Educ Stud Math
DOI 10.1007/s10649-015-9637-8

Between people-pleasing and mathematizing: South African learners' struggle for numeracy

Einat Heyd-Metzuyanim[1,2] **· Mellony Graven**[3]

Abstract The reported research attempts to trace possible reasons for third grade learners' limited progress in numeracy in a low socioeconomic status (SES) South African context. This is done through two lenses, both stemming from Sfard's commognitive (The term "commognition" has been offered by Sfard (2008) as an amalgam of "cognition" and "communication," thus expressing the unity of these concepts. Since its original appearance, some authors (including Sfard herself) have preferred using the word "communicational" to describe Sfard's framework. We chose to stick with "commognitive" because we believe it clearly points to the specific theoretical stance presented in Sfard (2008), whereas "communicational" might point to many other theories or frameworks that have something to do with human communication.) framework. One lens aims to analyze two learners' (Mina and Ronaldo (all names are pseudonyms)) mathematical and identity discourse both in one-on-one interviews and in a small group "math club" lesson led by the second author. The other examines the mathematical milieu in which these learners have participated through the analysis of a school mathematics lesson which exemplifies prevalent instructional practices in this milieu. Relying on the distinction between ritual and explorative participation, we show that while Mina was acting in an extremely ritualized manner, Ronaldo was more explorative in his actions. However, the milieu, as seen in the school lesson, encouraged almost exclusively ritual participation. Thus, while Mina was identified as a good student, Ronaldo was identified as an outcast or "troublemaker." We conclude by drawing implications to the tenacious nature of rituals in the mathematics classroom and the effects that these rituals may have on students' identities.

✉ Einat Heyd-Metzuyanim
einat.metz@gmail.com

1 Faculty of Education in Science and Technology, Technion—Israel Institute of Technology, Haifa, Israel

2 Learning Research and Development Center, University of Pittsburgh, Pittsburgh, PA, USA

3 Rhodes University, Grahamstown, South Africa

Published online: 13 October 2015

Keywords Commognition · Ritual learning · Explorative learning · Identity · Discourse · Struggling students · Mathematical milieu · Learning difficulties

1 Introduction

Ritual participation in mathematical learning, whereby learners' actions are geared toward pleasing others rather than for producing mathematical truths about the world, has mainly been studied from the view point of *learning* or the *learner* (Heyd-Metzuyanim, 2013, 2015; Sfard & Lavie, 2005). Recently, some work has been done on characterizing *ritual instruction* (Heyd-Metzuyanim, Tabach, & Nachlieli, 2015). However, the alignment of learners' ritual participation with the ritual instructional practices that are part of their milieu has not yet been studied directly. In the present study, we wish to examine a rather extreme case of both ritual participation, as seen in the actions of one South African third grade learner named Mina and an extreme case of ritual instruction, as seen in an episode of a third grade math teacher in a rural area of South Africa. To better understand the ritual properties of both the learning and the instruction, we shall compare them to another case that exemplifies some explorative properties. In the case of learning, this will be done by comparing Mina with Ronaldo, a learner who was more interested in math rather than in pleasing the teacher. In the case of instruction, we shall do this by comparing a lesson led by Mellony (the second author) in an out-of-school "math club" which included explorative goals, with a lesson in a third grade classroom that was characterized by strongly ritual routines. Thus, our goal in this work is to examine how the characteristics of ritual learning are aligned with ritual instruction and how students' identities and patterns of participation may be affected by ritual vs. explorative instruction.

2 Theoretical background

2.1 The context—issues of mathematics learning and instruction in South Africa

SA has two education systems operating. One is for the wealthier 20–25 % of the population, which is largely functional. Here, the performance scores of the mathematics learners compare favorably with other developed countries. The other, serving the majority of South African learners, is mostly from poor and economically disadvantaged background, where many schools are dysfunctional (Fleisch, 2008; Spaull, 2013). As Spaull (2013) cautions, it is thus problematic to talk about South African education in general or the "average South African learner." In this respect, we must be clear about the context in which this paper emerges. The schools, teacher, and learners that we refer to in this paper are from the second system of education. These two schools referred to under apartheid were "Coloured" schools[1] (under apartheid, there were four groups of racially segregated schools) and as such were provided with far fewer state resources than white schools. The recently introduced Annual National Assessments (ANAs) for the schools show that in general their learners perform poorly on

[1] In South Africa, racial classifications such as "Coloured" continue to be used to analyze the extent to which redress is occurring across previously disadvantaged racial groups in various sectors of society such as education and economic participation.

mathematics assessments. For example, the 2012 average for Mina and Ronaldo's school for grade 3 was 36 % (SANCP, 2015), compared to the national average of 41 % for grade 3 learners (DBE, 2012).

The mathematics results of the learners in this school, the national mathematics averages, and scores from numerous international and regional comparative studies that South African primary mathematics learners have participated in point to a crisis in mathematics education. Furthermore, this crisis and ongoing poor performance is despite the relative wealth of the country (compared with other African countries). Also, the government sees education as a vehicle for redressing apartheid-entrenched inequalities (Graven, 2014) and therefore prioritizes education by allocating relatively high levels of resources.

In the 2003 Trends in International Mathematics and Science Study (TIMSS), SA was the lowest performing of 50 countries, had the largest variation in scores, and the highest percentage of learners achieving below the low international benchmark. While the average of learners in historically White schools was close to the international average, scores of learners in African schools were almost half of that and these worsened from TIMSS 1999 to TIMSS 2003 accentuating the inequality of performance between the wealthier and poorer schools (Reddy, 2006). This begs investigation into the complex way in which South Africa's recent apartheid history continues to play out in education.

Graven (2014) and Graven and Heyd-Metzuyanim (2014) have argued that an aspect that is largely absent in the literature that might account for the extreme nature of SA results relates to the prevalence of passive reliance on teacher authority in SA classrooms. Research with a range of Eastern Cape[2] learners indicated that learners tend to equate mathematical success with teacher dependence, compliance, and careful listening rather than relating it to independent thinking, problem solving, or making sense of mathematics (see Graven, Hewana, & Stott, 2013). On the other hand, reform notions of mathematical competence require active participation and sense making (Department of Basic Education, 2011). In this respect, passive compliant learning dispositions are likely to be a stumbling block to developing conceptual understanding that requires some level of learner agency to develop.

While there are differences in the way individual teachers teach, the predominance of superficial and concrete presentation of mathematical concepts has been noted to predominate South African primary mathematics classrooms, particularly in previously disadvantaged schools (Hoadley, 2012; Schollar, 2008; Taylor & Vinjevold, 1999). Hoadley (2012) concludes from drawing on a range of research that the descriptive features of SA primary classrooms include dominance of oral discourse with limited reading and writing opportunities, classroom interaction patterns that privilege the collective (chorusing), limited feedback or evaluation of student responses, the majority of learners learning in an additional[3] language, and whole class learning. Furthermore, she notes low levels of cognitive demand and the predominance of concrete over abstract meaning.

This background provides the context in which Mina and Ronaldo's learning stories emerge.

[2] The Eastern Cape is one of nine South African provinces and is situated in the South Eastern part along the coast. It is one of the poorest provinces with among the lowest education results across the Annual National Assessments (DBE, 2012).

[3] South Africa is a multilingual society and many learners speak more than three languages. Thus, for many learners, the medium of instruction at their school is their third or fourth language rather than a second language.

2.2 Ritual vs. explorative participation in mathematical learning

For the study of Mina and Ronaldo's mathematical learning in the context of instruction practices prevalent in South African rural schools, we have chosen to use Sfard's commognitive framework since it provides a comprehensive set of conceptual tools for capturing both mathematical discourse, social participation patterns, and identity narratives (Heyd-Metzuyanim & Sfard, 2012; Sfard, 2008; Sfard & Prusak, 2005). Rooted in sociocultural views of learning, this framework provides an appropriate platform for linking between students' individualized practices and the cultural narratives about learning that surround them. The commognitive framework, in which the main tenant is that thinking can be thought of as an internalized type of public discourse, enables studying mathematical cognition, classroom norms, and students' identity using a unified theoretical toolset that relies on a single set of theoretical and philosophical assumptions. Such a unified conceptual toolset is generally missing from cognitive frameworks that have tended to the individual student, frequently dichotomizing affect and cognition and back-grounding the students' sociocultural environment.

In her conceptualization of mathematical learning as becoming a central participant in the mathematical discourse, Sfard (2008) points to the fact that there are two distinct types of such participation: ritual participation,[4] which goal is first and foremost to connect with (or please) others; and explorative participation, which goal is to produce mathematical narratives for their own sake.

Ritual participation, besides being defined by different goals, is also characterized by different routines. Ritual routines often rely on syntactic rules (such as "multiply nominator by nominator and denominator by denominator") without any reference to the object manipulated by the routine (i.e., the fractions). Explorative routines, on the other hand, are aimed at producing narratives about mathematical objects, often treating its different realizations (such as ½ or 0.5) as interchangeable. This difference may often go unnoticed to the untrained eye. For instance, Ben-Yehuda, Lavy, Linchevsky, and Sfard (2005) studied the discourse of two low-achieving high school girls, who both had a personal history of educational impoverishment. They showed how one of the girls, Talli, who identified herself as "good in math," and was identified by the teacher as having "good potential," was actually performing calculations in a strictly ritual and syntactic way. That is, she was relying solely on externally given rules of replacing digits with other digits, without showing any signs of objectification. Objectification is talking about mathematical objects as entities in the world. For instance, talking about "the number four" as existing on its own, instead of saying "four is the number-word that I end up with when counting one, two, three, four." Sfard and Lavie (2005) have postulated that objectification is a necessary step toward developing a more sophisticated mathematical discourse. For instance, a child has to objectify the number "four" before she can talk about "four plus four is eight." The second girl in Ben-Yehuda and her colleagues' study, Mira, was a girl who identified herself and was identified by her teacher as very "weak." Her mathematical discourse, however, was much more objectified than that of Talli, and yet she restrained herself from relying on her own object-mediated routines of calculation because she deemed them as "inappropriate" for her age (for instance, she was embarrassed about counting with her fingers). The authors thus postulated that the reasons for why she was not able to progress

[4] We are using here the term "ritual" in a very specific sense, as will be defined in the next few paragraphs. Though this use has some resemblance to the colloquial use of the term (as in religious rituals), it also differs from it in many ways. For more on the definition of ritual, see Heyd-Metzuyanim (2015) and Sfard and Lavie (2005).

satisfactorily in math had to do with her identity and with the norms she was attempting to follow of how mathematics "should" be done. Both Mira and Talli were performing at a lower than expected level. However, while Mira seemed to have the potential to move forward, based on her strong sense of numbers as mathematical objects, Talli seemed to be stuck at a ritual stage from which she made no sign of wanting to advance.

Another case of "getting stuck" in the ritual phase has been shown by Heyd-Metzuyanim (2015) in the case of a girl (Idit) who between the age of 12 and 14 (seventh–ninth grades) deteriorated considerably in her achievements in mathematics while concomitantly developing a strong anxiety of the subject. In particular, the study examined the relationship between Idit's *subjectifying* (communicating about herself) and *mathematizing* (communicating about mathematical objects). The findings revealed that Idit, in fact, had a strong understanding of whole numbers and mostly talked about them in an objectified fashion. Fractions, however, were a "black box" for her, and her performance with them was purely ritualistic. Over the 2 years between seventh and ninth grades, Idit became more and more ritualistic in her mathematizing, along with increased negative subjectifying (such as saying "I'm not good with fractions" or expressing embarrassment while attempting to solve mathematical problems). This process was hypothesized to be a result of the interaction between the girl's shallow and syntactic grasp of mathematical concepts needed for advancement in algebra, the mathematical milieu (school and home) that encouraged ritual participation and that gave her limited access to any type of explorative mathematizing, and the identity models that were available for her as a mathematics learner.

The link between identity narratives and ritual participation has been shown in another case study (Heyd-Metzuyanim, 2013) which exemplified an extreme case of ritual participation, exhibited by a very low achieving seventh grader named Dana. The study revealed that this ritual participation was actually co-constructed by the student and her teacher, who happened to be the author herself. This co-construction was a result of the way both Dana and the first author identified her as "weak" in mathematics and as needing constant directions. In rare instances where the teacher tried to break away from this pattern, Dana either insisted strongly on going back to it or acted in ways that made the teacher identify her as a "clown" who has no clue about math. Dana's identity as "weak" was constructed and reconstructed by all the significant narrators around her, including her school teacher, her school counselor, and her mother. In addition, the curriculum followed at school at the time was very traditional and consisted mainly of "drill and practice" types of instruction. All these factors contributed to an escalating cycle of failure that neither Dana nor her teacher in the out-of-school course could break away from.

Both the cases of Idit and Dana described above point to the complexity of failure in mathematics and the fact that none of it is solely dependent on the student or on the educational setting which she is part of. Gresalfi (2009) shows this aptly in a study that examined students' dispositions in mathematics lessons in very different classrooms. She found that students' actions were neither the sole product of the opportunities they got to engage exploratively, nor were they the sole product of students' individual dispositions. They were a combination of both.

In light of the above, and in the context of South African learners' perpetuated failure, we will examine a case of two learners, both of whom had some opportunities to learn in a math club outside a mostly ritual learning environment. We ask: How is ritual participation reinforced by the discourse that surrounds the learner and what actions need to be taken up by the learner to engage in explorative learning?

3 The study

3.1 Participants and setting

The brief of the South African Numeracy Chair (SANC) is to research sustainable ways forward to the challenges in mathematics education. The SANC project team consists of doctoral and masters students researching in the field of numeracy education under the supervision of the Chair (second author) while simultaneously working toward the development goals of improving the numeracy proficiency of learners in schools in the broader Eastern Cape area. As part of the project's ongoing international collaboration, the first author visited with the project (and the club) in 2012 and early 2013.

Two key intervention projects are part of the Chair initiative. The first is the teacher development program, the Numeracy Inquiry Community of Leader Educators (NICLE) which began in 2011 with over 50 teachers from 14 schools in the broader Grahamstown area. Mina and Ronaldo studied in one of these schools. The second, a key learner intervention project, introduced in 2011, is after-school mathematics clubs with between 6 and 12 learners from various NICLE schools. These clubs aim to develop the learners' mathematical fluency through encouraging more active, independent (less teacher dependent), conceptual, and explorative participation (see Graven, 2011). The two learners in this study participated in one of the six clubs run by the SANC project team in 2012. Mellony coordinated this club from July 2012 when the original club facilitator was no longer able to. The club ran at an afterschool development center that supports "at-risk" learners from three of the nearby schools through providing afternoon meals and care. The clubs ran for about an hour every week after school during term time. Participation in the clubs by SANC project members enables powerful learning, which then feeds into the NICLE teacher program. To support this learning ongoing data gathering and research, analysis of data takes place in the clubs (see for example Graven, 2012; Graven et al., 2013).

The two learners in this paper have been chosen for the following reasons. First, they were both regular participators in the club and we had almost full sets of data about their participation (some of the other learners were absent for several periods of time). Both learners came from economically disadvantaged backgrounds and live in the same community near the school and club. Second, their patterns of participation in the math clubs differed quite significantly in ways that seemed related to their different learning gains. In particular, one of them, whom we call Ronaldo, had the name of a "troublemaker" within the community of educators in his surrounding, yet achieved relatively well in SANCP assessments. In contrast, his peer Mina was considered a "good student" despite her weaker mathematical performance. While Mina's patterns of participation aligned with the more prevalent practices promoted in SA primary classrooms noted above, Ronaldo's participation patterns provide a useful counterpoint that highlights the way in which different patterns of participation can enable or constrain learning. These differences in participation patterns gave us an opportunity to study closely the relationship between sociocultural norms and advancement in mathematics in the case of these two learners.

3.2 Tools of data collection

The SANC project works toward improving the teaching and learning of numeracy based on the notion of mathematical (or numeracy) proficiency as involving the development of five interrelated strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, Swafford, & Findell, 2001). In order to research the nature of learning (and to track numeracy progress) within the project schools and clubs, the SANC project team developed a range of data collection methods including videoing of participating teacher lessons; regular videoing of club sessions; one-to-one learner proficiency interviews; learner disposition interviews; written assessments and interviews of learners' mastery of four basic operations; and written reflections of teachers, previous club facilitator, and development center staff. Parental/ caregiver permission was obtained for all club learners for participation in the clubs and in the research process.

Below, we briefly explain the data collection tools that we report on here.

3.2.1 Video

The video of the teacher that we share below was taken in October 2011 toward the end of her first year of participation in the NICLE teacher development program. The videos were used to inform the design of the NICLE program and enabled teacher lesson reflection. Additionally, videos of lessons informed research on participating teachers' evolving practices over time. The lesson excerpt is chosen because it exemplifies strongly ritualized practice widely noted in the SA landscape and across NICLE teacher lessons in year 1 of the program, albeit in more nuanced forms. It should be noted, however, that comparative analysis of teacher lessons in year 1 and year 4 shows that for all teachers, there were observable shifts away from ritualized practices toward increased focus on sense making (SANCP, 2015). Permission for the research was obtained from relevant authorities and participation in the research was voluntary.

Three of the nine club sessions between June 2012 and November 2012 were video recorded with the assistance of a postgraduate student. One of these recordings was of Mellony facilitating the club, while the other two were of the other club facilitator. This session was transcribed for the purpose of informing future research on investigating club learner dispositions.

3.2.2 Comments of teachers, club facilitators, and the development center coordinator

The club ran from January 2011 to November 2011. During this time, there were three club facilitators. The first facilitator ran the weekly sessions for the first 6 months and then left due to competing work commitments. Thereafter, the second author (Mellony) and another facilitator (a retired mathematics teacher from a NICLE school) facilitated the club sessions, at times jointly and at other times individually for the second half of the year. During the course of the club, Mellony gathered a range of written comments from Mina and Ronaldo's teachers and club facilitators. The club facilitators were asked to write freely about their impression of the club learners during their work with them. Their school teacher was asked to write about whether the club has had any influence on the learners' ways of participating in class. Additionally, we used e-mail correspondence for capturing Mellony's impressions and reflections of the club learners following various sessions. Finally, we (Einat and Mellony) jointly interviewed the coordinator of the after-school development center in February 2013 as to her impression of the club learners.

3.2.3 Mathematical proficiency assessments

The SANC project used Wright, Martland, Stafford, & Stanger's (2006) one-to-one interview assessments, designed for year 1–3 learners to assess learners' levels of competence in various aspects of a Learning Framework in Number (LFIN), combined with items from the assessment of year 3 learners used in the Effective Teachers Of Numeracy study conducted in England in the 1990s (Askew, Brown, Rhodes, Johnson, & William, 1997). The assembled instrument (of 24 questions in total) combines questions (sometimes adapted) from both these key works. Individual questions were grouped together to constitute a full picture for a particular LFIN aspect. The instrument was translated into Afrikaans and isiXhosa as these languages are widely spoken in the broader Grahamstown area. Learners were interviewed in their language of learning and teaching (LOLT) with translation into their home language where necessary.

In this paper, we report on data derived from interviews with two learners in one club. The instrument was administered individually to each club learner in January 2012 and again in November 2012. Interviews lasted between 45 and 60 min.

4 Findings

4.1 Mina and Ronaldo's mathematizing in pre- and postindividual interviews

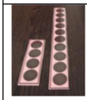
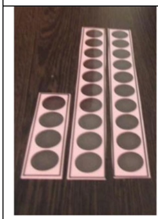
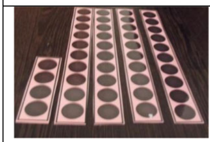
Throughout the course of the study (February 2012–November 2012), Mina and Ronaldo maintained a steady difference in their mathematical skills. This could be seen in the summary of their pre- and postassessments (see Appendix), which, though it skims over many details of their specific solutions, still provides a general view of the two learners numeracy skills. While Mina scored 61.8 % correct answers at the beginning of the program and 79.8 % at the end of it, Ronaldo scored 80.9 and 95.5 %, respectively.

The items in the assessments were designed for identifying the need for remediation in 7–11 year olds (i.e., grades 1–4) and are based on the premise that 7–11 year olds lacking these skills will struggle to progress further mathematically. Mina was 9 years 4 months and Ronaldo was 10 years 0 months at the start of 2012, yet both of them attended third grade. Both of them advanced and to similar extents (at least as quantified by the coding of their answers as right or wrong). Yet observations revealed that Mina was participating differently than Ronaldo during the club lessons. In particular, she seemed to be more inclined to try and please the teacher rather than produce mathematical narratives. Before moving to show this difference in more detail, we shall demonstrate the difference between the two learners in their objectification of number.

4.1.1 Commognitive analysis of excerpts from Mina and Ronaldo's postcourse interviews

Several instances, taken from Ronaldo's postclub interviews show that, by the end of that year, he had already objectified numbers in the realm of tens and hundreds. In other words, he was able to compose and decompose numbers into hundreds, tens, and units and use this composition to facilitate addition routines. Table 1 compares Mina and Ronaldo's answers to a series of tasks that exemplify this skill, referred to as "base ten arithmetical strategies" (Wright, Martland, Stafford, & Stanger, 2006, p. 8).

Table 1 A comparison of Mina and Ronaldo's responses to "base ten" questions

Task/question	Ronaldo's response	Mina's response
	"Four"	"Four"
	Ronaldo: Must I count all? Intrv: How many altogether? Ronaldo: (reorients the strips through 90° so they are lying horizontally rather than vertically as the interviewer had laid them. He then touches each dot on the ten dot strip counting quietly one to ten) 14	Mina: (Mina touches each dot on the ten dot strip mouthing each number from 1 to 10 as she touches each dot): ten Intrv: How many all together? Mina: (counts on fingers) 4; 5; 6; 7; 8; 9; 10; 11; 12; 13; 14.
	24	14; 15; 16; 17; 18; 19; 20; 21; 22; 23; 24; (touching each dot on the new ten dot strip as she counts)
	44	24; 25; 26; 27; 28; 29; 40; 41; 42 (shakes her head) no it was 42 (then moves finger back to the bottom of the same strip touching each dot as she counts) 45; 46; 47; 48; 49; 50; 51; 52; 53; 54 (then touches the final ten dot strip that was placed down saying 64) 64.
4 dots strips and 7 tens strips (total 74)	10; 20; 30; 40; 50; 60; 70; 74	Mina: 64; 74; 84; 94 (Starts from the last strip. Then touches each new dot strip at the top as she counts.) Interv: And do you think you could have counted them right from the beginning in 10's? Mina: 4; 14 ;24; 34; 44; 54; 64; 74
Add 10 to 92	10 plus 92. 102	92 (pauses, then starts counting on her fingers) 92; 93; 94; 95; 96; 97; 98; 99; 100; 101; 102
Add 10 to 294	Writes on paper 294 + 10 = 304 Once he is done says 304 and looks up.	(Counts on her fingers) 294; 295; 296; 297; 298; 299; (pauses and looks at Interviewer briefly) 300; 301; 302; 303; 304

In the above comparative table, we can see that where Ronaldo did not need any assistance in seeing that adding a strip of 10 dots adds 10 to the number, Mina went back to counting with every addition of a strip. Only after she went through several of these additions, hearing the pattern that emerges ("fifty four, sixty four") did she relinquish her routine of counting each one. Apparently, however, the "adding 10" routine was still very much in her Zone of Proximal Development (Vygotsky, 1978), that is, it could be performed together with an expert but not by herself. When faced with the next question: "Can you add 10 to 92? 92 plus 10, what would that be?" Mina went back to counting on from 92, using her fingers as aids for tracking the 10 that is to be added. In contrast, Ronaldo answered the same exact question with "10 plus 92. 102." His quick answer and no apparent signs of counting show that he most probably relied on the "adding 10" routine and had completely individualized it. There were other indicators from across the interviews that while Ronaldo had developed some flexible routines for dealing with numbers in the realm of 10–100, Mina had not. Some of them are summarized in Table 2.

4.2 Differences in Mina and Ronaldo's identities as mathematical learners

One might think that, due to the marked differences in mathematical skills, Mina and Ronaldo's identity stories would also be very different. Indeed, they were, yet not necessarily in the expected direction. Both learners, in their "disposition interview" claimed to "love math." However, while Ronaldo was quite reserved and did not talk much about himself as a math learner, Mina excitedly talked about her engagement with math:

> I'm loving maths. It's so nice to be in maths class… I have a sticker in my book (good work sticker). When I am big I will study it, maybe become a teacher, it will be fun for me to do maths with children. With other children we play school in class and I give then maths problems. It's lovely to do maths. [November, 2012]

Even more surprising than the discrepancy between Mina's first person identity and the one we were able to tell about her (based on her mathematical interviews) is the discrepancy

Table 2 Examples of differences between Mina and Ronaldo's routines for solving arithmetic problems in the realm of 10–100

	Question	February	November
Mina	16+10=?	Draws 16 and 10 lines then counts all from the beginning	Counts on 10 from 16 in ones
Ronaldo		Counts on in ones	Answers instantly 26
Mina	So then what is 16+9?	Draws again 16 lines and 9 lines then counts them all.	Counts on 9 from 16
Ronaldo		Counts on 9 using his fingers	Answers: 6+9 is 15 so 15+10 is 25.
Mina	Three lots of three or three multiplied by three equals?	Verbal (not instantly): 6	Written routine: 1×3=3; 2×3=6; 3×3=9; 4×3 is 12 and so answered 12 thus failing to stop the routine at the appropriate place
Ronaldo		Instant: 9	Instant: 9

between our story and the story of other significant narrators around these two learners. This discrepancy was particularly evident in the stories told about Ronaldo who some teachers saw as "a brilliant little boy," while others only noted his behavior problems. Following is a comparison of the stories told by Mellony and Jane,[5] the first teacher of the club who led it during the first 6 months.

Ronaldo—Jane's (first club facilitator) story

> This little chap really worries me. He always comes to maths club looking very untidy and his eyes are always watery. It is difficult to get him positively involved in maths games and activities because his behaviour is a problem. He doesn't work well with the other children in groups and often gets himself into trouble for unacceptable behaviour, …. His understanding of numeracy is very limited to a little rote counting backwards and forwards. I don't feel like I am making any progress with his learning of the value of numbers. [Ronaldo] needs daily learning support to improve his knowledge of numbers and fill in the huge gaps which are evident. (written in a letter to Mellony, on 26 February 2013)

Ronaldo—Mellony's story

The notes by Mellony were made after the club sessions (February–November) and as a reflection on her first encounters with Ronaldo.

> He (Ronaldo) … caused some frustration with other students as he consistently shouted out answers or grabbed the cards when it was other learners' turns. … I noted his passion and his speed of thinking and he stood out for both his behaviors that caused tension and arguments and for his quick thinking—although I was also concerned that he dominated and took away other learners' chance to think for themselves … Ronaldo wasn't there to comply—he was there because the mathematics of the activity grabbed him … I saw the possibility of him becoming one of Xtown's maths stars possibly winning AMESA maths challenge competitions at a later point.

Later, Mellony reflected on the reasons that she got so enthusiastic about Ronaldo's mathematical potential:

> This view of Ronaldo was influenced by my perception of his reasons for participation in the club being almost solely about his interest in the mathematical activities themselves. I had worked with a student from a similar school and a similarly disadvantaged context 3 years before who had also seemed a social outsider in my club (he worked alone while all others worked together in groups). Rather than focus on the mathematics in the school curriculum, he would bring questions and activities related to mathematical challenges or university mathematics content such as complex numbers. This student had achieved 100 % for mathematics in his exit year (grade 12) and had won the provincial student prize for the top student and I saw Ronaldo as having this similar potential.

Several points are worth noting in relation to the marked differences between Jane and Mellony's stories about Ronaldo. One is that they seem to imply a very different view of what being a "good mathematics learner" is all about. While Mellony valued "passion,", "speed of thinking," and the fact that Ronaldo "wasn't there to comply," Jane did not differentiate

[5] All teacher names, except that of Mellony, are pseudonyms.

between Ronaldo's inadequate social behavior and his mathematical skills. In particular, her remark that "His understanding of numeracy is very limited to a little rote counting backwards and forwards" stands in sharp contrast to the comparatively high numerical skills that were seen in both of his interviews, as well as in Mellony's club lesson analyzed below. Another point is the "identity model" that Mellony had, in the form of a former student of hers who had become a "math star" in previous years. This identity model, to which Ronaldo fitted based on similarity between his actions and the older student actions as a young learner, enabled Mellony to designate Ronaldo to follow the "math star" footsteps.

In contrast to Ronaldo, who was identified by Jane as having only very limited numerical skills, Mina received a much warmer evaluation:

Mina—Jane's story

> Mina is a lovely little girl who always comes to maths club eager to see what the activities are going to be. She helps me unpack the maths equipment and games. She has a sound understanding of number value and is able to calculate fairly accurately. She is always ready to listen and participate in the games and carry out my instructions. She has the potential to make good progress because she has the willingness to participate and learn. …. Mina always has a positive attitude and enjoys a little competition.

Though Jane's story about Mina resembles Mellony's in describing her social behavior (being eager to come to the club, ready to comply with all the instructor's requests), it is markedly different in the description of her mathematical skills. Thus, according to the mathematical interview (done a few months *after* Jane had worked with Mina), Mina was very far from having "a sound understanding of number value" (though she might have been pretty accurate in calculations, if only those were done on her fingers or with other concrete materials).

We bring these surprisingly different stories not to critique or say anything about the ability of teachers such as Jane, who only have limited encounters with students, to accurately identify learners' skills. Rather, we wish to point to the possibility that Jane's evaluation of Mina and Ronaldo was an indicator of a certain mathematical culture (or set of meta-mathematical norms), according to which "good learners" are enthusiastic, rule-following, and accurately calculating learners while "shouting out" and not obeying the social rules tend to be associated with low performing learners.

Having described Mina and Ronaldo's individual mathematizing and their identity stories, we now turn to examine how these mathematical skills and identity stories play out in one club session in which Mellony taught the group for the second time (the first session was not video recorded).

4.3 Mina and Ronaldo's participation in the club session

To examine the differences between our two focal students' participation patterns in mathematical learning, we looked closely at two episodes taken from a single session where Mellony taught Mina and Ronaldo (as well as Alice and Bev, two other students who are not the focus of this analysis). The first episode was taken from the beginning of the lesson, during which Mellony introduced a "spider game": a number is written in the middle, and learners are asked to come up with different sums that produce this number and write these on the "legs" of the "spider." Following is a short transcript taken from this episode:

1.	Mellony	If I put 10 in the middle (*Draws a circle*, *10 in the middle*, *and rays coming out of the circle*) what different sums can you give me?
2.	Mina	Ten plus ten! Ten plus ten (gestures toward Mellony). Ten plus ten
3.	Mellony	Well no I want the ANSWER to be ten.
4.	Mina	..Oh One plus nine! (Answers quickly. Points with her pen, makes a coquettish face toward Mellony)
5.	Mellony	Do it. You write on yours (points to Ronaldo) you write on yours (points to Bev). Do you agree with her, one plus nine? (leans over to Ronaldo)
6.	Ronaldo	(Nods)
7.	Mellony	Do you agree with her? (turns left to the girls) Does 1 plus 9 give you an answer of?
8.	Bev + others	Yes, Ten.
9.	Mellony	So put ten in the middle and give me a few [sums].
10.	Mina	(jumps up) [uhh null] plus-
11.	Alice	(shoots up her finger) [---]
12.	Mellony	Uh uh! You're right, you're right. (Mina squints looking at Alice's board, then turns to writing on her own board). Write your own sums, (Everyone is writing and looking down) Let's see.
13.	Ronaldo	(While writing and looking down) Eight plus seven
14.	Mellony	what is 8 plus 7 –
15.	Bev	(Raises her head, looks at Mellony and then leans back laughing. Then immediately goes back to writing on her board).
16.	Ronaldo	Fifteen (looking at Mellony)
17.	Mellony	So ja but then the answer is not 10 hey (points at the 10 in the middle of Ronaldo's board). I like your thinking 8+7 is a good sum and you got the right answer I am impressed (.) but at the moment I want you to give me some sums that make 10. Can you give me some sums that make 10?
18.	Ronaldo	Zero plus ten?
19.	Mellony	Right. I like it. [I] like it
20.	Camera man	[Mmm!] (Ronaldo looks toward camera man with a satisfied look)

Though the above episode is very short (only 52 seconds), some interesting features of the discourse in this group can already be gleaned from it. First, both teacher and students are preoccupied not just with the mathematics, but also (and perhaps more so) with the subjectifying messages, many which can be seen in facial expressions and direction of gaze. These subjectifications include the teacher's encouragements seen in statements such as "I like your thinking" [17],[6] in Mina's coquettish smile [4] which communicates something like *aren't you happy with my answer*, in Bev's laugh about Ronaldo's mistake [15] and even in the camera man's (a math teacher and graduate student) impressed humming [20] that elicits a satisfied gaze from Ronaldo. All of these communicative actions make clear that participants in this episode are not only preoccupied with the mathematical task. Rather, they are also

[6] Numbers in square brackets [] refer to the line number in the transcript.

preoccupied with what others (and especially the teacher) think about them. This is probably a natural characteristic of learning-teaching situations. However, we shall show in the next episodes that Mina's preoccupation with subjectifying leads her to different forms of participation than Ronaldo. Whereas Ronaldo continues to focus and rely on his own mathematical routines and narratives to come up with new narratives that would result in positive affirmation from the teacher and satisfy her requirements, Mina tries to achieve a teacher-pleasing goal with whatever means she has, including copying from others and building on nonmathematical cues that she tries to glean from the teachers' communication.

4.3.1 Adding 25+ 36 episode

1.	Mellony	Okay I am going to give you a sum and I want you to see. (Pauses to write a circle with 61 in the center and a line with 25+36 radiating from it]. I am telling you that 25 plus 36 (pointing to the 25+36] is 61. Do you want me to prove it to you?
2.	Ronaldo	Yes (*Bev and Mina nod*)
3.	Mellony	Okay 25.
4.	Ronaldo	I know! I know (starts writing on the board)
5.	Mellony	Have you ever done these Flard cards[7]? (*unpacks the packet of Flard cards* (*arrow cards*))
6.	Bev & Mina:	[Yes]
7.	Ronaldo	(*louder and at same time as the other learners*) 2 plus 3 is 5, 5 plus 6 is 11 (*pointing to the 2 and the 3 in the written sum 25+36*).
8.	Mellony	Okay. [2 plus 3 is 5] and 5 and 6 is 11. But is this a 2 or is this two 10's?
9.		(*Mina is attentive but tapping her pen*, *Mellony puts hand over pen to stop the tapping while continuing to talk*.)
10.–25.		*Mellony points the learners to the fact that the 2 stands for* "*two tens.*" *She then proceeds to pick up Flard cards that would signify the addition of 25 and 36*, *namely the cards 20*, *5*, *30 and 6*.
26.	Mellony	Thirty six (*lays out 30 card and the 6 below it*). Alright. Now look here. You said something interesting Ronaldo. You said 2 and 3 is 5 (*points at the 2 and the 3 of the sum 25+36 written on her board*) but is that a 2 (*points at 2 in* '*20*')?

27.	Ronaldo	No
28.	Mellony	What is this?
29.	Ronaldo	Tens
30.	Mellony	It's 2 10's. The 20 plus 30 is
31.	Ronaldo	[50]
32.	Bev	[50]
33.		(*Alice and Mina watch silently*)

[7] Flard cards are sets of cards with the following numbers on them: 1; 2; 3; …9 and 10; 20; 30; …90, and 100; 200; 300 …. 900, and 1000; 2000; 3000; … 9000. See line 26 of the excerpt for a photograph. They are also called number builder cards and arrow cards. Combining cards allows children to build up numbers and they are a recommended teaching resource in the South African curriculum for the Foundation Phase (grades 1–3).

34.	Mellony	Fifty. Do you agree? Twenty plus thirty is fifty and then you said 5 plus 6 is
35.	Ronaldo	11!
36.	Mellony	And now what are we going to do with this eleven (rubs her cheek). What I'm going to do, [its ten and one] (*looks down to find the 10 card and the 1 card to make 11*)
37.	Mina	(Jumps up) You're gonna take—So you're gonna take the 1 away (points at the Flard cards) then the one, then you just put a 6 and you add the 6 to…
38.	Mellony	Okay. Show me. Yeah. This 5 and 6 is 11 now what? (puts the 10 card and the 1 card together) There is 11. Now what?
39.	Mina	Then you take away the 11 (moves the 10 card back) and then you put the 10 (takes the 6 card) and then you put the 6 there (puts the 6 card together with the 1 *so they now look together as 61*)
40.	Mellony	Oh. I see. I see.. I see why you are wanting the 6. The 6 for 60 but that is a 6 hey So let's see you said 5 plus 6 is 11. Am I right?
41.	Ronaldo	Yes

This episode shows the remarkable difference between Ronaldo's and Mina's mathematizing around the problem of adding 25 and 36. Ronaldo sees these two numbers as being easily separated into tens and units, which thereafter can be combined separately. Though his objectification of numbers such as 25 and 36 is not completely clear from his first statement [7], it becomes clearer from his agreement with the teacher's clarification of the fact that 2 stands for "20" and 3 stands for "30" and from his statement that combining these two produces "50" [31]. It can also be derived from the ease in which he calculates in his head that the sum of tens (50) and sum of units (11) can be recombined to produce 61, by which he reaches his fast solution. In fact, this swiftness interferes with the teacher's original plan to carefully "prove" to the students (by using the Flard cards) that 25+36 equals 61.

In contrast, Mina's view of 25 and 36 and the ways they combine to form 61 is very different. For her, it involves some complex manipulation of the digits in ways that have to be cleverly figured out. Her suggestion [37, 39] to take away the 1 from the 11 and put it together with the 6 from the 36 shows that for her, the relative place of the digits does not make much difference, neither does the number from which they are taken (25, 36, or 11), as long as these digits can be combined to form the requested answer (61) as given in line 1. Such a routine is a fine example of *syntactic mediation* (Ben-Yehuda et al., 2005; Heyd-Metzuyanim, 2015). The fact that Mina is so excited about her suggestion (see her "I know! I know!" and the excited expression on her face [37]) shows that she is quite confident that such magical tricks and manipulations on digits are the meta-rules that govern the mathematical discourse.

After having shown, together with Ronaldo, that 25+36 equals 61, Mellony moves forward. She asks the learners to repeat the "spider game," this time producing different sums of 61. Ronaldo is the first to respond. He writes on his board "20+40+1."

Mellony	Can you see what Ronaldo has done here (pointing to Ronaldo's board)
Bev	20 plus 40 plus 1.
Mellony	Does it give you 61? (Bev nods Mina is writing on her board) Do you like his answer? (Bev nods again)
	(Some distraction from a kid who is peeking from the window)
Mellony	OK (looking at boards then points to Bev's board) Very nice you are also using his idea of three

Ronaldo	No its 51 there! (*Shouting and pointing to Bev's board*)
	Mellony asks Ronaldo to "*explain nicely*" *to Bev her mistake. Ronaldo's explanation is inaudible. Bev discusses with Mellony her solution and then corrects it*
Ronaldo	I did another three (*Mina still looking at Mellony, lying on her arms then glances at Ronaldo's board where he is showing Mellony that he had written 10+50+1*)
Mellony	Pardon, Well done Alright. I like it. I like it. (Bev and Mina look over at Ronaldo's board)
Mina	*Writes a sum on her board. Probably the one on Ronaldo's board, since it gets no particular attention from the teacher.*
Mellony	(*Looking at Alice's board*) Ooh, I like it! Let's see, let's see what Alice … has done. Alice has done 60 plus 1. Can you look here quickly. Look here quickly. Alice has done 60 plus 1 which a lot of you have done and then she has done 1 plus 60. What do you notice about those two of Alice's?
Mina	(jumping up). She did turn it around.
Mellony	She turned them around. Very good. (*pointing toward Mina*) Now look what Alice has done. 25 plus 36 that is the one that I gave hey then Alice says 36 plus 25
Mina	And she turn it around (stands up from the seat and looks at the sums on the board pointing to them while saying this with wide eyes and looking now at Mellony).
Mellony	Do you agree with her? (Mina looking at Mellony nodding) Can she do that?
Mina	Yes (slowly and cautiously)
Mellony	Why can she do that?
Mina	(Mina looks at Mellony, shakes her head slightly) No she can't do that she did turn it around
Mellony	Are you sure? Work it out.
Mina	(softly) no she can't
Mellony	When she did 36+25?
Mina	She did turn it around
Mellony	Ya—when you said 9+1 and 1+9 did they give you the same answer?
Mina	Yes
Mellony	*Mellony repeats a few sums and their opposites while showing them on her fingers and switching hands around as she swaps the order of the sums.* (*2+3*, *3+2*; *4+5*, *5+4*) *The learners*, *including Mina*, *chorus enthusiastically their answers*.
Mellony	Does it make a difference which way we add them?
Mina	no (standing waving her arm to indicate confidently no)
Mellony	So is it right of Alice to have done this? 36 plus 25? Do you all agree?
Ronaldo	Yes (loudly, somewhat impatiently, as if wanting to go on)

This episode is notable for several reasons. First, it shows Mina's disengagement and ritual participation in the task of finding different sums that add to 61. Having no independent routines that can lead to the solution of this task, she mainly relies on peeking at her peers' boards and refrains from any creative suggestions. In contrast, Ronaldo is constantly seeking to come up with new narratives, including adding three addends instead of two (20+40+1; then 50+10+1). He also monitors Bev's board, but not for copying her solutions, rather for pointing out her mistake. The second point worth mentioning is that at the first occasion where Mina has the skills to re-engage, she does so enthusiastically. Since she has already successfully engaged in a conversation about the permissibility of "turning sums around" (this can be seen in her earlier participation in the "sums of 10 episode"), she jumps up at the teacher's questions that seems to be aimed at eliciting a similar rule (seen in the facilitator's question "what do you notice about these two" which refers to the sums 60+1, 1+60, and then to 36+25). Yet, despite her earlier assertions that "turning numbers around" provides identical sums, the facilitator's questioning tone makes Mina hesitate and then revise her answer completely. When faced with the two sets of rules, mathematical ones (turning numbers around is permissible) and subjectifying rules (teacher repeat questioning usually indicates you were wrong), Mina chooses to follow the subjectifying rules. She thus changes her answer to "no she can't," oddly retaining the original description of Alice's acts ("She did turn them around").

5 Widening the lens—ritual instruction in the school

To understand better the milieu in which ritual participation such as that of Mina is fostered and perpetuated, we now move to examine a classroom lesson from a school in the SANC project, which is in the same community as the development center and whose learners attend the center. We focus on a short episode that, from our experience, is exemplary of the teaching practices seen in schools such as those that Mina and Ronaldo attend. Since this is not the actual classroom of the two learners, we cannot draw direct connections between it (or between the teacher) and Mina or Ronaldo's mathematical skills. However, since Mellony's experience with viewing many lessons in this district has shown this type of teaching to be quite prevalent, we wish to examine the connections that might be made between these teaching practices and ritual participation in mathematical learning. For reasons of space, we do not bring the lesson's transcription in detail. Rather, we describe the most important utterances and happenings in the class that are relevant for our analysis.

5.1 Episode from Mrs. Xolile's classroom

:: Marks prolonged speech. ^ Marks heightened, singing tone

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1. *Teacher walks around the room. Learners have Flard cards spread on the table. Their task is to find numbers according to the teachers' questions.*
2. T: Check there in your numbers. Can you tell me in your numbers, the biggest number. Check your numbers, don't look at me. Check your numbers. Can you tell me the biggest number there? (*Moves around students' desks*)
3. T: The ^biggest number there^ If you can't pronounce it you just show me. Put them here. Can you show me, can you show me the biggest number. The biggest number alright. The first one to get it, the first one to get it (hup hup hup?). Good, what number is that? (probably raises '9000' card up in the air, though out of camera)
4. Children chorus: nine thousand (teacher asks again, choir repeats: "9,000")
5. T: Very good very good. Can you tell me what number is before 9000? Can you tell me. Check it check it check it. Don't put it wrong. What number is before 9000?
6. Children chorus: 8000

Teachers asks the learners to "build for her" the number 387 from Flard cards.
A learner (Thabo) comes up to the front of the class, shows his cards (300, 80, 7) put together. Then the teacher asks for a student who can "break the number on the board." A learner comes up to the board, writes 300+ 80+7

7. T: Right thank you, is that the same as that one, is that the same as Thabo has shown us? Is that the same?
8. Children chorus: no
9. T: Is it the same?
10. Children chorus: Yes
11. T: This one is saying no (points to Nandi), what is the correct one? Come, Nandi. You are saying no.
12. Why are you saying no? Why are you saying no Nandi? No reason ne Nandi. Can you please go and write another name of that number Nandi. Another name now. Another name of this, or this one (points to 387 on the board).

13. *Nandi comes up to the board, writes on it very slowly and carefully. 45 s go by. The class is quiet. Students move their cards quietly on their tables.*
14. T: (while Nandi is still writing) Are you- look at what Nandi is doing?
15. Quiet chorus: yes… (*22 more seconds go by as Nandi finishes writing*)
16. Okay, can you read what number she has written here? (*points to the first word written by Nandi*)
17. Learners chorus: three (*voices trail off*)
18. T: (*points to the other two words*) Is that correct?
19. Chorus: no (learners hands go up offering to come up and write)
20. T: Then come help Nandi (scans all the students raising their hands) I know, but not all of you can come and help Nandi. Right Miqobo (*points to Miqobo to come to the board, keeping Nandi standing next to her*). Help Nandi please.
21. (*Miqobo comes up and slowly writes on the board. 30 s go by.*)
22. T: Right. Let's read it now, let's read it now.
23. *Learners chorus along with the teacher, as she points to the number and says it out loud: "300 and 87." Then they chorus when she points to each digit being "hundreds", "tens" and "units."*
24. T: Very good. Okay go and sit down Nandi (*gently guides two learners back to their seats, then stops Nandi and turns her to the board*). Did you see that you did a mistake here? (*points to the number words Nandi wrote on the board. Nandi nods*) Don't do that again, ha? (*Gestures Nandi back to her seat*).
25. T: Okay, that's good, where is my duster. (The teacher erases the "incorrect" writing of Nandi on the board leaving the rest).
26. T: Right. Now let us look at our numbers again, let's look at our numbers again. Can you tell me the number between 50 and 52? Ah, 50 and 60? 50 and 60 when you are counting in 5's? When you are counting in 5's what is the number between 50 and 60 when you are counting in 5's? (*Only three learner hands go up*)
    Yes (*points to one of the learners raising his hand*).
27. Learner (stands up): 55
28. T: Okay, let's give him a hand. (Claps. Learners join in clapping). Very very good.

To an external observer, the most pronounced characteristic of this lesson is probably the highly structured way in which the teacher and students produce questions and answers, despite almost no mathematizing taking place. Thus, no routines are being explicated for arriving at the facts that are stated by individual learners or chorused by the whole class. In fact, the chorus quality of most of the students' responses implies that students have already been well trained to picking up certain cues (some of them difficult to detect to any foreign eye) that signal they should be raising their voice and chorusing a certain answer. This is particularly evident in turns [5–6] where the learners chorus "8,000" to the question "which number comes before 9000", despite 8000 not being the number that, in fact, comes before 9000 (8999 is). That much of what is cuing learners to chorus a certain answer does not have to do with anything mathematical can be seen in turns 8–10. Here, the learners seem to interpret the question "is that the same?" as signaling the answer should be "no." However, when the teacher repeats the question, they reinterpret it as signaling the answer should be "yes" (either way, there are only two answers possible—"no" or "yes"). Nothing has changed in the mathematics written on the board, or in any mathematical reasoning given for one answer or the other (actually, mathematical reasoning is absent from this whole episode). Therefore, the only cues that the learners can build on are contextual cues having to do with the type of question, the activity that came before it, and teachers' tone of voice.

Another remarkable feature of this excerpt is the heavy emphasis made on identifying. Students stand up to say their answers and almost every "correct" response receives a clapping applause. The only incorrect response picked up by the teacher is that of Nandi, who mistakenly answered “no” in the previous chorus described above. Nandi is also the only one who is brought to the board without volunteering to do so. Her mistake is made public, but not in a matter that would communicate erring is legitimate. Quite the opposite occurs. The class is encouraged to chorus Nandi’s written number as wrong [19] and a more capable learner is invited to “help” Nandi by writing the correct answer on the board. The fact that the word “help” in this context is devoid of its original meaning can be seen in the absence of any interaction between the other learner (Miqobo) and Nandi. Neither is Nandi being really “helped” by the teacher. Rather, she is simply pointed to the board, required to acknowledge she made a mistake, and warned “not to do that again.” If this treatment may seem harsh to some external eyes, we should add that the whole situation does not seem to be highly degrading. Thus, Nandi does not express distress (though her solemn face might indicate she is not happy about the situation) and other learners are not seen to be mocking her. Still, it is clear that such an “outing” of making a mistake, without paying any attention to what the mistake actually was, or how it should be fixed, puts the focus solely on the identifying aspect of making such a mistake.

Finally, the ritual character of this lesson can be seen in the absence of any authority given to the students to come up with their own narratives or routines. The sole person responsible for determining if an answer is right or wrong is the teacher. Learners are not required to provide their reasoning for any mathematical fact they come up with. All mathematical facts are purely addressed to the teacher, in the aim of getting applause or at least a positive evaluation.

6 Discussion and conclusions

Despite the fact that the two stories (those of Mina and Ronaldo and that of Mrs. X) are not entirely connected (neither Mina nor Ronaldo have been learning in Mrs. X’s class), we claim that looking at these two cases greatly informs us about the question we started out with. That is, *what are the discourse characteristics surrounding learners that encourage ritual participation*? Our claim is based on two justifications: one, that from our experience, Mrs. X’s lesson was quite typical of the math instruction to which Ronaldo and Mina had been exposed. Second, there are clear lines that can be drawn between this type of instruction and the ritual activity seen in the learners’ (mainly Mina’s) performance both in the interviews and in the club lesson. Table 3 aligns the main findings about the ritual vs. explorative nature of Mina and Ronaldo’s activity with the characteristics of the two instructional settings (the club and the school classroom).

As can be seen in the table, every ritual characteristic of a learners’ activity can be aligned with a ritual form of instruction. This is most obvious for the “goals” characteristic. As activity theorists have pointed out (Roth & Radford, 2011; Roth & Tobin, 2007), goals (or “motives”) should not be thought of only as individual incentives. Rather, they are characteristics of activity systems, or *discourses*. It is in this sense that we speak about an “identifying discourse,” in which the goal is to produce narratives about self, and “mathematical discourse,” in which the goal is to produce narratives about the world (using mathematical words). Thus, the “problem” so to speak, does not

Table 3 Signs of ritual vs. explorative learning and instruction

	Learning		Instruction	
	Mina	Ronaldo	Mellony	Mrs. X
Goals	Ritual (pleasing teacher)	Combined (pleasing teacher as well as being mathematically correct and inventive)	Ultimately explorative, but uses ritual goals as "baits" for engaging students	Solely ritual (clapping for successful students. Reprimanding student for making mistake without explaining what the mistake is)
Flexibility of routines and mathematical objects	Rigid: only counting is used for addition; syntactic routine used for 25+36. Numbers are not treated as objects separable into tens and units.	Some flexibility. Searches for alternative routines for sums; treats numbers as separable into hundreds, tens, and units.	Encourages flexibility (Task affords multiple ways of coming up with a sum).	Ignores routines (except of counting). Only pays attention to final narratives. No alternative realizations for mathematical objects are given to students.
Authority/addressees	Others (relies solely on the clues from the teacher or on writings of others she identifies as more knowledgeable)	Mixed. Addresses the teacher with any mathematical claim but also makes claims of his own (like pointing to the error in Bev's board)	Mixed. Tries to overlay authority to students ("do you agree with her?") but does not insist on articulating reasons. Encourages learners to come up with "their own" solutions.	Solely teacher-centered. The addressee is only the teacher and she is the only one who decides if something is "right" or "wrong." When the class is asked if a solution is right, it is "staged" rather than introduced for genuine argumentation.

lie solely in the fact that Mina is aiming at pleasing the teacher. It lies in the fact that the discourse in which she has participated so far (up to the point of joining the club led by Mellony) mostly promotes identity-related goals and neglects mathematical goals. Similarly, the fact that Mina attempts to draw on subjectifying cues (such as those hinted by tone of voice or manner of question) is quite understandable in light of the ubiquity of these cues in classrooms such as Mrs. X's and the way learners are trained to rely on them for chorusing back "correct" answers. Finally, Mina and her friends' (Bev and Alice) avoidance of any mathematical authority and their reluctance to provide mathematical arguments that would justify their claims can be linked to the lack of such requests for justification in their classrooms and even the unpleasant consequences that may be brought upon anyone who disagrees with the teacher (as in Nandi's case).

In light of these prevalent norms in Mina and Ronaldo's environment, Ronaldo's partially explorative participation in the club lesson is even more noteworthy. We cannot help but hypothesize that his general conduct, which tended to defy social norms, was actually of assistance to him in his mathematical development. Thus, it was the "good girl," Mina, who adhered strictly to the rules and who attempted to please the teacher with every act, that fell into the trap of ritualized mathematizing, while Ronaldo, who seemed to care much less about these rules, was able to engage exploratively. In other words, Mina, who was an avid participant in the identifying discourse surrounding her in school mathematics had much more trouble letting go of her ritual patterns in a relatively explorative environment such as the math club than Ronaldo, who showed less interest in the identifying discourse to begin with.

At this point, it is worth reconsidering the hypothesis put forward by Sfard and Lavie (2005) regarding the *inevitability* of the ritual phase in mathematical learning. This hypothesis was put forward based on empirical evidence of young children (aged 4–5) learning to communicate about numbers, together with a theoretical argument regarding the inherent paradoxical nature of mathematical learning. Sfard and Lavie ask: how can learners talk about mathematical objects they have yet no experience with? (see also Sfard, 2008 for an elaboration of this point). Sfard and Lavie's answer is that for the process of mathematical objectification to start, learners have to initially imitate the teacher or expert, and only after gaining some experience with talking about the mathematical objects, can they become explorative and flexible in operating with them. This, in essence, is the basis for their conceptualization of ritual participation. However, in our case study, it does not seem Mina's ritual form of participation *led* her (or was a necessary step) to a more exploratory phase. Manipulating the digits in 25+36 in syntactic ways that were ignoring their place value was not a necessary stage toward objectifying numbers. It was a *shortcut*, a way to overcome the difficulty of objectifying numbers and their decimal structure, not a stage toward achieving this objectification.

We therefore suggest ritual can be divided into two types: natural, or *necessary* ritual, and *extended* ritual, or "ritual gone wrong." Mina's case (as were Idit's and Dana's cases in Heyd-Metzuyanim, 2013, 2015) is a case of the "ritual gone wrong." From the initial necessary stages where imitating and following the rules of grown-ups is necessary, these learners extended the reliance on external, nonmathematical cues to steer their mathematical discourse far too long. In other words, they failed to make the move to an exploratory stage that was due much earlier. When one considers that, in essence, ritual imitation is necessary in mathematics only in the relatively rare cases where one starts talking about new mathematical objects (such as the move from natural to rational numbers, positive numbers to negative numbers, etc.). It then becomes clear that ritual participation should be considered as necessary only in rare cases. In all other cases, ritual participation is a more likely evidence of a "vicious cycle" (Heyd-Metzuyanim, 2015) that is fed by repeated failure to communicate effectively with the teacher on the one hand (in mathematical language) and the growing necessity of the learner to please the teacher and thereby repair her mathematical identity that was impaired by this failure.

An alleviation of this "vicious cycle" would only be likely in an educational environment that consistently highlighted other meta-rules for mathematical activity, meta-rules that highlight the value of describing the world mathematically as a goal in itself. Yet, as could be seen from Mellony's attempt to "change the rules," such a change is far from simple. For one, because the learners are very active in reinstituting the familiar rules brought from their schools. They avoid any negotiation of mathematical meaning, making it difficult for the teacher to gain access to their idiosyncratic mathematical routines and narratives. Second, because learners such as those seen in the club's episode are already so marginalized from mathematics, engaging them with it seems to require much emphasis on identifying (as seen in Mellony's repeated encouragements, positive evaluations, etc.). Such emphasis on identifying may, in itself, promote ritual participation as it highlights identity-goals. The only way out of this cycle seems to be a gradual dismantling of the "identifying scaffolds," those extra-mathematical goals that have to do with "pulling the students in" to begin with. Indeed, Dweck (1986, 2000) has long been advocating the minimization of praise for students' ability and instead maximizing praise for actions and perseverance. However, such a recommendation is easier said than done in a culture where students have been accustomed to praise being a result of "ability" rather than any explicit action. For such students, lack of praise in general, and praise of their ability in particular, would mean they are not living up to their designated identities as "successful students" and could lead to disengagement.

Zooming out into a much wider lens, the problematic nature of the move from ritual to explorative mathematics instruction can be seen in the many attempts (and often failures) of reforming math instruction both in South Africa and worldwide (Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne, 2006; McCloskey, 2014; Sztajn, 2003). While most reform efforts have generally aimed at shifting mathematics instruction to be more explorative, the evidence shows that teachers often remain at the ritual level, either sticking mainly to the instruction of procedures or assimilating the form of the new curriculum without its function or content.

Such a wide view of the successes (and failures) of attempting to improve mathematics instruction may lead one to give up any hope of change. However, we do not share this pessimism. Rather, we rely on Giddens' (1984) idea that "in many contexts of social life there occur processes of selective 'information filtering' whereby strategically placed actors seek reflexively to regulate the overall conditions of system reproduction either to keep things as they are or to change them" (p. 27–28). We believe a project such as that led by the second author can be thought of as such a "strategically placed actor." Being set by an academic institution, it is informed by updated research in mathematics education, which forms the necessary "selective information filtering." Nevertheless, it is also very much a part of the life of learners, being based in after-school clubs at a center where children spend their afternoon hours. In addition, it aims at providing professional development for teachers in schools, thus "reflexively regulating the overall conditions of system reproduction." Yet, as our analysis has shown, for such change to occur, one must not only aim at changing teaching practices, but also take into account the histories of learning that learners bring to the classroom.

Appendix

Table 4 Summary of mathematical proficiency assessment for Mina and Ronaldo at the beginning of the course (February 2012) and at the end of the course (November 2012)

Task no.	Task number and description	No. of subparts in task	Mina February	Mina November	Ronaldo February	Ronaldo November
1	Numeral identification	10	8	7	7	10
2	Number line representation	2	1	2	0	2
3	Forward number word sequences	3	2	3	2	3
4	Backward number word sequences	3	1	2	3	3
5	Number word before	6	4	6	6	6
6	Number word after	6	5	6	6	6
7	Sequencing numerals	2	1	1	2	2
8	Perceptual counting	2	2	2	2	2
9	Counting in incrementing 10s	5	5	3	4	5
10	Addition/subtraction with 10s	4	0	3	3	4
11	Addition/subtraction with 100s	4	0	2	1	4
12	Horizontal +/− sentences	4	2	3	3	3
13	Word problems	3	0	1	1	2
14	Number stories	1	0	1	0	0
15	Noncounting by 1s	6	3	6	5	6
16	Number combinations	6	5	6	6	6
17	Visible items in an array (subitising)	3	3	3	3	3
18	Visible items in an array (subitising)	2	2	1	2	2
19	Visible items in an array (subitising)	2	2	1	2	2
20	Equal grouping of visible items	3	3	3	3	3
21	Equal grouping of visible items	2	3	3	3	3
22	Equal grouping of visible items	3	4	3	3	3
23	Times tables	4	0	2	3	4
24	½ and ¼ of a collection	2	0	1	2	1
	Number of Wright et al. questions	63				
	Number of Askew et al. questions	25				
	Total number of questions	86	56=65 %	71=83 %	72=84 %	85=99 %

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