possible ways forward.

**Conceptual framing of the research and development project**

While the design of NICLE and research of teacher learning is based on Lave and Wenger’s (1991) and Wenger’s (1998) work on learning within communities of practice, a socio constructivist perspective guides the SANCP research and development work on student mathematics learning. Thus Wenger’s four component (i.e. practice, meaning, community and identity) theory of learning as involving learning as doing, experience, becoming, and belonging has guided NICLE activities, practices and its ethos, as well as providing a framework for researching the nature of teacher learning within NICLE (see for example, Pausigere & Graven, 2013; Pausigere & Graven, 2014). This perspective foregrounds the interrelationship of the four components in communities of practice as follows:

- a community of practice is a living context that can give newcomers access to competence and also invite a personal experience of engagement by which to incorporate that competence into an identity of participation… a well functioning community of practice is a good context to explore radically new insights without becoming fools or stuck in some dead end. A history of mutual engagement around a joint enterprise is an ideal context for this kind of leading-edge learning, which requires a strong bond of communal competence along with a deep respect for the particularity of experience. When these conditions are in place, communities of practice are a privileged locus for the creation of knowledge (Wenger, 1998, p.214).

On the other hand research and development work of student mathematics learning is based on a socio constructivist perspective of mathematics learning drawing primarily on the work of Steffe & Cobb (1988) and Wright and colleagues (Wright, Martland, Stafford & Stanger, 2006; Wright, Martland & Stafford, 2006, Wright, 2013). In this respect, mathematics is viewed as a progressive subject whereby learning requires learners to actively construct knowledge during participation in mathematical activity and these concepts progressively build on earlier concepts. Mathematical proficiency is defined by drawing on Kilpatrick, Swafford and Findell’s (2001) five-stranded definition of mathematical proficiency in which the rope metaphor captures the way in which the five interrelated strands of Conceptual Understanding,
Graven: Going back in order to go forward in mathematics learning

Procedural Fluency, Strategic Competence, Adaptive Reasoning and Productive Disposition work together to enable mathematical proficiency. The strands of proficiency are defined as follows:

- **conceptual understanding**: comprehension of mathematical concepts, operations, and relations
- **procedural fluency**: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **strategic competence**: ability to formulate, represent, and solve mathematical problems
- **adaptive reasoning**: capacity for logical thought, reflection, explanation, and justification
- **productive disposition**: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy (Kilpatrick et al., 2001, p. 5)

NICLE sessions emphasized mathematical progression both assisting teachers in recognizing the level at which learners are at as well as developing and using activities and resources that push for progression. A guiding principle was that teaching should be ‘focused just beyond the ‘cutting edge’ of the child’s current knowledge’ (Wright, Martland and Stafford, 2006, 4). Thus NICLE sessions drew on progressive frameworks such as Wright et al.’s Learning Framework in Number and research informed structured teaching resources for enabling mathematical recovery. Additionally, a connectionist approach to teaching and learning was emphasized drawing particularly on the effective teachers of numeracy study (Askew, Brown, Rhodes, Johnson, & William, 1997). One of the learner assessments used in the Askew et al (1997) study was adapted, with permission, for our second learner assessment.

Due to the interrelated nature of the strands mathematical proficiency assessments should address all these aspects. While mathematical activity involves all these strands working together certain assessment items enable the assessment of specific strands more than others. With this in mind, in order to assess the progression of mathematical proficiency (or lack thereof) of learners in our schools we conducted three assessments, which collectively provided a rich picture of learner mathematical proficiency and information as to the level and stages of numeracy learning learners were at.

The first was a four-operations basic assessment administered to all grade 3 and 4 learners in all core participating schools. This primarily assessed procedural fluency but through analysis of learner methods provided some evidence of levels of conceptual understanding. The second was an orally administered but written response assessment (administered to all grade 4 learners) which included a range of questions, which clearly required adaptive reasoning and strategic competence. The third was an orally administered but written response learner questionnaire (administered to all grade 3 and 4 learners) that focused on gathering data explicitly on learner mathematical dispositions. So for example learners were told Sam is really good at maths and were asked to describe Sam in the mathematics class by completing the sentence, Sam is…. (see Graven, 2012; Graven & Heyd-Metzuyanim, 2014). For the purposes of this paper I focus on the data gathered in the first two instruments, focused on the first four strands of proficiency, across grade 4 learners from 2011 – 2014 in nine of our ten core participating schools (one school was a junior primary school and thus did not have a grade 4 class in 2011). I therefore discuss these instruments in more detail below.

**Methodology**

As indicated above the project has adopted a mixed method approach to gathering data involving gathering of both qualitative and quantitative data. Qualitative data gathering techniques included case study learner and teacher interviews, learner task based interviews, lesson observations, teacher questionnaires, among others. The range of mathematical assessments used across grade 3 and 4 learners in the participating schools provided largely quantitative data on learner performance. Selection of schools invited for participation in the research and development project was done in consultation with the district officials in the Department of Education based on the SANC project brief that we work with schools serving mostly learners from the poorer end of the socio-economic spectrum. Our schools
included township, ex model C (previously for people classified as ‘white’ under apartheid), ex HoR (previously schools designated for people classified as ‘coloured’ under apartheid) schools and one farm school. The majority of schools are non fee-paying schools and the language of learning and teaching in the schools are English, Afrikaans and isi-Xhosa (up to grade 4). Permission for the research was obtained from the Eastern Cape Department of Education, principals, teachers and parents. Teacher and school participation was voluntary and thus while in some schools all grade 3 and 4 teachers participated regularly across the five years in other schools only one or two teachers participated and in some cases teachers from other grades elected to participate. Annual learner performance data was gathered for learners in ten core participating schools which were defined as those schools who had at least one grade 3 and 4 regularly participating teacher in NICLE in the first year. [i.e. the two schools who left the project in the first year and the schools that joined NICLE after 2011 were not considered core schools and annual data was thus not collected for these schools]. For several of the core schools, as can be seen in Table 3 below, from 2012 to 2014 not all grade 3 and 4 teachers in the core schools continued to participate in NICLE after the first year. In addition to that growing numbers of learners in some NICLE schools, particularly in grade 4, meant that many of the grade 4 learners had not been in NICLE grade 3 classes the previous year. Such factors reduced the effect of NICLE participation on the grade 4 results. Since the data included in this paper is learner performance data I focus here on describing those instruments and the way in which they were administered.

The first instrument administered to learners was a four operations assessment. This instrument was administered in April/May 2011 each year. This assessment was based on an instrument used by Brombacher and Associates across several African countries (with permission). This written assessment includes five items on each of the four operations ranging from simple single digit sums to two and three digit sums that become progressively harder. For example the five sums for addition are: 3+4 = ; 8+6 = ; 23+18 = ; 55+67= ; and 104 + 97 = .

The second instrument, administered annually in August to all grade 4 learners, was adapted from the year 3 learner instrument used in the *Effective Teachers Of Numeracy* study conducted in England in the nineties (Askew, Brown, Rhodes, Johnson, & William, 1997). This instrument included a range of items that assessed procedural fluency, conceptual understanding, strategic competence and adaptive reasoning.

For the purposes of this paper I report on the results of learner performance of those items in this assessment that particularly assessed the strands of conceptual understanding, strategic competence and adaptive reasoning. The data for these assessment items are given in Table 2 in the findings section below. In addition I report on the four operations assessment results, which primarily assessed procedural fluency. The data for this instrument is given in Table 1 below.

The items that focused on conceptual understanding included locating the numbers 38 and 262, and placing 289 on number lines; looking at pictures of boxes of cupcakes and bags of apples with some additional apples and cupcakes outside and being asked to quickly calculate how many in each picture after being told there are 5 cupcakes in a box and ten apples in a bag. (The picture is only shown for a short amount of time so one to one counting is not possible). The item that focused on adaptive reasoning provided learners with a number fact sum and asked learner to then calculate another sum using that fact to help them. So for example they were told 86 + 57 = 143 and asked to calculate 86 + 56; 57 + 86; 860 + 570 and 143-86. Several word problems were considered top provide focus on strategic competence. These included for example ‘18 people are on a bus. At the first stop 8 people got on and 3 got off. How many are on the bus now?’ The instrument was translated into Afrikaans and isiXhosa as these languages are widely spoken in the participating schools.
Findings

Results from the four operations assessment conducted across grade 3 and 4 learners allowed tracking of the progress of a cohort of learners from grade 3 to grade 4. Here however I focus on the results of nine of the ten core participating school’s grade 4 results each year. (Recall one core school was a junior primary school and had no grade 4 class in 2011). A limitation of this is that each year there is a new cohort of grade 4 learners in each school (except for a small number of learners who repeat the grade) and the number of learners each year thus changes (in most cases upwards). Participating schools have simply been numbered (without identification of the type of school) in order to maintain anonymity. Schools are listed in the same order for all tables. The tables are presented together so that performance across the assessments can be analysed against other contextual information included in tables 3 and 4. Table 3 provides the number of grade 3 and 4 teachers who participated in NICLE from 2011-2014 out of the total possible number of grade 3 and 4 teachers at each school. It is important to note that learner performance data was collected for all grade 3 and 4 learners in all classes in core participating schools irrespective of whether the class teacher participated in NICLE or not. Table 4 provides the number of Grade 4 learners in each school. Three different groups of schools are noted from table 3 in terms of regular and long term participation in NICLE. That is it shows that for some NICLE schools:

- all grade 3 and 4 teachers attended in all years (i.e. school 1, 3, 6, 7, & 9);
- only some of the grade 3 but all grade 4 teachers attended in all years (i.e. schools 2 and 8)
- there were no grade 3 or 4 participating teachers in NICLE except for the first year (i.e. school 4 and 5).

Table 1: Grade 4 learner performance on 4 operations in May 2011 and 2014 (focused on PF)

<table>
<thead>
<tr>
<th>School</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011 ave %</td>
<td>46</td>
<td>40</td>
<td>79</td>
<td>43</td>
<td>45</td>
<td>35</td>
<td>79</td>
<td>43</td>
<td>41</td>
</tr>
<tr>
<td>2014 ave %</td>
<td>52</td>
<td>56</td>
<td>77</td>
<td>42</td>
<td>32</td>
<td>40</td>
<td>80</td>
<td>41</td>
<td>53</td>
</tr>
<tr>
<td>% improvement/decrease</td>
<td>6%</td>
<td>16%</td>
<td>-2%</td>
<td>-1%</td>
<td>-13%</td>
<td>5%</td>
<td>1%</td>
<td>-2%</td>
<td>8%</td>
</tr>
</tbody>
</table>

Table 1 indicates general improvements in learner performance on this instrument in most schools except for schools 3, 4 and 5 although for school 4 the 2011 were results were noted by the teacher as ‘unusually’ high as the result of both a very small number of grade 4 learners and a particularly strong group that year.

Table 2: Grade 4 learner performance on items of CU/AR & SC in August 2011 and 2014

<table>
<thead>
<tr>
<th>School</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011 ave %</td>
<td>23</td>
<td>22</td>
<td>55</td>
<td>31</td>
<td>31</td>
<td>22</td>
<td>76</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>2014 ave %</td>
<td>64</td>
<td>52</td>
<td>78</td>
<td>47</td>
<td>38</td>
<td>28</td>
<td>78</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>% improvement</td>
<td>41</td>
<td>30</td>
<td>23</td>
<td>16</td>
<td>7</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2 similarly shows improvement across most schools except for schools 8 and 9 where results remained unchanged from 2011 to 2014. Table 3 and table 4 below provide some further information that provides possible explanations for the strong improvement in some schools and little improvement of declining performance in others.
Table 3: number of participating teachers in NICLE per grade (of total possible teachers) who attended from 2011-2014

<table>
<thead>
<tr>
<th>School</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gr 3</td>
<td>3/3</td>
<td>0/1</td>
<td>1/1</td>
<td>0/1</td>
<td>0/1</td>
<td>3/3</td>
<td>1*/1</td>
<td>1/2</td>
<td>1*/1</td>
</tr>
<tr>
<td>Gr 4</td>
<td>3/3</td>
<td>1/1</td>
<td>1/1</td>
<td>0/1</td>
<td>0/1</td>
<td>1/1</td>
<td>1/1</td>
<td>1/1</td>
<td>1/1</td>
</tr>
</tbody>
</table>
* In these cases there was a participating grade 3 teacher each year but not the same teacher as different teachers taught the grade in subsequent years. Thus the attending grade 3 teachers from school 7 and 9 changed over the years due to teachers moving schools or shifting the grades they taught in the school over the years.

It is important to note that in school 4, 6 and 8 the grade 3 numeracy teachers in a school is higher than the number of grade 4 mathematics teachers since in these schools the grade 4 mathematics teachers teach only mathematics rather than teaching across the curriculum as is the case with all Foundation Phase numeracy teachers in the study.

Table 4: Number of learners in grade 4 per school in 2011 and 2014

<table>
<thead>
<tr>
<th>School</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>82</td>
<td>40</td>
<td>7</td>
<td>40</td>
<td>29</td>
<td>75</td>
<td>24</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>2014</td>
<td>96</td>
<td>33</td>
<td>10</td>
<td>53</td>
<td>39</td>
<td>77</td>
<td>19</td>
<td>52</td>
<td>45</td>
</tr>
<tr>
<td>% change</td>
<td>17%</td>
<td>-1.8%</td>
<td>42.9%</td>
<td>32.5%</td>
<td>34.5%</td>
<td>0.03%</td>
<td>-2.1%</td>
<td>36.8%</td>
<td>32.4%</td>
</tr>
</tbody>
</table>

Table 4 shows that while almost half the schools have experienced only small changes in class sizes over time (i.e. schools 1,2,6 and 7) others have experiences large growth in class size.

Discussion of findings

The greatest visible improvement is for School 1 in the CU, SC and AR assessment items (a 41% improvement). This school had all six grade 3 and 4 teachers as regular participants without any changes in teachers or the grades they taught from 2011. The most notable drop in results for the 4 operations assessment (-13%) was for School 5 even while showing some improvement (7%) in assessment items focused on conceptual understanding, adaptive reasoning and strategic competence. For this school the grade 3 teacher participated in 2011 while the grade 4 teacher was the principal and was unable to attend. However since the grade 2 and grade 6 teacher were regular participants across 2011 to date the school remained a core participating school.

Noticeably average improvement across the nine schools in the latter three strands is far greater (13.9%) than the average improvement in procedural fluency (2.4%) possibly as a result of a focus on these aspects within NICLE. Also noticeable is the differential improvement across schools. That is, for some schools there is strong improvement across both assessments (e.g. school 1 and 2) while for others there is a decrease in performance in the four operations assessment (i.e. school 4, 5 and 8). The differential improvement is not surprising given the differential participation in NICLE as indicated by the three sets of schools above.

The average performance improvement/decrease for these three sets of schools for the 4 operations assessment is 4.4%; 7% and -7% respectively. On the other hand the performance improvement on these three categories of schools for the assessment items on CU, SC and AR are: 14.4%; 15% and 11.5%.

While the sample of schools is too small for any definitive argument as to the advantage of buy-in by all focal grade teachers, looking across tables 1 to 3, it seems reasonable to assume that for the learners in the first two sets of schools NICLE participation had some positive effect on learner performance over
Graven: Going back in order to go forward in mathematics learning

time. The drop in results of schools 4, 5 and 8 on the 4 operations assessment is of concern and points to
the need for intervention in grade 4 at these schools. However participation in NICLE is entirely voluntary
and this limits the possibility of influencing the learning of the grade 3 and 4 learners in this school. It
points however to a need, in future project work, to only include core schools in which all teachers in the
particular targeted grades attend. The difficulty here is that if some teachers are unable or unwilling to
attend then other teachers in that school that wish to participate will be disadvantaged.

While buy-in of all focal grade teachers in schools is likely to improve the impact of long term teacher
development projects such as NICLE there are several other factors, revealed through qualitative
interviews with teachers and principals that hamper the influence of teacher development on learner
performance. While it is beyond the scope of this paper to focus on all influencing factors, of which
conditions of poverty is a key challenge; here I highlight just three issues that have arisen strongly in
teacher interviews.

The first is that student and teacher populations are not stable in some schools and there is increasing
pressure for schools (especially the ex HOR NICLE schools) to accept increasing numbers of learners
from neighboring township schools. Many of these learners move to these schools in grade 4 (i.e. the
start of the intermediate phase) having completed their foundation phase at school in isi-Xhosa. In such
cases learners have had little exposure to the languages of learning and teaching (i.e. English and/or
Afrikaans) in these schools. So for example schools 4, 5, 8 and 9 all show significant increases in number
of grade 4 learners (all greater than 30%) without an increase in the number of teachers. Of interest these
four schools have the lowest improvements across both instruments with schools 4, 5 and 8 showing a
decrease in performance in the four operations assessment. [School 3 also has a high percentage
increase although the increase is only 3 learners as this farm school has multi-grade classes.]

Additionally several NICLE principals have reported difficulties with the process of replacing teachers
when they leave due to bureaucratic inefficiencies and in some cases this led to classes of mathematics
learners without a teacher for several months at a time. Teachers explained that the increasing numbers
of learners in classrooms result in several challenges including addressing the language and conceptual
backlog of learners from other schools and the challenges of managing large classes (See Robertson &
Graven, in press).

The second is that across schools there are several learners per grade who are considered, or have been
formally assessed, as Learners with Special Education Needs (LSEN). So for example, 10 out of 36
learners in the Grade 3 class in school 9 have been identified and assessed as LSEN. While NICLE
focused on supporting teachers to recover foundational understanding in learners from previous grades
as assessments indicated that in most of our schools the majority of learners required such recovery, it
did not support teachers in working specifically with special needs learners who cannot read or write,
among other challenges. Such learners need targeted and regular support, in addition to usual classroom
tuition, in order to support them in benefitting from the work done in class. NICLE teachers have reported
that such learners continue to perform extremely poorly on assessments such as NICLE assessments
and the ANAs due to reading, language and writing challenges and that this has a negative impact on the
average learner performance improvement across assessments.

The third is that the system of ‘teacher support’ works against teachers focusing on recovery of
foundational concepts expected to be in place from earlier grades. Thus the curriculum assessment
standards and the weekly schemes of work provided work against teachers being able to be locally
responsive to the needs of learners. Thus elsewhere (Graven, submitted), drawing on interview data with
teachers, I have argued that the system tends to ignore the reality of the enormous learning gaps of the
majority of our learners in poorer quintile schools face. This is contradictory as one of the reasons given
for participating in national, regional and international comparative studies is so as to identify what the
problems are and to address these problems (see DBE, 2014). Yet departmental workshops and school
visits that check compliance of teaching according to the provided schemes of work tend to work against encouraging teachers to revisit work from earlier grades where necessary. Given that extreme learning gaps of South African learners in poorer schools have been identified across analyses of a range of mathematics assessments including regional SACMEQ assessments (Spaull 2013), international TIMSS assessments (Reddy, 2006; Reddy et al. 2015) and our national Annual National Assessments (DBE, 2014) this failure to support teachers in redressing these gaps is highly problematic.

On a final note, a key limitation of the results for the 4 operations instrument is that the way results are recorded, while showing improvements, fails to show shifts in learner methods towards more efficient and conceptual strategies. Such shifts are not always evident in improved results in terms of the accuracy of learner answers. In the 2014 assessments learners tended to finish much earlier than the required time (we believe due to using increasingly efficient strategies), which was not the case in 2011 where a strong predominance of concrete methods of working involving tally lines was noted. However these shifts away from concrete 1-1 working was not always evident as an improvement in performance on the 4 operations instrument as learner responses are captured as right or wrong. Thus in some cases we noted that learners were more prone to making errors when flexibly breaking up calculations than with 1-1 methods they had mastered. While concrete 1-1 methods are slow and indicate a problematically low level of mathematical reasoning, I have noted in my working with learners that they tend to trust these methods for ensuring accuracy (I have noted high levels of accuracy even with much higher number ranges). Below is an example of how a learners shift from 1-1 working towards a place value based strategy leads can lead to a drop in performance on subtraction items rather than an increase. Such errors were widely noted in the 2014 scripts for learners who used the breaking up method for subtraction problems. Some of the problem here is the extension of the breaking up method used in addition to subtraction but without conceptually considering what for example 3-8 means or how one might work with this:

<table>
<thead>
<tr>
<th>2.3</th>
<th>23 - 18 = 5</th>
</tr>
</thead>
</table>

Figure 1: An example of concrete methods of working widely noted in 2011 scripts

<table>
<thead>
<tr>
<th>2.3</th>
<th>23 - 18 = 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 - 10 = 10</td>
<td></td>
</tr>
<tr>
<td>3 - 8 = 5</td>
<td></td>
</tr>
<tr>
<td>10 + 5 = 15</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: An example of ‘breaking up’ strategy for subtraction widely noted in 2014 scripts

The orally administered Askew et al. (1997) assessment discussed above was thus useful in supplementing data gathered from the four operations assessment as several of these questions, did not allow for one to one concrete methods of working and assessed across the strands of proficiency more explicitly.
Concluding remarks
In this paper I have argued that we need to focus on addressing the enormous learning gaps in foundational knowledge of learners in the majority of our poorer schools. A national strategy is needed for providing support for teachers in order to enable this. Such support requires both resources for remediation and teacher development opportunities. Since mathematics is a hierarchical subject that requires learners to actively construct knowledge on existing foundations, ignoring these gaps is likely to entrench if not exacerbate the existing performance gaps in mathematics between the rich and poor. In this paper I shared empirical findings of grade 4 learner performance across a long term research and development project that focused on supporting grade 3 and 4 numeracy and mathematics teachers in developing foundational knowledge of learners (not achieved in earlier grades). The findings point to differential improvements across three groups of schools that can be distinguished based on the varying degrees of teacher participation in the NICLE development program. Strong improvements in some schools, and limited improvements (or in some cases declining performance) in others, point to the successes and challenges of improving performance of learners across a wide range of schools. The empirical data suggests that stronger performance improvements are linked to regular and long term participation in NICLE by teachers even while a range of other factors influence results. The paper has illuminated that even though one notes overall average improvements across project schools, individual schools have a wide range of differing factors challenging improvements. This points to the need to analyse changing patterns of school results against the changing local conditions over time.

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References


